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# Time decay of the magnetic properties of type-II superconductors in the presence of large pinning centers

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## Abstract

We present a study of the time decay of magnetic states in type-II superconductors. The mean escape time of flux quanta from the pinning centers is calculated by considering the well-known washboard potential and a pinning potential appropriate to the case of pinning center dimensions  $l$  much larger than the coherence length  $\xi$ . We find that, in both cases, the attempt frequency in the Arrhenius formula depends on the current density  $J$ . Finally, by plotting the  $E(J)$  curves, we show that the dissipation due to flux creep mechanisms goes to zero much faster in the second case, where  $l \gg \xi$  is assumed. © 1997 Elsevier Science B.V.

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## 1. Introduction

During the last few years the study of vortex dynamics of the high- $T_c$  superconductors has been a matter of extensive theoretical and experimental investigation. Indeed, several theoretical models have been proposed [1–4], where a non-linear increase of the creep barrier with decreasing current density is present. Moreover, from magnetic relaxation measurements, Maley et al. [5] report a sharp increase of the effective activation energy  $U$  as the current density  $J$  decreases. Analogous results have been found by Zeldov et al. [6] from transport measurements. These experimental results can be analyzed within the collective pinning theory [7]. In this particular model, though, the effective barrier height grows indefinitely as  $J$  goes to zero. Other models [8] predict the same diverging behavior for vanishingly small current den-

sities. In order to analyze the flux creep problem in detail, in the present work we study the mean escape time from the pinning centers of a single flux quantum, by starting from the general expression of the Arrhenius formula as derived from the classical theory of stochastic processes. In Section 2 we therefore start by studying the problem of the mean escape time from a washboard potential well, while in Section 3 we introduce a pinning potential  $U_j^{(H)}$  appropriate to the case of pinning center dimensions  $l$  much greater than the coherence length  $\xi$ . The choice of the potential  $U_j^{(H)}$  is justified by means of a phenomenological approach. We find that, while the attempt frequency  $\nu(J)$  goes to a finite value  $\nu_0$  as  $J$  goes to zero in the case of the washboard potential, for  $l \gg \xi$ ,  $\nu(J)$  goes to zero for vanishingly small current density values. In Section 4 the resulting curves of the electric field  $E$  due to creep mechanisms are plotted in terms of the

current density  $J$  for both types of potential. Conclusions are drawn in the last section.

## 2. The washboard potential

The problem of the current dependence of the mean escape time from a potential well of a single flux quantum can be tackled by adopting the same assumptions as in the usual derivation of the Arrhenius formula. Indeed, following Gardiner [9], the attempt frequency can analytically be expressed as the geometric mean of the curvatures of the potential well at the local minimum  $x_{\min}$  and at the local maximum  $x_{\max}$  of the potential as follows,

$$\nu(J) = \sqrt{U''_J(x_{\min})U''_J(x_{\max})}/2\pi\beta, \quad (1)$$

where  $\beta$  is the damping constant. Therefore, by starting from the following shape of the usual washboard potential, shown in Fig. 1,

$$U_J(x/\lambda)/U_0 = 1 - \cos(x/\lambda) - (J/J_0)x/\lambda, \quad (2)$$

where  $\lambda$  is some characteristic length, we find that the Arrhenius formula may be explicitly expressed in terms of the current density  $J$  in the following simple way,

$$1/\tau = \nu(J) \exp(-\Delta U_J/k_B T), \quad (3)$$

where  $k_B$  is the Boltzmann constant,  $\Delta U_J$  is the potential barrier height, and  $\nu = \nu(J)$  is the attempt frequency.

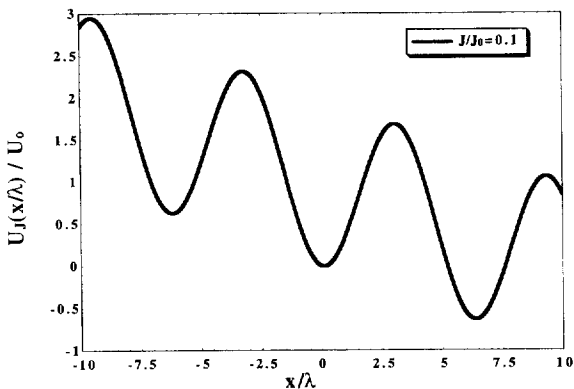


Fig. 1. The shape of the washboard potential  $U_J(x/\lambda)/U_0$  for the current density ratio value  $J/J_0 = 0.1$ .

By expressing the curvature of the potential energy at adjacent extrema as a function of  $J$ , we find

$$\nu(\gamma) = U_0 \sqrt{1 - \gamma^2}/2\pi\lambda^2\beta, \quad (4)$$

where  $\gamma = J/J_0$ . In Fig. 2 we report, for  $\beta = 1$ , the current density dependence of the attempt frequency  $\nu$  having a finite value for  $J \rightarrow 0$ . In the inset we show the current dependence of the resulting potential barrier height  $\Delta U_J$ , where

$$\Delta U_J = U_J(x_{\max}) - U_J(x_{\min}) = U_0 \left[ 2\sqrt{1 - \gamma^2} + 2\gamma \arcsin(\gamma) - \pi\gamma \right], \quad (5)$$

for the value of the ratio  $U_0/k_B T = 10$ . In this figure the horizontal line  $\Delta U_J = k_B T$  is traced. This line marks the lower limit of the range of validity of Eq. (3), since the Arrhenius formula is derived under the assumption of  $k_B T \ll \Delta U_J$ . Therefore, care must be taken in applying these results for  $J$  close to  $J_0$ .

## 3. A different pinning potential

In extreme type-II superconductors the coherence length may be small when compared to the characteristic pinning site dimension  $l$  as, for example, in the case of high- $T_c$  superconductors. Therefore, one needs to modify the traditional way of looking at the depinning mechanisms, which appear when a current  $J$  is applied in the direction orthogonal to the magnetic field  $H$ . Let us here consider the pinning barrier associated with a single vortex, which can be depinning from a single pinning site. The interaction potential between a vortex and a pinning site can be taken to be proportional to the Ginzberg–Landau superconducting order parameter [10]  $|\psi| \approx |\psi_\infty| \tanh(ax/\xi)$ , where  $\xi$  is the coherence length, and  $a$  is a constant of the order of unity. Therefore, the pinning potential  $U_J^{(H)}$  can be taken in the form

$$U_J^{(H)}(x)/U_0 = \tanh[(x - l)/\xi] - \tanh[(x + l)/\xi] - \epsilon_0(J/J_0)x/\xi, \quad (6)$$

where  $2l$  is the finite size of the pinning center, and  $\epsilon_0 = \Phi_0 J_0 L/U_0$ ,  $\Phi_0$  being the elementary flux quantum, and  $L$  the vortex line length. The linear term in Eq. (6) is introduced to take account of the Mag-

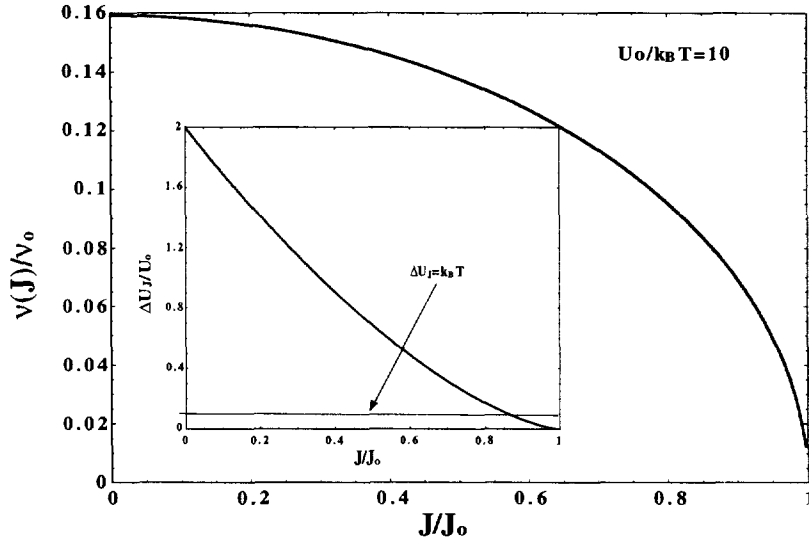


Fig. 2. The characteristic frequency  $\nu(J)/\nu_0$  curve as a function of the current density  $J/J_0$ . In the inset  $\Delta U_J/U_0$  versus  $J/J_0$  is shown for a value of the ratio  $U_0/k_B T = 10$ .

nus force on the flux lines due to the current density  $J$ . In Fig. 3 we show the potential energy shape for  $J/J_0 = 0.1$  using two different values of the ratio  $l/\xi = 4, 10$ . We numerically evaluate the extrema of the potential curve  $U_J^{(H)}(x)$  and the potential barrier  $\Delta U_J^{(H)} = U_J^{(H)}(x_{\max}) - U_J^{(H)}(x_{\min})$ . In this way, the generalized attempt frequency  $\nu(J)$  can be obtained by Eq. (1).

In Fig. 4 and in the inset we report the  $\nu$  versus  $J$  dependence for the values of the ratio  $l/\xi = 4, 10$  and the current dependence of  $\Delta U_J^{(H)}$  for the same

values of the ratio  $l/\xi$ , respectively. We find that, for pinning centers larger than the coherence length  $\xi$ , the characteristic frequency  $\nu$  goes to zero for vanishingly small current density values.

#### 4. Comparison between the two types of potentials

The analysis of non-linear characteristics,  $E$ - $J$  curves, has been shown to be a powerful tool for the study of the dissipation properties of high- $T_c$  superconductors. For this reason, in this section, the analysis of the  $E$ - $J$  curves will be reported. Having analytically and numerically derived the generalized attempt frequency  $\nu(J)$  in the Arrhenius formula, for the cases of the washboard potential  $U_J$  and of the pinning potential  $U_J^{(H)}$ , respectively, we will now discuss the transport properties due to flux creep mechanisms in these materials. The electric field  $E_{\text{creep}}$  can be written as follows,

$$E_{\text{creep}} = E_0(\nu(J)/\nu_0) \exp(-\Delta W/k_B T), \quad (7)$$

where  $E_0 = 2a_0 H \nu_0$ ,  $a_0$  being the distance between two pinning centers, and  $\Delta W$  is the energy barrier height.

Let us first calculate  $E_{\text{creep}}$  for the washboard potential case. Using Eq. (4) for  $\nu(J)$  and Eq. (5) for  $\Delta W$ ,

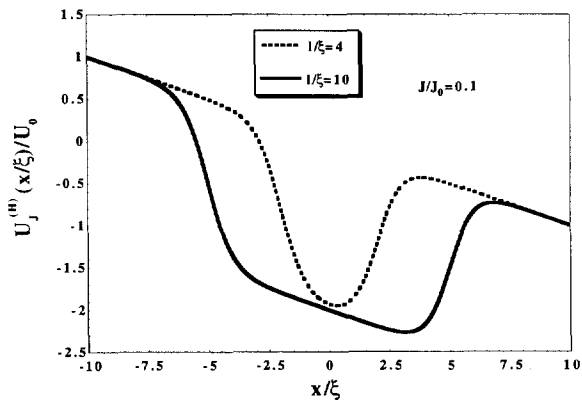


Fig. 3. The shapes of the pinning energy  $U_J^{(H)}(x)/U_0$  for  $l/\xi = 4, 10$  and  $J/J_0 = 0.1$ .

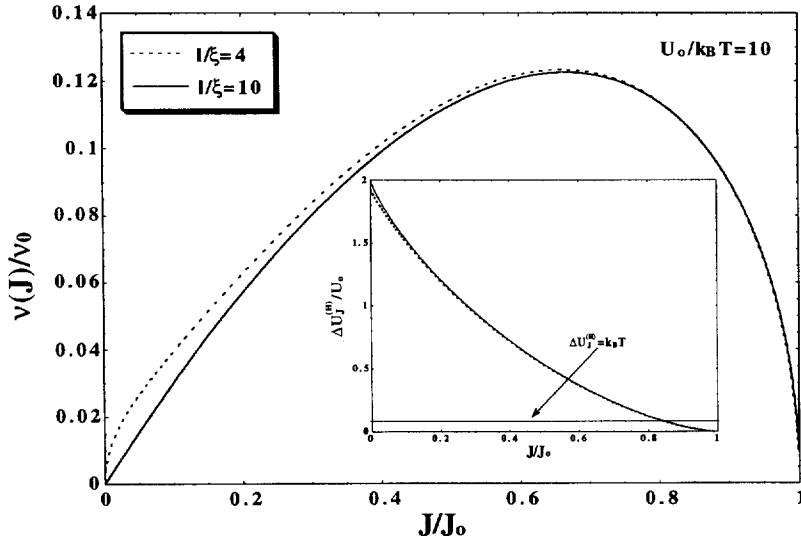


Fig. 4. The characteristic frequency  $\nu(J)/\nu_0$  curve as a function of the current density  $J/J_0$  for  $l/\xi = 4, 10$ . In the inset  $\Delta U_J^{(H)}/U_0$  versus  $J/J_0$  is shown for the value of the ratio  $U_0/k_B T = 10$ .

we obtain an analytic expression for the electric field  $E_{creep}$ . In Fig. 5 we show the  $E_{creep}$  versus  $J$  dependence, and compare it with the corresponding quantity calculated without taking into account the current density dependence of the frequency  $\nu$ . Here the ratio  $U_0/k_B T$  has been fixed to 10. In Fig. 5 we notice only a slight difference between the two curves.

The  $E_{creep}$  versus  $J$  curves can be calculated, by the same type of procedure as adopted in the case of the washboard potential, for superconductors having large pinning center sizes. This time, however, only a

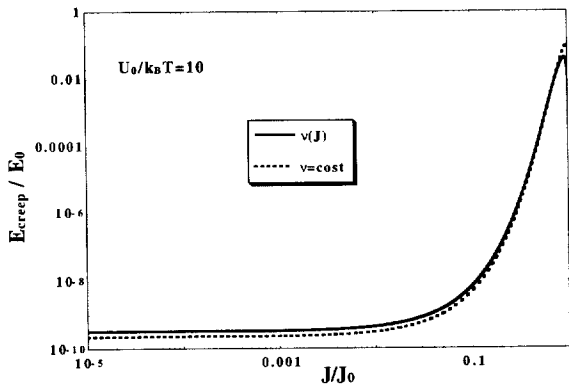


Fig. 5. The electric field  $E_{creep}/E_0$  curves due to creep mechanisms as a function of the current density  $J/J_0$  are shown for a value of the ratio  $U_0/k_B T = 10$  with  $\nu = \nu(J)$  and  $\nu = \cos t$ .

numerical evaluation of the curves can be presented, as explained in Section 3. In Fig. 6 the  $E_{creep}$  versus  $J$  curves are shown for  $\nu = \cos t$  and for  $\nu = \nu(J)$ , for two values of the ratio  $l/\xi$ . This figure shows that for  $l/\xi \gg 1$  and  $J \rightarrow 0$ , much smaller values of  $E_{creep}$  are obtained for  $\nu = \nu(J)$  than for  $\nu = \cos t$ .

The introduction of two different potentials allows us to compare the resulting curves of the electric field  $E_{creep}/E_0$  in terms of the current density  $J/J_0$  in the two cases discussed in the two previous sections. These have been plotted in Fig. 7, where  $\nu = \nu(J)$ ,  $U_0/k_B T = 10$  and, in the case of the potential choice  $U_J^{(H)}$ ,  $l/\xi = 4, 10$ . Finally, we obtain, for  $l/\xi \gg 1$  and vanishingly small current densities, that the dissipation phenomena due to flux mechanisms goes to zero much faster by considering the potential energy  $U_J^{(H)}$ .

### 5. Conclusions

We have analyzed the problem of the time decay of the magnetic properties of type-II superconductors for a modified Arrhenius formula in which the explicit current density  $J$  dependence of the attempt frequency  $\nu$  is taken into account. In particular, the  $\nu$  versus  $J$  dependence has been analytically calculated in the case of the washboard potential and has been numerically

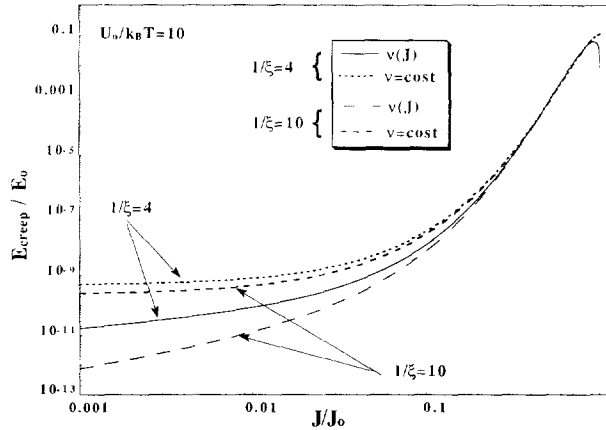


Fig. 6. The electric field  $E_{\text{creep}}/E_0$  curves due to creep mechanisms as a function of the current density  $J/J_0$  are shown for a value of the ratio  $U_0/k_B T = 10$  and  $l/\xi = 4, 10$  with  $\nu = \nu(J)$  and  $\nu = \text{cost}$ .

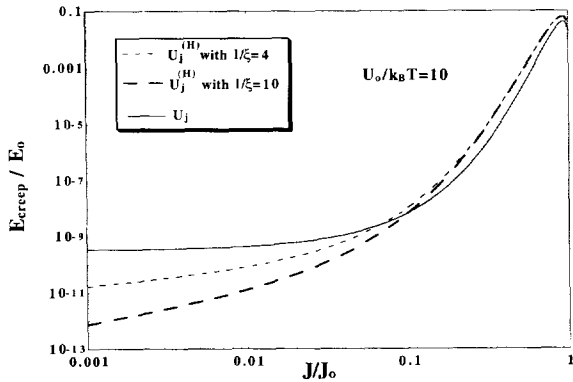


Fig. 7. The electric field  $E_{\text{creep}}/E_0$  curves due to creep mechanisms as a function of the current density  $J/J_0$  are shown for the value of the ratio  $U_0/k_B T = 10$  with  $\nu(J)$ . The solid line refers to the washboard potential, the two dashed lines to the potential  $U_j^{(H)}$  for  $l/\xi = 4, 10$ .

evaluated in the case of a potential  $U_j^{(H)}$  appropriate for extreme type-II superconductors in which the average pinning center size  $l$  is large with respect to the coherence length  $\xi$ .

The  $E$ - $J$  characteristics have been derived for both types of potentials, and it has been shown that the dissipation due to the creep mechanisms for the washboard potential does not very strongly depend on the  $J$  dependence of the prefactor of the Arrhenius formula

$\nu$ . In the case of  $l/\xi \gg 1$ , instead, the  $\nu$  versus  $J$  dependence significantly affects the dissipation phenomena due to flux creep mechanisms. Finally, we notice that, for decreasing values of  $J$ ,  $E_{\text{creep}}$  calculated for the potential  $U_j^{(H)}$  goes to zero much faster than in the case of the washboard potential.

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