Radiation Detector 2018/19 (SPA6309)

Particle Propagation 1 (Leo, Chapter 2)

Relativistic kinematics

Relativistic $\beta = \frac{v}{c} \equiv v$, and relativistic $\gamma = \frac{1}{\sqrt{1-\beta^2}}$

Energy, $E = \sqrt{p^2c^2 + m^2c^4} \equiv \sqrt{p^2 + m^2} = \gamma m$ and momentum $p = \beta\gamma m = \beta E$

kinetic energy $T = E - m$. At $T > m$, relativistic kinematics are important. For the non-relativistic limit ($T << m$), $T \sim \frac{p^2}{2m}$.

Some example of relativity:

In [4]:
```
import numpy as np
m=0.938 #GeV, mass of proton
print "speed of 1 GeV proton is ",100*np.sqrt(1.0**2-m**2)/1.0,"
print "speed of 10 GeV proton is ",100*np.sqrt(10.0**2-m**2)/10.
print "speed of 100 GeV proton is ",100*np.sqrt(100.0**2-m**2)/1
print "speed of 1000 GeV proton is ",100*np.sqrt(1000.0**2-m**2)
```

```
speed of 1 GeV proton is  34.6635254987 % of speed of light
speed of 10 GeV proton is  99.5591060627 % of speed of light
speed of 100 GeV proton is  99.9956006832 % of speed of light
speed of 1000 GeV proton is  99.9999560078 % of speed of light
```

On the other hand, γ of 1, 10, 100, and 1000 GeV protons are 1.066, 10.66, 106.6, and 1066. The speed is not an important concept to describe high energy particles, because all of them are just ~c. Instead, γ is more useful.

The Cross Section (Leo, 2.1.1)

The cross section is the "measure of the probability for a reaction to occur" (and related very little with the "cross section area"). Number of scattered particles N_s in $d\Omega$ solid angle is estimated using total cross section σ.

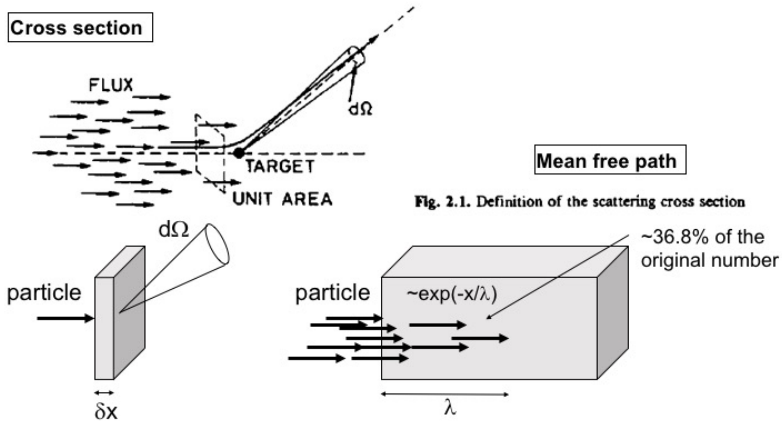


Fig. 2.1. Definition of the scattering cross section

$$\frac{dN_s}{d\Omega} = \frac{d\sigma}{d\Omega} \times \Phi \times t \times T$$

- $\frac{d\sigma}{d\Omega}$: differential cross section
- Φ: flux, number of particles passing through unit area ($/cm^2/s$)
- t: exposure time
- T: number of target particles

From the measurement with known Φ, t, and T, one can obtain the differential cross section.

$$\frac{d\sigma}{d\Omega} = \frac{1}{tT\Phi}\frac{dN}{d\Omega}$$

Total number of interaction is $N_{tot} = \sigma(cm^2) \times \Phi(\#/cm^2/sec) \times t(sec) \times T(\#)$ where $\sigma = \int \frac{d\sigma}{d\Omega} d\Omega$

Mean free path (Leo, 2.1.2)

Now, we assume only 1 beam particle hit the large target material, where density N is given. If the target has thickness δx, then the probability of having an interaction per second in δx is $\sigma(cm^2) \times N(\#/cm^3) \times \delta x(cm)$

Define $P(x)$ to be the probability NOT have any interactions for a particle to propagate distance x in the target material. Also the probability to have an interaction for a particle propagating from x to $x + \delta x$ is $\frac{1}{\lambda}\delta x$ ($\lambda \equiv 1/(N\sigma)$). Then the probability not to have an interaction between x and δx is (eq. 2.6),

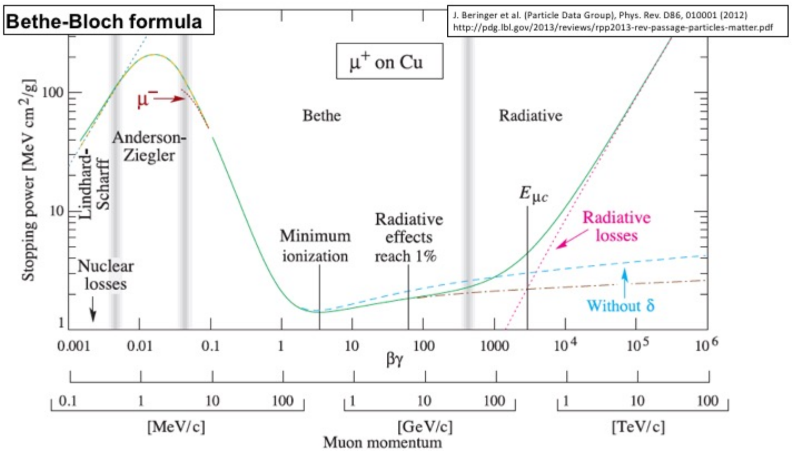
$P(x + dx) = p(x) - \frac{1}{\lambda} dx \cdot P(x)$

by solving this, one gets

$P(x) = C \cdot exp(\frac{-x}{\lambda})$

This means, number of particles surviving during the propagation of a material is an exponential law, and after travelling distance λ, the number of particles decreases to 1/e or 36.8% of the original number. This λ is called **mean free path.**

Bethe-Bloch formula (Leo, 2.2.2)



How much a charged particle loses its energy by propagating in a material is described by **Bethe-Bloch formula** (2.26)

$$\frac{dE}{dx} = 2\pi r_e^2 m_e c^2 \frac{z^2}{\beta^2} \frac{N_A Z\rho}{A} \left[ln(\ldots) - 2\beta^2\right] \propto \frac{1}{\beta^2}$$

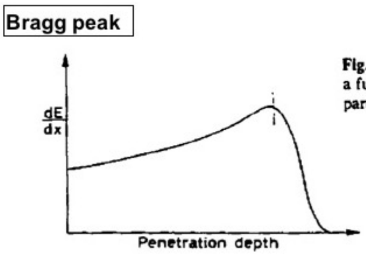
The main energy loss process is the **ionization.** Charged particles knock out electrons in a material, and through this process charged particles lose their energy. If the particle has higher energy than its mass, $\beta \sim 1$, this means dE/dx is almost constant for high energy particles. Such particle is called **minimum ionizing particle (MIP)**, and for a rule of thumb, **MIP loses 2 MeV per cm in 1**g/cm^3 **material**. In above example, muon energy loss in copper is shown with the unit of **stopping power** ($\frac{1}{\rho}\frac{dE}{dx}$). By multiplying $1g/cm^3$, you get ~2MeV/cm around 100 MeV/c to 1 TeV/c muon. Therefore, a muon is MIP in this energy region. In fact, a muon is MIP most of the case, and we use this approximation a lot in this course.

If the particle energy is really high, it loses energy by **radiative** process or **bremsstrahlung** ("radiative" is a confusing word because any emission of energy is a kind of radiation, but bremsstrahlung is often just called radiation or radiative loss). This is a relativistic effect only happens for extremely relativistic particles (kinetic energy >> mass). Clearly, the radiative loss is more important for light particles, such as electrons.

Cosmic muons

Majority of primary cosmic rays are protons. When protons hit the earth atmosphere, they generate showers of particles. Since muons have a long lifetime, **cosmic muons** dominate cosmic rays on the earth surface. Typical cosmic muon energy is **4 GeV**, so they are MIPs. Typical cosmic muon flux is roughly "10 muon hits on your palm per second" or **1 per second per 10cm**^{2}. Indeed, this rough estimation is about right for most applications.

Bragg peak (Leo, 2.2.3)



2. Passage of Radiation Through Matte

Fig. 2.5. A typical Bragg curve showing the variation of *dE/dx* a: a function of the penetration depth of the particle in matter. Th particle is more ionizing towards the end of its path

From Bethe-Bloch formula, you see one counterintuitive result..., when the particle is slowing down in material, dE/dx is increasing. Then, the particle goes even slower, lose even more energy and so on. Namely, a particle would lose most of the energy at the point it stops. This is called **Bragg peak**, and it provides the basic idea of proton or heavy ion cancer therapy. For a heavy particle, Bragg peak is sharper, so it penetrates the body without damaging cells, and release all energy at a spot.