Neutrino Interaction Physics

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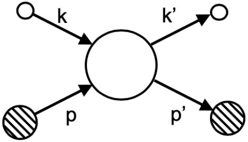
Overview

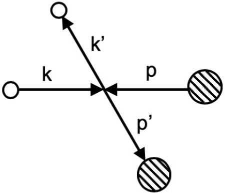
Intro - Lab frame and CMS frame kinematics

$$
\begin{aligned}
k &= k^\mu = (E, \boldsymbol{k}) \\
k' &= k'^\mu = (E', \boldsymbol{k'}) \\
p &= p^\mu = (E_p, \boldsymbol{p}) \\
p' &= p'^\mu = (E'_p, \boldsymbol{p'})
\end{aligned}
$$



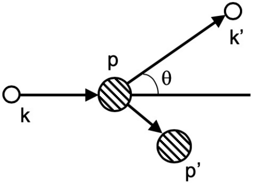
CMS frame

$$
\begin{aligned}
k &= k^\mu = (E, 0, 0, p_i) \\
k' &= k'^\mu = (E', \boldsymbol{p_f}) \\
p &= p^\mu = (E_p, 0, 0, -p_i) \\
p' &= p'^\mu = (E'_p, -\boldsymbol{p_f})
\end{aligned}
$$



Lab frame (neutrino)

$$
\begin{aligned}
k &= k^\mu = (E, 0, 0, E) \\
k' &= k'^\mu = (E', \boldsymbol{k'}) \\
p &= p^\mu = (M, 0, 0, 0) \\
p' &= p'^\mu = (E'_p, \boldsymbol{p'})
\end{aligned}
$$



Intro - Mandelstam variables

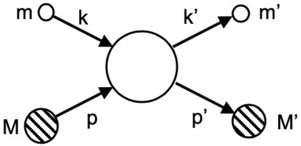
$$
\begin{aligned}
s &= (p+k)^2 = m^2 + M^2 + 2p\cdot k = (p'+k')^2 = m'^2 + M'^2 + 2p'\cdot k' \\
u &= (p'-k)^2 = m^2 + M'^2 - 2p'\cdot k = (k'-p)^2 = m'^2 + M^2 - 2k'\cdot p \\
t &= (k-k')^2 = m^2 + m'^2 - 2k\cdot k' = (p'-p)^2 = M^2 + M'^2 - 2p\cdot p' \\
s+t+u &= m^2 + m'^2 + M^2 + M'^2
\end{aligned}
$$

Lab frame (ν_μ-e elastic)

$$
\begin{aligned}
s &= M^2 + 2p\cdot k = M^2 + 2ME \\
u &= M^2 - 2p'\cdot k = M^2 - 2ME' \\
t &= -2k\cdot k' = 2M(E'-E) \\
s+t+u &= 2M^2
\end{aligned}
$$

Lab frame (CCQE)

$$
\begin{aligned}
s &= M^2 + 2p\cdot k = M^2 + 2ME \\
u &= M^2 - 2p'\cdot k = m^2 + M^2 - 2M \\
t &= m^2 - 2k\cdot k' = 2M(E'-E) \\
s+t+u &= m^2 + 2M^2
\end{aligned}
$$



Intro - CMS frame cross-section formula

$$d\sigma = \frac{|\mathcal{M}|^2}{F} \cdot dLips = \frac{|\mathcal{M}|^2}{4p_i\sqrt{s}} \cdot dLips$$

$$\begin{aligned} dLips &= (2\pi)^4\delta^4(k' + p' - k - p)\frac{d^3\boldsymbol{k'}}{(2\pi)^3 2E'}\frac{d^3\boldsymbol{p'}}{(2\pi)^3 2E'_p} \\ &= \frac{1}{4\pi^2}\frac{d^3\boldsymbol{k'}}{4E'E'_p}\delta^4(E' + E'_p - E - E_p) \end{aligned}$$

In CMS frame, $s = (k+p)^2 = (E+E_p)^2 - (\boldsymbol{k}+\boldsymbol{p})^2 = (E+E_p)^2 = W^2 = (E'+E'_p)^2$, and $|\boldsymbol{k}| = |\boldsymbol{p}| = p_i$, $|\boldsymbol{k'}| = |\boldsymbol{p'}| = p_f$. Thus, $dW = \frac{p_f}{E'}dp_f + \frac{p_f}{E'_p}dp_f$. Using spherical intergral, $\int d^3\boldsymbol{k'} \sim \int |\boldsymbol{k'}|^2 d|\boldsymbol{k'}| d\Omega$,

$$\begin{aligned} dLips &= \frac{p_f^2 dp_f d\Omega}{16\pi^2 E'E'_p}\delta(W - E' - E'_p) \\ &= \frac{p_f^2 d\Omega}{16\pi^2 E'E'_p}\frac{E'E'_p}{p_f(E'+E'_p)}dW\delta(W - E' - E'_p) = \frac{p_f}{16\pi^2\sqrt{s}}d\Omega \end{aligned}$$

Thus, you can derive CMS frame cross section formula.

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\frac{p_f}{p_i}|\mathcal{M}|^2$$

In the fixed target experiments (like neutrino experiments), we don't measure cross-sections in CMS frame so we don't use this formula.

Intro - Lab frame cross-section formula

$$d\sigma = \frac{|\mathcal{M}|^2}{F} \cdot dLips = \frac{|\mathcal{M}|^2}{4EM} \cdot dLips$$

For k', use spherical integral and integrate over angles $d^3\boldsymbol{k'} = |\boldsymbol{k'}|^2 d|\boldsymbol{k'}| d\phi dcos\theta = 2\pi|\boldsymbol{k'}|E' dE' dcos\theta$, and for p' you take derivative to make $\frac{d^3\boldsymbol{p'}}{2E'_p} = d^4p'\delta(p'^2)\theta(E'_p)$ and take integral with d^4. Also $dQ^2 = -2E|\boldsymbol{k'}|dcos\theta$.

$$\begin{aligned}
dLips &= (2\pi)^4\delta^4(k' + p' - k - p)\frac{d^3\boldsymbol{k'}}{(2\pi)^3 2E'}\frac{d^3\boldsymbol{p'}}{(2\pi)^3 2E'_p} \\
&= \frac{d^3\boldsymbol{k'}}{8\pi^2 E'} d^4p'\theta(E'_p)\delta(p'^2)\delta^4(k' + p' - k - p) \\
&= \frac{|\boldsymbol{k'}|^2 d|\boldsymbol{k'}|}{4\pi E'} dcos\theta\delta((k + p - k')^2) = \frac{|\boldsymbol{k'}|dE'}{4\pi}\frac{dQ^2}{2E|\boldsymbol{k'}|}\delta((\cdots) - 2ME') \\
&= \frac{dQ^2}{8\pi E}\frac{1}{2M}\delta((\cdots) - E') = \frac{dQ^2}{16\pi ME}
\end{aligned}$$

Thus, you can derive CMS frame cross section formula.

$$\frac{d\sigma}{dQ^2} = \frac{1}{64\pi M^2 E^2}|\mathcal{M}|^2$$

We use this formula to derive neutrino cross-sections.

Intro - Nucleon form factor

Homework 1
Assuming the charge distribution of a nucleon is spherically symmetric, $\rho(\boldsymbol{x}) = \rho_0 e^{-Mr}$, taking the Fourier transformation of this derive the form factor ($|\boldsymbol{q}| \equiv q$),

$$F(q^2) = \int \rho_0 e^{-Mr} e^{i\boldsymbol{q}\cdot\boldsymbol{x}} d^3x \equiv \frac{F(0)}{\left(1 + \frac{q^2}{M^2}\right)^2}$$

Intro - Totally anti-symmetric tensor

dimention 3, Levi-Civita symbol

$$
\begin{aligned}
\epsilon_{ijk} &= \epsilon_{jki} = \epsilon_{kij} \text{ (cyclic rule)} \\
\epsilon_{ijk} &= -\epsilon_{jik} \\
\epsilon_{ijk}\epsilon_{abc} &= \begin{vmatrix} \delta_{ia} & \delta_{ib} & \delta_{ic} \\ \delta_{ja} & \delta_{jb} & \delta_{jc} \\ \delta_{ka} & \delta_{kb} & \delta_{kc} \end{vmatrix} \\
\epsilon_{ijk}\epsilon_{ibc} &= \delta_{jb}\delta_{kc} - \delta_{jc}\delta_{kb} \\
\epsilon_{ijk}\epsilon_{ijc} &= 2\delta_{kc} \\
\epsilon_{ijk}\epsilon_{ijk} &= 6(\delta_{ii} = \delta 11 + \delta_{22} + \delta_{33} = 3) \\
\vec{A} \times (\vec{B} \times \vec{C}) &= \epsilon_{abi}\epsilon_{ijk} A_b B_j C_k = (\delta_{aj}\delta_{bk} - \delta_{ak}\delta_{bj}) A_b B_j C_k = B_a A_b C_b - C_a A_b B_b \\
&= \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})
\end{aligned}
$$

dimention 4

$$
\begin{aligned}
\epsilon_{ijkl}\epsilon_{abcd} &= -\begin{vmatrix} \delta_{ia} & \delta_{ib} & \delta_{ic} & \delta_{id} \\ \delta_{ja} & \delta_{jb} & \delta_{jc} & \delta_{jd} \\ \delta_{ka} & \delta_{kb} & \delta_{kc} & \delta_{kd} \\ \delta_{la} & \delta_{lb} & \delta_{lc} & \delta_{ld} \end{vmatrix} \\
\epsilon_{ijab}\epsilon_{ijcd} &= -2(\delta_{ac}\delta_{bd} - \delta_{ad}\delta_{bc})
\end{aligned}
$$

Intro - Gamma and trace algebra

$$g_{\mu\nu}g^{\mu\nu} = 4,\ \gamma_5\gamma_5 = I_4,\ \{\gamma_5, \gamma^\mu\} = 0,\ \sigma_{\mu\nu} = \frac{1}{2}\{\gamma_\mu, \gamma_\nu\}$$
$$\gamma^{0\dagger} = \gamma^0,\ \gamma^0\gamma^0 = I_4,\ \gamma_5^\dagger = \gamma_5,\ \gamma_5\gamma_5 = I_4$$
$$\gamma^0\gamma^{\mu\dagger}\gamma^0 = \gamma^\mu,\ \gamma^0\gamma_5\gamma^0 = -\gamma_5$$

$$\begin{aligned}
tr[I_4] &= 4 \\
tr[ABC] &= tr[BCA]\text{(cyclic rule)} \\
tr[\text{odd number of }\gamma s] &= 0 \\
tr[\gamma_\mu\gamma_\nu] &= 4g_{\mu\nu} \\
tr[\not{k}\not{k}'] &= 4k\cdot k' \\
tr[\gamma_\mu\gamma_\nu\gamma_\alpha\gamma_\beta] &= 4[g_{\mu\nu}g_{\alpha\beta} - g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha}] \\
tr[\gamma_\mu\not{k}\gamma_\nu\not{k}'] &= 4[k_\mu k'_\nu + k_\nu k'_\mu - (k\cdot k')g_{\mu\nu}] \\
tr[\gamma_5] &= 0 \\
tr[\gamma_5\gamma_\mu\gamma_\nu] &= 0 \\
tr[\gamma_5\gamma_\mu\gamma_\nu\gamma_\alpha\gamma_\beta] &= 4i\epsilon_{\mu\nu\alpha\beta} \\
tr[\gamma_5\gamma_\mu\not{k}\gamma_\nu\not{k}'] &= 4i\epsilon_{\mu\alpha\nu\beta}k^\alpha k'^\beta
\end{aligned}$$

Intro - Integral

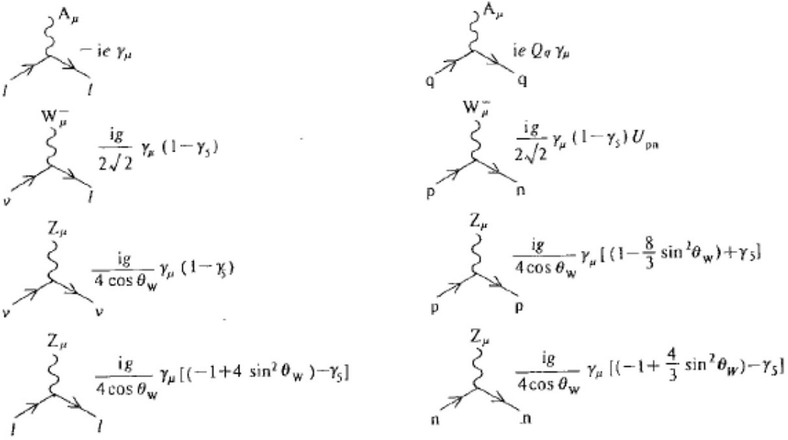
3-dimensional integral can be done either in cartesian corrdinate or spherical coordinate.

$$\int_{-infty}^{\infty}\int_{-infty}^{\infty}\int_{-infty}^{\infty} d^3p = \int_{0}^{\infty} dp \int_{-1}^{1} dcos\theta \int_{0}^{2\pi} d\phi$$

3-dimensional integral can be expanded to 4-dimensional integral, using $\delta(E_P^2 - |\vec{k}|^2) = \frac{1}{2E_p}[\delta(E_p - |\vec{k}|) + \delta(E_p + |\vec{k}|)]$

$$\int \frac{d^3p}{2E_p} = \int d^4p\Theta(E_p)\delta(p^2)$$

Intro - Feynman diagrams for weak interactions



$\nu_\mu - e$ - Cross-section $\nu_\mu + e \to \nu_\mu + e$

Define 4 spinors, $\nu(k)$, $\nu(k')$, $e(p)$, $e(p')$, for muon neutrino and electron. Then the neutrino current is,

$$\frac{g}{2cos\theta_w}\bar{\nu}(k')\frac{1}{2}\gamma_\mu(1-\gamma_5)\nu(k)$$

the electron current is,

$$\frac{g}{2cos\theta_w}\bar{e}(p')\gamma_\mu(g_V - g_A\gamma_5)e(p)$$

where $g_V = -\frac{1}{2} + 2sin^2\theta_w$ and $g_A = -\frac{1}{2}$, and the propagator is $\sim \frac{1}{Q^2+M_Z^2} \sim \frac{1}{M_Z^2}$.

Then put all together,

$$\begin{aligned} M &= \frac{g^2}{8cos^2\theta_w M_Z^2}[\bar{\nu}(k')\gamma_\mu(1-\gamma_5)\nu(k)][\bar{e}(p')\gamma^\mu(g_V - g_A\gamma_5)e(p)] \\ &= \frac{G_F}{\sqrt{2}}J_\mu j^\mu \end{aligned}$$

Here, $cos^2\theta_w M_Z^2 = M_W^2$ and $\frac{g^2}{8M_W^2} \equiv \frac{G_F}{\sqrt{2}}$.

Then, square of the matrix element is

$$|\mathcal{M}|^2 = \left(\frac{G_F}{\sqrt{2}}J^\mu j_\mu\right)\left(\frac{G_F}{\sqrt{2}}j_\nu^\dagger J^{\nu\dagger}\right) = \frac{G_F^2}{2}\left(J^\mu J^{\nu\dagger}\right)\left(j_\mu j_\nu^\dagger\right) \equiv \frac{G_F^2}{2}L^{\mu\nu}W_{\mu\nu}$$

Thus, the spin average of matrix element square is,

$$\overline{|\mathcal{M}|}^2 = \frac{G_F^2}{2}\frac{1}{2}\sum_{spin}L^{\mu\nu}l_{\mu\nu} = \frac{G_F^2}{4}\overline{L}^{\mu\nu}\overline{l}_{\mu\nu}$$

Homework 2
Show,

$$\begin{aligned}
\overline{L}_{\mu\nu} &= tr[\bar{\nu}(k')\gamma_\mu(1-\gamma_5)\nu(k)][\bar{\nu}(k)\gamma_\nu(1-\gamma_5)\nu(k')] \\
&= 8[k_\mu k'_\nu + k'_\mu k_\nu - (k\cdot k')\gamma_{\mu\nu}] - 8i\epsilon_{\mu\nu\alpha\beta}k^\alpha k'^\beta \\
&\equiv L^S_{\mu\nu} + L^A_{\mu\nu}
\end{aligned}$$

The first 3-terms are symmetric in $\mu \leftrightarrow \nu$, but the last term is antisymmetric in $\mu \leftrightarrow \nu$. A symmetric term times an anti-symmetric term is 0. This is useful so we calculate these terms separately.

$\nu_\mu - e$ - Leptonic tensor of electron

$$
\begin{aligned}
\bar{l}_{\mu\nu} &= tr[\bar{l}(p')\gamma_\mu(g_V - g_A\gamma_5)l(p)][\bar{l}(p)\gamma_\nu(g_V - g_A\gamma_5)l(p')] \\
&= tr[\gamma_\mu(g_V - g_A\gamma_5)(\not{p} + M)\gamma_\nu(g_V - g_A\gamma_5)(\not{p}' + M)] \\
&= tr[g_V\gamma_\mu\not{p} - g_V M\gamma_\mu - g_A\gamma_\mu\gamma_5\not{p} - g_A M\gamma_\mu\gamma_5] \times \\
&\quad [g_V\gamma_\nu\not{p}' - g_V M\gamma_\nu - g_A\gamma_\nu\gamma_5\not{p}' - g_A M\gamma_\nu\gamma_5] \\
&= (g_V^2 + g_A^2)tr[\gamma_\mu\not{p}\gamma_\nu\not{p}'] + M^2(g_V^2 - g_A^2)tr[\gamma_\mu\gamma_\nu] \\
&\quad +2g_V g_A tr[\gamma_5\gamma_\mu\not{p}\gamma_\nu\not{p}'] \\
&= 4(g_V^2 + g_A^2)[p_\mu p'_\nu + p'_\mu p_\nu - (p\cdot p')\gamma_{\mu\nu}] + 4M^2(g_V^2 - g_A^2)g_{\mu\nu} \\
&\quad -8i\epsilon_{\mu\nu\alpha\beta}p^\alpha p'^\beta \\
&\equiv l^S_{\mu\nu} + l^A_{\mu\nu}
\end{aligned}
$$

$\nu_\mu - e$ - Matrix element

Surviving terms are $L^S_{\mu\nu} l^{S\mu\nu}$ and $L^A_{\mu\nu} l^{A\mu\nu}$.

$$
\begin{aligned}
|\mathcal{M}|^2 &= \frac{G_F^2}{4} \cdot \Big[32[k_\mu k'_\nu + k'_\mu k_\nu - (k \cdot k') g_{\mu\nu}] \times \\
&\quad \{(g_V^2 + g_A^2)[p^\mu p'^\nu + p'^\mu p^\nu - (p \cdot p')\gamma^{\mu\nu}] + M^2 (g_V^2 - g_A^2) g^{\mu\nu}\} \\
&\quad -64 g_V g_A \epsilon_{\mu\nu\alpha\beta} \epsilon^{\mu\nu\gamma\delta} k^\alpha k'^\beta p_\gamma p'_\delta \Big] \\
&= 8 G_F^2 \Big[(g_V^2 + g_A^2)[2(k \cdot p)(k' \cdot p') + 2(k \cdot p')(k' \cdot p)] + M^2 (g_V^2 - g_A^2)[-2(k \cdot k')] \\
&\quad -2 g_V g_A [-2(\delta^\gamma_\alpha \delta^\delta_\beta - \delta^\gamma_\beta \delta^\delta_\alpha) k^\alpha k'^\beta p_\gamma p'_\delta] \Big] \\
&= 16 G_F^2 \Big[(g_V^2 + g_A^2)[(k \cdot p)(k' \cdot p') + (k \cdot p')(k' \cdot p)] - M^2 (g_V^2 - g_A^2)(k \cdot k') \\
&\quad -2 g_V g_A [(k \cdot p)(k' \cdot p') - (k \cdot p')(k' \cdot p)] \Big]
\end{aligned}
$$

Using $c_L \equiv \frac{1}{2}(g_V + g_A), c_R \equiv \frac{1}{2}(g_V - g_A)$,

$$
\begin{aligned}
|\mathcal{M}|^2 &= 16 G_F^2 [(g_V^2 + g_A^2)(k \cdot p)(k' \cdot p') + (g_V^2 - g_A^2)(k \cdot p')(k' \cdot p) - M^2 (g_V + g_A)(g_V - g \\
&= 16 G_F^2 [c_L^2 (2k \cdot p)(2k' \cdot p') + c_R^2 (2k \cdot p')(2k' \cdot p) - 2M^2 c_L c_R (2k \cdot k')] \\
&= 16 G_F^2 [c_L^2 (s - M^2)^2 + c_R^2 (u - M^2)^2 + 2M^2 c_L c_R t]
\end{aligned}
$$

$\nu_\mu - e$ - cross section

The cross section formula

$$
\begin{aligned}
d\sigma &= \frac{|\mathcal{M}|^2}{F} dLips = \frac{|\mathcal{M}|^2}{4E_\nu M} dLips \\
&= \frac{|\mathcal{M}|^2}{4E_\nu M} \int \frac{d^3\boldsymbol{k'}}{(2\pi)^3 E'_\nu} \frac{d^3\boldsymbol{p'}}{(2\pi)^3 E'_e} (2\pi)^4 \delta^4(k+p-k'-p') \\
&= \frac{|\mathcal{M}|^2}{4E_\nu M} \int \frac{d^4k' \cdot 2\pi |\boldsymbol{p'}|^2 d|\boldsymbol{p'}|}{8\pi^2 E'_e} \Theta(E'_\nu)\delta(k'^2)\delta^4(k+p-k'-p') \\
&= \frac{|\mathcal{M}|^2}{4E_\nu M} \int \frac{|\boldsymbol{p'}|^2 d|\boldsymbol{p'}| dcos\theta}{4\pi E'_e} \Theta(E'_\nu)\delta((p+k-p')^2) \\
&= \frac{|\mathcal{M}|^2}{4E_\nu M} \int \frac{|\boldsymbol{p'}| E'_e dE'_e dcos\theta}{4\pi E'_e} \Theta(E'_\nu) \times \\
&\quad \delta((E_\nu + M - E'_e)^2 - (\boldsymbol{k} - \boldsymbol{p'})^2) \\
&= \frac{|\mathcal{M}|^2}{4E_\nu M} \int \frac{|\boldsymbol{p'}| E'_e dE'_e dcos\theta}{4\pi E'_e} \Theta(E'_\nu) \frac{1}{2E_\nu |\boldsymbol{p'}|} \delta(\cdots - cos\theta) \\
&= \frac{|\mathcal{M}|^2}{4E_\nu M} \frac{dE'_e}{8\pi E_\nu} = \frac{|\mathcal{M}|^2}{32\pi M E_\nu^2} dE'_e
\end{aligned}
$$

$\nu_\mu - e$ - cross section

$$
\begin{aligned}
\frac{d\sigma}{dE'_e} &= \frac{|\mathcal{M}|^2}{32\pi M E_\nu^2} \\
&= \frac{16 G_F^2}{32\pi M E_\nu^2}[c_L^2(s - M^2) + c_R^2(u - M^2) + 2M^2 c_L c_R t] \\
&= \frac{G_F^2}{2\pi M E_\nu^2}[c_L^2(2ME_\nu)^2 + c_R^2(2ME'_\nu)^2 + 2M^2 c_L c_R \cdot 2M(E'_\nu - E_\nu)] \\
&= \frac{2MG_F^2}{\pi}\left[c_L^2 + c_R^2\left(\frac{E'_\nu}{E_\nu}\right)^2 - c_L c_R M\left(1 - \frac{E'_\nu}{E_\nu}\right)\right] \\
&= \frac{2MG_F^2}{\pi}\left[c_L^2 + c_R^2(1-y)^2 - c_L c_R M\frac{My}{E_\nu}\right]
\end{aligned}
$$

Thus, the total cross-section is,

$$
\sigma = \int_0^1 \frac{d\sigma}{dE'_e} dy = \frac{2MG_F^2 E_\nu}{\pi}\left[c_L^2 + \frac{1}{3}c_R^2 - \frac{1}{2}c_L c_R \frac{M}{E_\nu}\right] \propto E_\nu
$$

Total cross-section of $\nu_\mu - e$ scattering is a linear function of E_ν, and the value is $\sigma(\nu_\mu - e) = 1.55 \times 10^{-42}(E_\nu/1GeV)cm^2$, about 1,0000 times smaller than CCQE cross section at 1 GeV.

CCQE - Kinematic variables 1

$$
\begin{aligned}
K &\equiv k + k' \\
P &\equiv p + p' \\
q &\equiv k - k' = p' - p \\
Q^2 &\equiv -q^2 \\
\tau &\equiv \frac{Q^2}{4M^2} \\
P^2 &= (p + p')^2 = 2M^2 + 2p \cdot p' = 4M^2 - q^2 \\
2p \cdot p' &= 2M^2 - q^2 \\
K^2 &= (k + k')^2 = m^2 + 2k \cdot k' = 2m^2 - q^2 \\
2k \cdot k' &= m^2 - q^2 \\
P \cdot K &= (p + p')(k + k') = pk + p'k' + pk' + p'k \\
&= \frac{1}{2}\left[(s - M^2) + (S - M^2 - m^2) + (m^2 + M^2 - u) + (M^2 - u)\right] \\
&= s - u = 2M(E + E') - m^2 = 4ME + q^2 - m^2 \\
P \cdot q &= (p + p')(p' - p) = 0 \\
K \cdot q &= (k + k')(k - k') = -m^2
\end{aligned}
$$

CCQE - Kinematic variables 2

$$
\begin{aligned}
P_\mu P_\nu &= p_\mu p_\nu + p_\mu p'_\nu + p'_\mu p_\nu + p'_\mu p'_\nu \\
q_\mu q_\nu &= p'_\mu p'_\nu - p'_\mu p_\nu - p_\mu p'_\nu + p_\mu p_\nu \\
\frac{1}{2}(P_\mu P_\nu - q_\mu q_\nu) &= p_\mu p'_\nu + p'_\mu p_\nu
\end{aligned}
$$

CCQE - Reconstructed neutrino energy and Q^2

Homework 3

Assuming the CCQE kinematics and the CCQE interaction ($\nu + n \rightarrow l + p$), derive,

1. $E = \frac{ME' - 0.5m^2}{M - E' + \sqrt{E'^2 - m^2} cos\theta}$
2. $Q^2 = -m^2 + 2EE' - 2E\sqrt{E'^2 - m^2} cos\theta$

This implies you can calculate the initial neutrino energy E and Q^2 from measured outgoing lepton energy E' and the scattering angle θ.

CCQE - cross-section $\nu + n \to l + p$

Define 4 spinors, $\nu(k)$, $l(k')$, $n(p)$, $p(p')$, for neutrino, charged lepton, neutron, and proton. Then the leptonic current is,

$$J_\mu^{(l)} = \bar{l}(k')\gamma_\mu(1-\gamma_5)\nu(k)$$

and the hadronic current is,

$$J_\mu^{(h)} = \bar{p}(p')\left[F_1\gamma_\mu + \frac{1}{2M}F_2 i\sigma_{\mu\nu}q^\nu + F_A\gamma_\mu\gamma_5 + \frac{q_\mu}{M}F_P\gamma_5\right]n(p) \equiv \bar{p}(p')\Gamma_\mu n(p)$$

Use Gordon decomposition, $\bar{p}(p')\gamma_\mu n(p) = \bar{p}(p')\frac{1}{2M}\left[P_\mu + i\sigma_{\mu\nu}q^\nu\right]n(p)$, to erase $\sigma_{\mu\nu}$.

$$\begin{aligned} J_\mu^{(h)} &= \bar{p}(p')\left[(F_1+F_2)\gamma_\mu - \frac{F_2}{2M}P_\mu + F_A\gamma_\mu\gamma_5 + \frac{F_P}{M}q_\mu\gamma_5\right] \\ &\equiv \bar{p}(p')\left[a\gamma_\mu - bP_\mu + c\gamma_\mu\gamma_5 + dq_\mu\gamma_5\right] \end{aligned}$$

The matrix element is,

$$\begin{aligned} |\mathcal{M}|^2 &= \left(\frac{G_F}{\sqrt{2}}J^{\mu(l)}J_\mu^{(h)}\right)\left(\frac{G_F}{\sqrt{2}}J_\nu^{(h)\dagger}J^{\nu(l)\dagger}\right) = \frac{G_F^2}{2}\left(J^{\mu(l)}J^{\nu(l)\dagger}\right)\left(J_\mu^{(h)}J_\nu^{(h)\dagger}\right) \equiv \frac{G_F^2}{2}L^{\mu\nu}W_{\mu\nu} \\ \overline{|\mathcal{M}|}^2 &= \frac{G_F^2}{2}\frac{1}{2}\sum_{spin}L^{\mu\nu}W_{\mu\nu} = \frac{G_F^2}{4}\overline{L}^{\mu\nu}\overline{W}_{\mu\nu} \end{aligned}$$

CCQE - Leptonic tensor

$$\begin{aligned}
\bar{L}_{\mu\nu} &= tr[\bar{l}(k')\gamma_\mu(1-\gamma_5)\nu(k)][(l(k')^\dagger\gamma_0)\gamma_\nu(1-\gamma_5)\nu(k)]^\dagger \\
&= tr[\bar{l}(k')\gamma_\mu(1-\gamma_5)\nu(k)][\nu(k)^\dagger\gamma_0\gamma_0(1-\gamma_5)^\dagger\gamma_\nu^\dagger\gamma_0^\dagger l(k')] \\
&= tr[\bar{l}(k')\gamma_\mu(1-\gamma_5)\nu(k)][\bar{\nu}(k)(1+\gamma_5)\gamma_\nu l(k')] \\
&= tr[\bar{l}(k')\gamma_\mu(1-\gamma_5)\nu(k)][\bar{\nu}(k)\gamma_\nu(1-\gamma_5)l(k')] \\
&= tr[\gamma_\mu(1-\gamma_5)\not{k}\gamma_\nu(1-\gamma_5)(\not{k}'+m)] \\
&= tr[\gamma_\mu\not{k}\gamma_\nu(\not{k}'+m)-\gamma_\mu\not{k}\gamma_\nu\gamma_5(\not{k}'+m)-\gamma_\mu\gamma_5\not{k}\gamma_\nu(\not{k}'+m)+\gamma_\mu\gamma_5\not{k}\gamma_\nu\gamma_5(\not{k}'+m)] \\
&= tr[\gamma_\mu\not{k}\gamma_\nu\not{k}'-\gamma_\mu\not{k}\gamma_\nu\gamma_5\not{k}'-\gamma_\mu\gamma_5\not{k}\gamma_\nu\not{k}'+\gamma_\mu\gamma_5\not{k}\gamma_\nu\gamma_5\not{k}'] \\
&= 2tr[\gamma_\mu\not{k}\gamma_\nu\not{k}']+2tr[\gamma_5\gamma_\mu\not{k}\gamma_\nu\not{k}'] \\
&= 8[k_\mu k'_\nu+k'_\mu k_\nu-(k\cdot k')\gamma_{\mu\nu}]+8i\epsilon_{\mu\alpha\nu\beta}k^\alpha k'^\beta \\
&= 8[k_\mu k'_\nu+k'_\mu k_\nu+\frac{1}{2}(q^2-m^2)\gamma_{\mu\nu}]-4i\epsilon_{\mu\nu\alpha\beta}(k^\alpha k'^\beta-k'^\alpha k^\beta) \\
&= 8\cdot\left[\frac{1}{2}[(k_\mu+k'_\mu)(k_\nu+k'_\nu)-(k_\mu-k'_\mu)(k_\nu-k'_\nu)]+\frac{1}{2}(q^2-m^2)\gamma_{\mu\nu}\right] \\
&\quad -4i\epsilon_{\mu\nu\alpha\beta}[(k^\alpha+k'^\alpha)(k'^\beta-k^\beta)+(k^\alpha k^\beta-k'^\alpha k'^\beta)] \\
&= 4[K_\mu K'_\nu-q_\mu q'_\nu+(q^2-m^2)g_{\mu\nu}+i\epsilon_{\mu\nu\alpha\beta}K^\alpha q^\beta] \\
&\equiv L^S_{\mu\nu}+L^A_{\mu\nu}
\end{aligned}$$

The first term is symmetric in $\mu \leftrightarrow \nu$, but later term is antisymmetric in $\mu \leftrightarrow \nu$. A symmetric term times an anti-symmetric term is 0. This is useful so we calculate these terms separately.

CCQE - Hadronic tensor

$$
\begin{aligned}
\overline{W}_{\mu\nu} &= tr[p(p')^\dagger \gamma_0 \Gamma_\mu n(p)][n(p)^\dagger \Gamma_\mu^\dagger \gamma_0 p(p')] \\
&= tr[\bar{p}(p')\Gamma_\mu n(p)][n(p)^\dagger \Gamma_\mu^\dagger \gamma_0 p(p')] \\
&= tr[\bar{p}(p')\Gamma_\mu n(p)][\bar{n}(p)\gamma_0[a\gamma_\nu - bP_\nu + c\gamma_\nu\gamma_5 + dq_\nu\gamma_5]^\dagger \gamma_0 p(p')] \\
&= tr[\bar{p}(p')\Gamma_\mu n(p)][\bar{n}(p)[a\gamma_0\gamma_\nu^\dagger\gamma_0 - bP_\nu + c\gamma_0\gamma_5^\dagger\gamma_\nu^\dagger\gamma_0 + dq_\nu\gamma_0\gamma_5^\dagger\gamma_0]p(p')] \\
&= tr[\bar{p}(p')\Gamma_\mu n(p)][\bar{n}(p)[a\gamma_\nu - bP_\nu - c\gamma_5\gamma_\nu - dq_\nu\gamma_5]p(p')] \\
&= tr[\bar{p}(p')\Gamma_\mu n(p)][\bar{n}(p)[a\gamma_\nu - bP_\nu + c\gamma_\nu\gamma_5 - dq_\nu\gamma_5]p(p')] \\
&\equiv tr[\bar{p}(p')\Gamma_\mu n(p)][\bar{n}(p)\Gamma'_\nu p(p')] \\
&= tr[\Gamma_\mu n(p)(\not{P} + M)\Gamma'_\nu(\not{p}' + M)] \\
&= tr[\Gamma_\mu n(p)\not{P}\Gamma'_\nu\not{p}'] + Mtr[\Gamma_\mu n(p)\not{P}\Gamma'_\nu] + Mtr[\Gamma_\mu n(p)\Gamma'_\nu\not{p}'] + M^2 tr[\Gamma_\mu n(p)\Gamma'_\nu] \\
&\equiv W^1_{\mu\nu} + W^2_{\mu\nu} + W^3_{\mu\nu} + W^4_{\mu\nu}
\end{aligned}
$$

Hadronic tensor calculation takes time and space, so we split it to 4 parts. Let's start from the easiest one.

CCQE - Hadronic tensor, $W^4_{\mu\nu}$

$$\begin{aligned}
W^4_{\mu\nu} &= tr[(a\gamma_\mu - bP_\mu + c\gamma_\mu\gamma_5 + dq_\mu\gamma_5)(a\gamma_\nu - bP_\nu + c\gamma_\nu\gamma_5 - dq_\nu\gamma_5)] \\
&= M^2 tr\left[a^2\gamma_\mu\gamma_\nu - ab\gamma_\mu P_\nu + ac\gamma_\mu\gamma_\nu\gamma_5 - ad\gamma_\mu q_\nu\gamma_5\right. \\
&\quad -abP_\mu\gamma_\nu + b^2P_\mu P_\nu - bcP_\mu\gamma_\nu\gamma_5 + bdP_\mu q_\nu\gamma_5 \\
&\quad +ac\gamma_\mu\gamma_5\gamma_\nu - bc\gamma_\mu\gamma_5 P_\nu + c^2\gamma_\mu\gamma_5\gamma_\nu\gamma_5 - cd\gamma_\mu\gamma_5 q_\nu\gamma_5 \\
&\quad \left.+adq_\mu\gamma_5\gamma_\nu - bdq_\mu\gamma_5 P_\nu + cdq_\mu\gamma_5\gamma_\nu\gamma_5 - d^2q_\mu\gamma_5 q_\nu\gamma_5\right]
\end{aligned}$$

It looks awful, but using gamma matrix trace rule, you can identify many of them are zero,

$$\begin{aligned}
W^4_{\mu\nu} &= M^2 tr\left[a^2\gamma_\mu\gamma_\nu + b^2P_\mu P_\nu + c^2\gamma_\mu\gamma_5\gamma_\nu\gamma_5 - d^2q_\mu\gamma_5 q_\nu\gamma_5\right] \\
&= M^2 tr\left[(a^2 - c^2)\gamma_\mu\gamma_\nu + b^2P_\mu P_\nu - d^2q_\mu q_\nu\right] \\
&= 4M^2[(a^2 - c^2)g_{\mu\nu} + b^2P_\mu P_\nu - d^2q_\mu q_\nu]
\end{aligned}$$

CCQE - Hadronic tensor, $W^2_{\mu\nu}$ and $W^3_{\mu\nu}$

These 2 tensors have certain symmetry so it's good to calculate together.

$$W^2_{\mu\nu} = Mtr[(a\gamma_\mu - bP_\mu + c\gamma_\mu\gamma_5 + dq_\mu\gamma_5)\not{p}(a\gamma_\nu - bP_\nu + c\gamma_\nu\gamma_5 - dq_\nu\gamma_5)]$$

This time, we skip all zero terms (odd number of gammas, etc). Since there is one more gamma compared with $W^4_{\mu\nu}$, basically, terms survive and terms go zero are flipped.

$$\begin{aligned} W^2_{\mu\nu} &= Mtr[-ab\gamma_\mu\not{p}P_\nu - abP_\mu\not{p}\gamma_\nu - cd\gamma_\mu\gamma_5\not{p}q_\nu\gamma_5 + cdq_\mu\gamma_5\not{p}\gamma_\nu\gamma_5] \\ &= 4M[-abg_{\mu\alpha}p^\alpha P_\nu - abg_{\nu\alpha}p^\alpha P_\mu + cdg_{\mu\alpha}p^\alpha q_\nu + cdg_{\nu\alpha}q_\mu p^\alpha] \\ &= 4M[-ab(p_\mu P_\nu + p_\nu P_\mu) + cd(p_\mu q_\nu + q_\mu p_\nu)] \end{aligned}$$

Similarly,

$$\begin{aligned} W^3_{\mu\nu} &= Mtr[(a\gamma_\mu - bP_\mu + c\gamma_\mu\gamma_5 + dq_\mu\gamma_5)(a\gamma_\nu - bP_\nu + c\gamma_\nu\gamma_5 - dq_\nu\gamma_5)\not{p}'] \\ &= Mtr[-ab\gamma_\mu P_\nu\not{p}' - abP_\mu\gamma_\nu\not{p}' - cd\gamma_\mu\gamma_5 q_\nu\gamma_5\not{p}' + cdq_\mu\gamma_5\gamma_\nu\gamma_5\not{p}'] \\ &= 4M[-abg_{\mu\alpha}P_\nu p'^\alpha - abP_\mu g_{\nu\alpha}p'^\alpha - cdg_{\mu\alpha}q_\nu p'^\alpha - cdq_\mu g_{\nu\alpha}p'^\alpha] \\ &= 4M[-ab(P_\nu p'_\mu + P_\mu p'_\nu) - cd(q_\nu p'_\mu + q_\mu p'_\nu)] \end{aligned}$$

Thus,

$$W^2_{\mu\nu} + W^3_{\mu\nu} = 4M[-ab(P_\nu P_\mu + P_\mu P_\nu) + cd(-q_\mu q_\nu - q_\mu p_\nu)] = -8M(abP_\mu P_\nu + cdq_\mu q_\nu)$$

CCQE - Hadronic tensor, $W^1_{\mu\nu}$

Homework 4
Show,

$$
\begin{aligned}
W^1_{\mu\nu} &= tr[\Gamma_\mu n(p) \not{P} \Gamma'_\nu \not{p}'] \\
&= 2(a^2 + c^2)[P_\mu P_\nu - q_\mu q_\nu - (2M^2 - q^2) g_{\mu\nu}] + 2(2M^2 - q^2)(b^2 P_\mu P_\nu + d^2 q_\mu q_\nu) \\
&\quad +4iac\epsilon_{\mu\nu\alpha\beta} P^\alpha q^\beta
\end{aligned}
$$

CCQE - Hadronic tensor, summary

$$
\begin{aligned}
\overline{W}_{\mu\nu} &= W^1_{\mu\nu} + W^2_{\mu\nu} + W^3_{\mu\nu} + W^4_{\mu\nu} \\
&= 2(a^2+c^2)[P_\mu P_\nu - q_\mu q_\nu - (2M^2-q^2)g_{\mu\nu}] + 2(2M^2-q^2)(b^2 P_\mu P_\nu + d^2 q_\mu q_\nu) \\
&\quad +4iac\epsilon_{\mu\nu\alpha\beta}P^\alpha q^\beta - 8M(abP_\mu P_\nu + cdq_\mu q_\nu) \\
&\quad +4M^2[(a^2-c^2)g_{\mu\nu} + b^2 P_\mu P_\nu - d^2 q_\mu q_\nu]
\end{aligned}
$$

First, we put them together with $P_\mu P_\nu$, $q_\mu q_\nu$, $g_{\mu\nu}$. All these terms are symmetric with replacement of $\mu \leftrightarrow \nu$, but the term with $\epsilon_{\mu\alpha\nu\beta}$ is antisymmetric.

$$
\begin{aligned}
\overline{W}_{\mu\nu} &= 2[(a^2+c^2) - 4Mab + 4M^2b^2 - b^2q^2]P_\mu P_\nu + 2[-(a^2+c^2) - 4Mcd - d^2q^2]q_\mu q_\nu \\
&\quad +2[(a^2+c^2)q^2 - 4M^2c^2]g_{\mu\nu} - 4iac\epsilon_{\mu\nu\alpha\beta}P^\alpha q^\beta \\
&\equiv XP_\mu P_\nu + Yq_\mu q_\nu + Zg_{\mu\nu} + W^A_{\mu\nu} \\
&\equiv W^S_{\mu\nu} + W^A_{\mu\nu}
\end{aligned}
$$

We use $L^{S,mu\nu}W^A_{\mu\nu} = 0$ and ${}^{A,mu\nu}W^S_{\mu\nu} = 0$, so the surviving terms are $L^{S,\mu\nu}W^S_{\mu\nu}$ and $L^{A,\mu\nu}W^A_{\mu\nu}$.

$$
\overline{|\mathcal{M}|}^2 = \frac{G_F^2}{4}\overline{L}^{\mu\nu}\overline{W}_{\mu\nu} = \frac{G_F^2}{4}\left[L^{S,\mu\nu}W^S_{\mu\nu} + L^{A,\mu\nu}W^A_{\mu\nu}\right]
$$

CCQE - Antisymmetric term

Let's calculate antisymmetric part because it's easier

$$
\begin{aligned}
L^{A,\mu\nu}W^{A}_{\mu\nu} &= 4i\epsilon^{\mu\nu\alpha\beta}K_\alpha q_\beta(4iac\epsilon_{\mu\nu\gamma\delta}P^\gamma q^\delta) = -16acK_\alpha q_\beta P^\gamma q^\delta \epsilon^{\mu\nu\alpha\beta}\epsilon_{\mu\nu\gamma\delta} \\
&= -16acK_\alpha q_\beta P^\gamma q^\delta(-2(\delta^\alpha_\gamma\delta^\beta_\delta - \delta^\alpha_\delta\delta^\beta_\gamma)) \\
&= 32ac[(K\cdot P)q^2 - (K\cdot q)(q\cdot P)] \\
&= 32ac[(s-u)q^2 - 0] = 32(F_1+F_2)F_A(-Q^2)(s-u) = -128\tau(F_1+F_2)F_A \\
&= -128M^2(F_1+F_2)F_A\tau\frac{(s-u)}{M^2}
\end{aligned}
$$

In the CCQE Llewellyn-Smith formalism, cross-section has 3 terms. Here, we derive the B-term which is proportion to $(s-u)$.

CCQE - Symmetric term

$$
\begin{aligned}
L^{S,\mu\nu}W^{S}_{\mu\nu} &= 4[K_\mu K_\nu - q_\mu q_\nu + (q^2 - m^2)g_{\mu\nu}](XP_\mu P_\nu + Yq_\mu q_\nu + Zg_{\mu\nu}) \\
&= 4[X(K\cdot P)^2 + Y(K\cdot q)^2 + ZK^2 - X(q\cdot P)^2 - Yq^4 - Zq^2 \\
&\quad +(q^2 - m^2)(XP^2 + Yq^2 + 4Z)] \\
&= 4[X(s-u)^2 + Ym^4 + Z(2m^2 - q^2) - Yq^4 - Zq^2 \\
&\quad -(m^2 + Q^2)[X(4M^2 - q^2)^2 + Yq^2 + 4Z] \\
&= 4X(s-u)^2 + 4[Ym^4 + 2Zm^2 - YQ^4 + 2ZQ^2] \\
&\quad -(m^2 + Q^2)[X(4M^2 - q^2)^2 + Yq^2 + 4Z] \\
&= 4X(s-u)^2 + 4(m^2 + Q^2)[(Ym^2 - YQ^2) + 2Z - X(4M^2 + Q^2)^2 + YQ^2 - 4 \\
&= 4X(s-u)^2 + 4(m^2 + Q^2)[Ym^2 - 2Z - 4XM^2(1+\tau)]
\end{aligned}
$$

The first term is proportion to $(s-u)^2$, so this term will be the C-term. And the second part will become the A-term.

CCQE - Symmetric term - X, Y, Z term

$$
\begin{aligned}
X &= 2\left[4M^2\left(\frac{F_2}{2M}\right)^2 - 4M(F_1+F_2)\left(\frac{F_2}{2M}\right) + \right. \\
&\left. + (F_1+F_2)^2 + F_A^2 + \left(\frac{F_2}{2M}\right)^2 Q^2\right] \\
&= 2(F_2^2 - 2F_1F_2 - 2F_2^2 + F_1^2 + F_2^2 + 2F_1F_2 + F_A^2 + \tau F_2^2) \\
&= 2(F_1^2 + \tau F_2^2 + F_A^2) \\
Y &= -2[(F_1+F_2)^2 + F_A^2] - 8MF_A\left(\frac{F_P}{M}\right) + 2\left(\frac{F_P}{M}\right)^2 Q^2 \\
&= 2[-(F_1+F_2)^2 - F_A^2 - 4F_AF_P + 4\tau F_P^2] \\
&= 2[-(F_1+F_2)^2 - (F_A+2F_P)^2 + 4(1+\tau)F_P^2] \\
Z &= -2[(F_1+F_2)^2 + F_A^2]Q^2 - 8M^2F_A^2 \\
&= -8M^2[\tau(F_1+F_2)^2 + (1+\tau)F_A^2]
\end{aligned}
$$

CCQE - Symmetric term, summary

$$
\begin{aligned}
L^{S,\mu\nu} W^S_{\mu\nu} &= 4X(s-u)^2 + 4(m^2+Q^2)[Ym^2 - 2Z - 4XM^2(1+\tau)] \\
&= 8[F_1^2 + \tau F_2^2 + F_A^2](s-u)^2 \\
&\quad + 4(m^2+Q^2)\left[16M^2\tau(F_1+F_2)^2 + 16M^2(1+\tau)F_A^2\right. \\
&\quad \left. - 8M^2(1+\tau)(F_1^2+\tau F_2^2+F_A^2) + Ym^2\right] \\
&= 8[F_1^2 + \tau F_2^2 + F_A^2](s-u)^2 \\
&\quad + 32M^2(m^2+Q^2)\left[2\tau(F_1+F_2)^2 + (1+\tau)F_A^2 - (1+\tau)(F_1^2+\tau F_2^2) + Ym^2\right] \\
&= 8[F_1^2 + \tau F_2^2 + F_A^2](s-u)^2 \\
&\quad + 32M^2(m^2+Q^2)\Bigg[(1+\tau)F_A^2 - (1-\tau)F_1^2 + \tau(1-\tau)F_2^2 + 4\tau F_1 F_2 \\
&\quad - \frac{m^2}{4M^2}\left[(F_1+F_2)^2 + (F_A+2F_P)^2 - 4(1+\tau)F_P^2\right]\Bigg]
\end{aligned}
\tag{2}
$$

CCQE - Llewellyn-Smith formalism

Let's put everything together.

$$
\begin{aligned}
\frac{d\sigma}{dQ^2} &= \frac{1}{64\pi M^2 E^2}|\mathcal{M}|^2 = \frac{G_F^2}{256\pi M^2 E^2}\overline{L}^{\mu\nu}\overline{W}_{\mu\nu} \\
&= \frac{G_F^2}{256\pi M^2 E^2}\Big[8[F_1^2 + \tau F_2^2 + F_A^2](s-u)^2 \\
&\quad +32M^2(m^2+Q^2)\Big\{(1+\tau)F_A^2 - (1-\tau)F_1^2 + \tau(1-\tau)F_2^2 + 4\tau F_1 F_2 \\
&\quad -\frac{m^2}{4M^2}\left[(F_1+F_2)^2 + (F_A+2F_P)^2 - 4(1+\tau)F_P^2\right]\Big\} \\
&\quad -128M^2(F_1+F_2)F_A\tau\frac{(s-u)}{M^2}\Big] \\
&= \frac{G_F^2 M^2}{8\pi E^2}\Big[\frac{(m^2+Q^2)}{M^2}\Big\{(1+\tau)F_A^2 - (1-\tau)F_1^2 + \tau(1-\tau)F_2^2 + 4\tau F_1 F_2 \\
&\quad -\frac{m^2}{4M^2}[(F_1+F_2)^2 + (F_A+2F_P)^2 - 4(1+\tau)F_P^2]\Big\} \\
&\quad -4\tau(F_1+F_2)F_A\frac{(s-u)}{M^2} \\
&\quad +\frac{1}{4}(F_1^2+\tau F_2^2+F_A^2)\frac{(s-u)^2}{M^4}\Big] \\
&\equiv \frac{G_F^2 M^2}{8\pi E^2}\left[A(Q^2) - B(Q^2)\frac{(s-u)}{M^2} + C(Q^2)\frac{(s-u)^2}{M^4}\right]
\end{aligned}
$$

CCQE - anti-CCQE cross-section $\bar{\nu} + p \to l^+ + n$

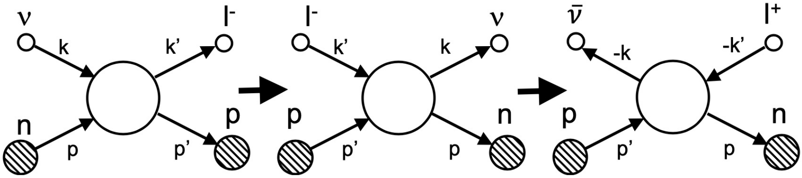
Cross-section for anti-neutirno can be derived by reordering of the diagram. By this operation,

$$
\begin{aligned}
s &= (k+p)^2 \to (k-p')^2 (s \to u) \\
t &= (k'-k)^2 \to (k'-k)^2 (t \to t) \\
u &= (k-p')^2 \to (k+p)^2 (s \to u)
\end{aligned}
$$

Thus, CCQE anti-neutirno cross-section is replacing s and u by $s \leftrightarrow u$.

$$
\frac{d\sigma}{dQ^2} = \frac{G_F^2 M^2}{8\pi E^2}\left[A(Q^2) + B(Q^2)\frac{(s-u)}{M^2} + C(Q^2)\frac{(s-u)^2}{M^4}\right]
$$

In Llewellyn-Smith formalism, this corresponds to change the sign of the B-term.



CCQE - Llewellyn-Smith formalism, summary

$$\frac{d\sigma}{dQ^2}\begin{pmatrix}\nu + n \to l + p \\ \bar{\nu} + p \to l^+ + n\end{pmatrix} = \frac{G_F^2 M^2}{8\pi E^2}\left[A(Q^2) \mp B(Q^2)\frac{(s-u)}{M^2} + C(Q^2)\frac{(s-u)^2}{M^4}\right]$$

$$A = \frac{(m^2+Q^2)}{M^2}\Big\{(1+\tau)F_A^2 - (1-\tau)F_1^2 + \tau(1-\tau)F_2^2 + 4\tau F_1 F_2$$
$$- \frac{m^2}{4M^2}[(F_1+F_2)^2 + (F_A+2F_P)^2 - 4(1+\tau)F_P^2]\Big\}$$

$$B = 4\tau(F_1+F_2)F_A$$

$$C = \frac{1}{4}(F_1^2 + \tau F_2^2 + F_A^2)$$

1. The A-term is important at low energy, and the C-term is important at high-energy, so the cross section is $\propto E^2$ at low energy, and $\propto$ *const* at high energy.
2. The B-term is proportion to F_A, which is negative. So the νCCQE cross section is larger than the $\bar{\nu}$CCQE cross-section.
3. In the $m^2/M^2 \to 0$ limit, the last term of A-term goes zero, meaning contribution of F_P (pseudo-scalar) goes zero. This is true for ν_e and ν_μ CCQE scatterings. For *tau* CCQE, $m^2/M^2 > 1$ and the F_P contribution is big.
4. Everything assumes isospin symmetry, $M = M_p = M_n$. This is true for many nuclear models.
5. All form factors are assumed to be real. This means there is no T-violation, or no CP-violation.

CCQE - total cross sections

The graphs show CCQE total cross-sections for muon neutrino - neutron scattering and muon antineutrino - proton scattering. The x-axis is in GeV, and the y-axis is in 10^{-38} cm^2.