Radiation Detector 2018/19 (SPA6309)

Signal transmission

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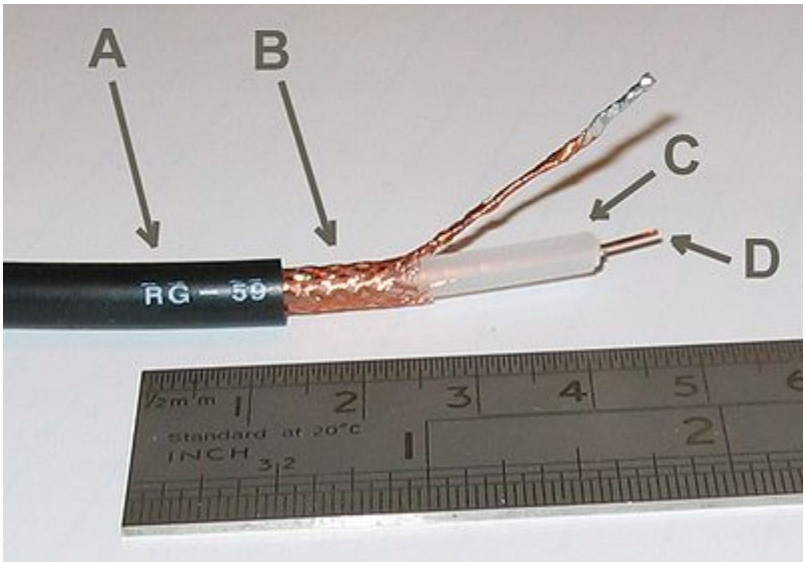
Coaxial cable (Leo, 13.1)



For the transmission of data (fast, weak pulse), **Coaxial cable** is commonly used. It has 4 basic elements; A. jacket B. shield C. insulator D. conductor

The inner **conductor** is often made by copper. The **dielectric** insulator separates conductor and outer **shield** which acts as a ground line. They are protected by **jacket** often made by plastic, such as PVC.

The cable has characteristic **capacitance C** and **inductance L** (13.1). Say conductor radius is a, and the shield radius is b, current is I, and the charge stored in a unit length is Q. From Gauss's law ($\nabla \cdot E = \rho/\epsilon$), $2\pi r E = \frac{Q}{\epsilon}$. Then stored energy is,

$$\frac{1}{2}\int E \cdot D r dr d\phi = \frac{1}{2\epsilon}\left(\frac{Q}{2\pi}\right)^2 2\pi log\left(\frac{b}{a}\right) \equiv \frac{Q^2}{2C}, C = \frac{2\pi\epsilon}{log(b/a)}$$

Similarly, from Ampere's law ($\nabla \times H = j$), $H = \frac{I}{2\pi r}$. Then stored energy is,

$$\frac{1}{2}\int H \cdot B r dr d\phi = \frac{\mu}{2}\left(\frac{I}{2\pi}\right)^2 2\pi log\left(\frac{b}{a}\right) \equiv \frac{LI^2}{2}, L = \frac{\mu}{2\pi}log\left(\frac{b}{a}\right)$$

Thus, **capacitance C** and **inductance L** are the function of the conductor radius a and the shield radius b, or more frankly C and L are the function of the cable size.

Ideal lossless cable (Leo, 13.3)

Coaxial cable makes **LC circuit**. In a small unit length of a co-acial cable Δz, the difference of ΔV and ΔI across Δz are,

$$\Delta V = -L\Delta z\frac{\partial I}{\partial t},\ \Delta I = -C\Delta z\frac{\partial V}{\partial t}$$

or

$$\frac{\partial V}{\partial z} = -L\frac{\partial I}{\partial t},\ \frac{\partial I}{\partial z} = -C\frac{\partial V}{\partial t}$$

By combining them (13.5),

$$\frac{\partial^2 V}{\partial z^2} = -LC\frac{\partial^2 I}{\partial t^2}$$

Using a trial solution $V = V(z)exp(i\omega t)$ and define $k^2 \equiv \omega^2 LC$ (13.6),

$$\frac{\partial^2 V}{\partial z^2} = -\omega^2 LCV = -k^2 V,\ V(Z) = V_1 exp(-ikz) + V_2 exp(ikz)$$

Thus, the full solution is (13.7)

$$V(Z, t) = V_1 exp[i(\omega t - kz)] + V_2 exp[i(\omega t + kz)]$$

There are 2 waves in the solution, the first part is traveling to $+z$ direction, and the second part is travelling to $-z$ directions (**reflection**). Also, k is a wave number and,

$$v = \frac{\omega}{k} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu\epsilon}}.$$

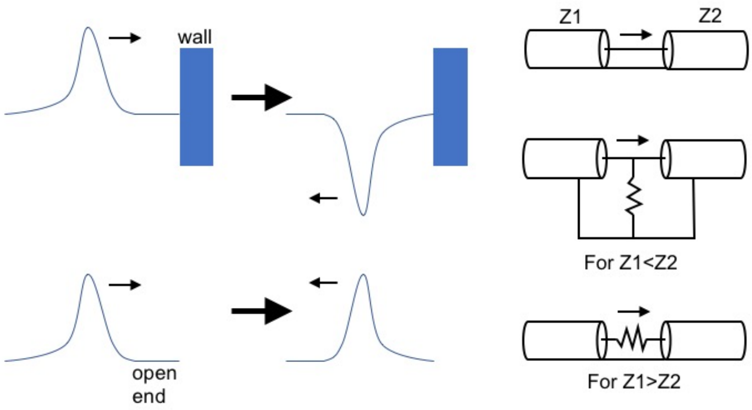
Thus the speed of the signal in the cable is determined from LC or $\mu\epsilon$. Here, **delay** ($T = 1/v = \sqrt{LC}$) is the time for the signal to propagate unit length. For a standard 50Ω cable, **the delay is 5ns/m (speed of the signal is 2/3c)**.

Characteristic impedance (Leo, 13.3.1)

$$Z_0 = \sqrt{\frac{L}{C}} \sim 60\sqrt{\frac{\mu/\mu_0}{\epsilon/\epsilon_0}}ln\frac{b}{a}\ \Omega$$

Importantly, **characteristic impedance** is independent of cable length, but depends on cable geometry logarithmically. This makes cable impedance to be around 50-200 Ω. Let's say if you want the cable to be 1000 Ω, diameter ratio should be $\sim 10^{11}$ and impractical. The most typical cables are **50 Ω impedance** and we assume impedance of cables and all devices are 50 Ω for the rest of this module. Impedance is the most important parameter for cables. Matching of impedance is essential to prevent signal **reflection**.

Reflections and impedance matching (Leo, 13.5)



In general, some part of the signal would reflect if the impedance of the cable is abruptly changed. The reflected signals can be understood from the analogy of "infinitely high impedance" and "zero impedance"

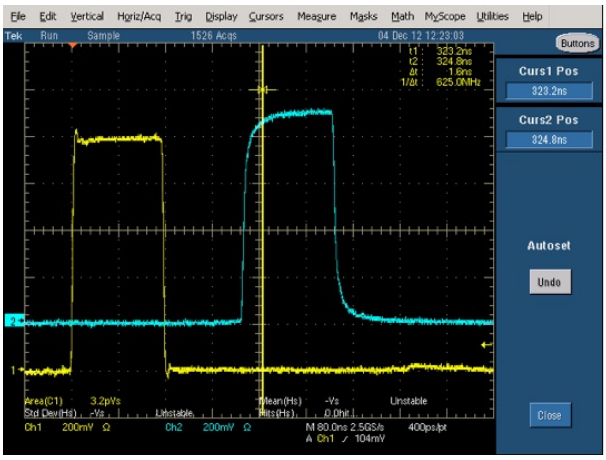
- infinitely high impedance - string connected to the wall → reflected wave will be reverted
- zero impedance - open end string → reflected wave will have the same sign

In reality, all impedances are finite, so there are no total reflections, but some parts of signals are reflected. Reflected signal can be reflected again and it will make "ringing" of the signal.

To prevent reflections at the cable connection, **impedance matching** is important. Namely, the impedance of new region "Z2" should be same with "Z1". If the impedance of Z2 is higher than Z1, add a resistor in parallel to adjust the impedance of Z2 "looks" lower for the signal coming from region Z1. On the other hand, if Z2 is lower than Z1, add a resistor in series to adjust the impedance of Z2 "looks" higher for the incoming signal. Clearly, this works only for signals go to one direction.

Sometimes reflection can be used to check cable length. By knowing the speed of the signal, one can measure cable length precisely from the reflected signal.

Losses in coaxial cables and pulse distortion (Leo, 13.6)



Cable has nonzero resistance, which generates heat and loses signal strength and distorts pulse shape. Although the loss of signal is usually a small effect, a high-frequency signal (>100kHz) is often distorted. For example, block pulse often loses the rising edge and falling edge by long propagation.