# Radiation Detector 2018/19 (SPA6309), Tutorial 3


Name:
ID:

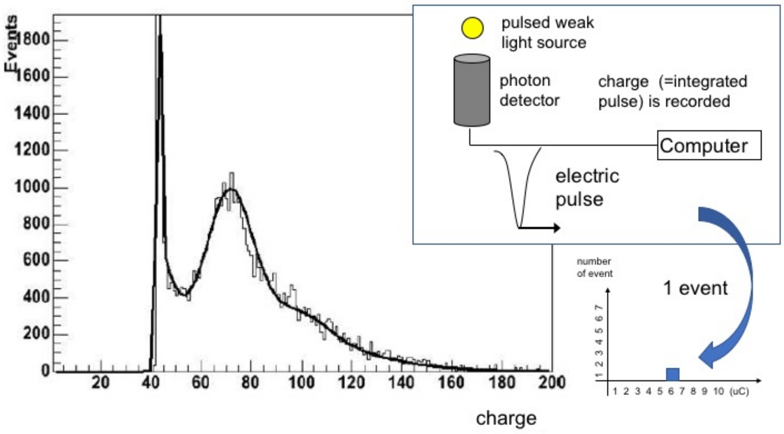
**Problem 1**

There is a photon detector in front of a very weak pulsed light source. The photon signal is converted to an electric pulse, then the integrated pulse (=electric charge, arbitrary unit) is recorded as a signal (1 event). After many hours of exposure, we obtain a distribution of charge. The detector is very sensitive and it can "count" individual photons (0 photon, 1 photon, 2 photons, etc). The data shows a large peak corresponds to "zero detection" (called **pedestal**). The next moderate mountain corresponds to a 1 photon detection.

[1] Why does a zero photon peak not deposit 0 charges?

[2] Why 1 photon makes round mountain signal, instead of a sharp peak?

[3] Estimate how many photons are detected on average in this setting.

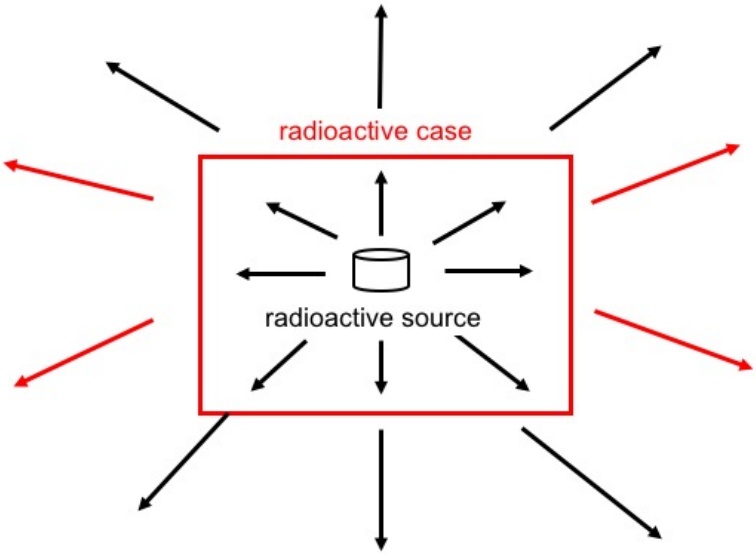



**problem 2**

Prove both the binomial distribution $B(x, n, p, q) = {}_nC_x p^x q^{(n-x)}$ and the Poisson distribution $P(x, \mu) = \frac{\mu^x e^{-\mu}}{x!}$ are correctly normalized.

**Problem 3**

A radioactive source decays with decay constant $\lambda_s$, and thus the probability to measure count $r_s$ in time $t$ is distributed with a Poisson distribution, $P(r_s, \lambda_s t) = \frac{(\lambda_s t)^{r_s} e^{-\lambda_s t}}{r_s!}$. This source is stored in the case for a long time, and because of this now the case itself is activated and emit radiation with decay constant $\lambda_b$ and count $r_b$ in a time $t$. What is the distribution of the total count rate $r$? is it a Poisson distribution?




**solution, problem 1**

[1] Even there is no signal, the computer records integrated electric noise. Therefore nonzero signal is not nonzero charge. It is an important feature for experiments to record "zero detection".

[2] There is a number of reasons the data doesn't have a sharp peak, for example, you can imagine the signal pulse may spread during the propagation, or perhaps part of signal gets lost and is not quite 1 photon equivalent charge etc..., but all of these are part of the detector resolution. Every detector has a finite resolution and every successive measurement causes the distribution to smear out due to the detector resolution.

[3] The zero measurement can be used to estimate the mean. By eye, the fraction which are zero detection events (area of distribution left side from the "valley") is about 1/5 (this is my eye-rolling estimation, you can get different numbers), this is lower than 0.368 (0 observation probability when the mean is 1), so we can expect more than 1 photon on average.

In [8]:
```
import numpy as np
mu=-np.log(1.0/5.0)
print "mean=",mu
```

```
mean= 1.60943791243
```

**solution, problem 2**

$\Sigma_{x=0}^{n} B(x,n,p,q) = \Sigma_{x=0}^{n} \, {}_nC_x p^x q^{(n-x)}$
$= \, {}_nC_0 p^0 q^n + \, {}_nC_1 p^1 q^{(n-1)} + \cdots + \, {}_nC_{(n-1)} p^{(n-1)} q^1 + \, {}_nC_n p^n q^0$
$= (p+q)^n = (p+1-p)^n = 1$

$\Sigma_{x=0}^{\infty} P(x,\mu) = \Sigma_{x=0}^{\infty} \frac{\mu^x e^{-\mu}}{x!}$
$= e^{-\mu} \Sigma_{x=0}^{n} \frac{\mu^x}{x!} = e^{-\mu} e^{\mu} = 1$

**solution, problem 3**

The probability to measure count $r$ is a product of 2 distribution $P(r_s, \lambda_s t)$ and $P(r_b, \lambda_b t)$. Now, sum of $r_s$ and $r_b$ should be $r$. By adding all possible $r_b$,

$P = \Sigma_{r_b=0}^{r} P(r-r_b, \lambda_s t) P(r_b, \lambda_b t)$
$= \Sigma_{r_b=0}^{r} \frac{(\lambda_s t)^{r-r_b} e^{-\lambda_s t}}{(r-r_b)!} \frac{(\lambda_b t)^{r_b} e^{-\lambda_b t}}{r_b!}$
$= \frac{1}{r!} \left[ \Sigma_{r_b=0}^{r} \frac{r!}{(r-r_b)! r_b!} (\lambda_s t)^{(r-r_b)} (\lambda_b t)^{r_b} \right] e^{-(\lambda_s+\lambda_b)t}$

The term inside of [] is the normalization condition of binomial distribution, $B(r_b, r, \lambda_s t, \lambda_b t)$ where $\Sigma_{r_b=0}^{r} B(r_b, r, \lambda_s t, \lambda_b t) = (\lambda_s t + \lambda_b t)^r$, thus,

$P = \frac{1}{r!} (\lambda_s t + \lambda_b t)^r e^{-(\lambda_s+\lambda_b)t} = P(r, (\lambda_s + \lambda_s) t)$. The total rate $r$ is Poisson distribution with decay constant $\lambda_s + \lambda_b$ (addition theorem of Pisson distributed variables).