Radiation Detector 2018/19 (SPA6309), Homework 3

Name:
ID:

Problem 1 (6 points)

Supernova type IIa (core-collapse supernova) produces significant amounts of detectable neutrinos and many people (including me!) are waiting for the next one within the Milky Way. Here, the rate of type IIa supernova is predicted to be 3 per century.

[1] Assuming a Poisson distribution, what is the probability of measuring 0, 1, 2 or 3 type IIa supernovae in the next 100 years? (4 points)

[2] Next, from the observation, people cannot find any type IIa supernova in 100 years observation. What is the upper limit of the rate of type IIa supernova observation with a 90% confidence level? (2 points)

problem 2 (3 points)

During an experiment, an array of 50 independent counters are monitoring background particles entering the detector to reject them (=veto counters). Each of them has a 99% efficiency to detect background particles, or 1% chance to fail to detect particles entering the detector. An experiment discards the data if 2 or more counters fail to reject background particles. What percentage of data from this experiment will be useful?

problem 3 (1 points)

There is a series of measurements about x, where mean is μ and variance is σ^2. The sample mean (average) of n measurements is $\hat{\mu}$ and the sample variance is $\hat{\sigma}^2$.

As expected, the expectation value of sample mean $\hat{\mu}$ is the mean μ.

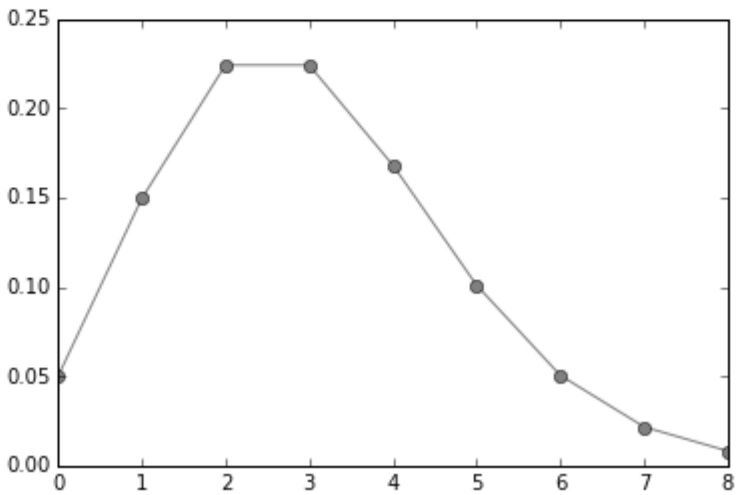
$$< \hat{\mu} >= \left\langle \frac{1}{n} \sum_{i=1}^{n} x \right\rangle = \frac{1}{n} \sum_{i=1}^{n} < x >= \frac{1}{n} \cdot n\mu = \mu$$

Prove expectation value of sample variance $\hat{\sigma}^2$ is the variance σ^2.

Solution

In [6]:
```
%matplotlib inline
import math
import numpy as np
import matplotlib.pylab as plt
a_list=[0,1,2,3,4,5,6,7,8]
b_list=[]
mu=3
for n in a_list:
    b_list.append(mu**n*math.exp(-mu)/math.factorial(n))
    print "Prob. to detect",n,"is",b_list[n]
prob_bin = np.linspace(0,8,9)
plt.plot(prob_bin,b_list,marker='o',linestyle='-',color='grey',l
plt.show()
```

```
Prob. to detect 0 is 0.0497870683679
Prob. to detect 1 is 0.149361205104
Prob. to detect 2 is 0.224041807655
Prob. to detect 3 is 0.224041807655
Prob. to detect 4 is 0.168031355742
Prob. to detect 5 is 0.100818813445
Prob. to detect 6 is 0.0504094067225
Prob. to detect 7 is 0.0216040314525
Prob. to detect 8 is 0.00810151179468
```



In [11]:
```
Lo=-math.log(0.1)
print "90%CL of the rate of 0 type IIa supernova is ",Lo,"per 10
```

```
rate of 0 type IIa supernova is  2.30258509299 per 100 year
```

Even after 100 years measurement, the upper limit is 2.3 supernova per century (90%CL), so you cannot eliminate the statement of saying "3 supernovae per century" very much after 100 years observation!

Using binomial distribution, total probability is sum of 0 counter failure and 1 counter failure.
$P = B(50; 50, 0.99) + B(49; 50, 0.99) = (0.99)^{50} +_{50} C_{49}(0.99)^{49} \cdot 0.01 = 0.605$
so 91% of data will be useful.

In [15]:
```
P=0.99**(50)+50*0.99**(49)*0.01
print P
```

```
0.910564686904
```

$$< \hat{\sigma}^2 >= \frac{1}{n-1}\Sigma_{i=1}^{n} \left\langle [(x_i - \mu) - (\hat{\mu} - \mu)]^2 \right\rangle$$
$$= \frac{1}{n-1}\Sigma_{i=1}^{n} \left\langle [(x_i - \mu) - \frac{1}{n}\Sigma_{j=1}^{n}(x_j - \mu)]^2 \right\rangle$$
$$= \frac{1}{n-1}\Sigma_{i=1}^{n} \left\langle (x_i - \mu)^2 \right\rangle - \frac{2}{n(n-1)}\Sigma_{i=1}^{n}\Sigma_{j=1}^{n} \left\langle (x_i - \mu)(x_j - \mu) \right\rangle + \frac{1}{n^2(n-1)}\Sigma_{i=1}^{n} \left\langle \left[\Sigma_{j=1}^{n}\right.\right.$$

Here, use
$\left[\Sigma_{j=1}^{n}(x_j - \mu)\right]^2 = \left(\Sigma_{j=1}^{n}(x_j - \mu)\right) \cdot \left(\Sigma_{j=1}^{n}(x_j - \mu)\right) = \Sigma_{i=1}^{n}\Sigma_{j=1}^{n}(x_i - \mu)(x_j - \mu)$
for the second and the third terms,

$$= \frac{n\sigma^2}{n-1} - \frac{2}{n(n-1)}\Sigma_{i=1}^{n}\Sigma_{j=1}^{n} \left\langle (x_i - \mu)(x_j - \mu) \right\rangle + \frac{1}{n(n-1)}\left\langle \left[\Sigma_{j=1}^{n}(x_j - \mu)\right]^2 \right\rangle$$
$$= \frac{n\sigma^2}{n-1} - \frac{1}{n(n-1)}\Sigma_{i=1}^{n}\Sigma_{j=1}^{n} \left\langle (x_i - \mu)(x_j - \mu) \right\rangle$$

Here, you need to split the second term for $j = i$ and $j \neq i$. Note, expectation value of products of $j \neq i$ term is zero.

$$= \frac{n\sigma^2}{n-1} - \frac{1}{n(n-1)}\Sigma_{i=1}^{n} \left\langle (x_i - \mu)^2 \right\rangle - \frac{1}{n(n-1)}\Sigma_{i=1}^{n}\Sigma_{j\neq i}^{n} \left\langle (x_i - \mu)(x_j - \mu) \right\rangle$$
$$= \frac{n\sigma^2}{n-1} - \frac{1}{n(n-1)}\Sigma_{i=1}^{n} \left\langle (x_i - \mu)^2 \right\rangle$$
$$= \frac{n\sigma^2}{n-1} - \frac{n\sigma^2}{n(n-1)} = \sigma^2$$

Alternatively, one could use $< x_i^2 >= \sigma^2 + \mu^2$ and $< \hat{\mu}^2 >= \frac{1}{n}\sigma^2 + \mu^2$ to find the same answer.