Tests of Lorentz and CPT Violation with Neutrinos

Teppei Katori
Queen Mary University of London
NExT meeting, University of Southampton, UK, Nov. 27, 2013

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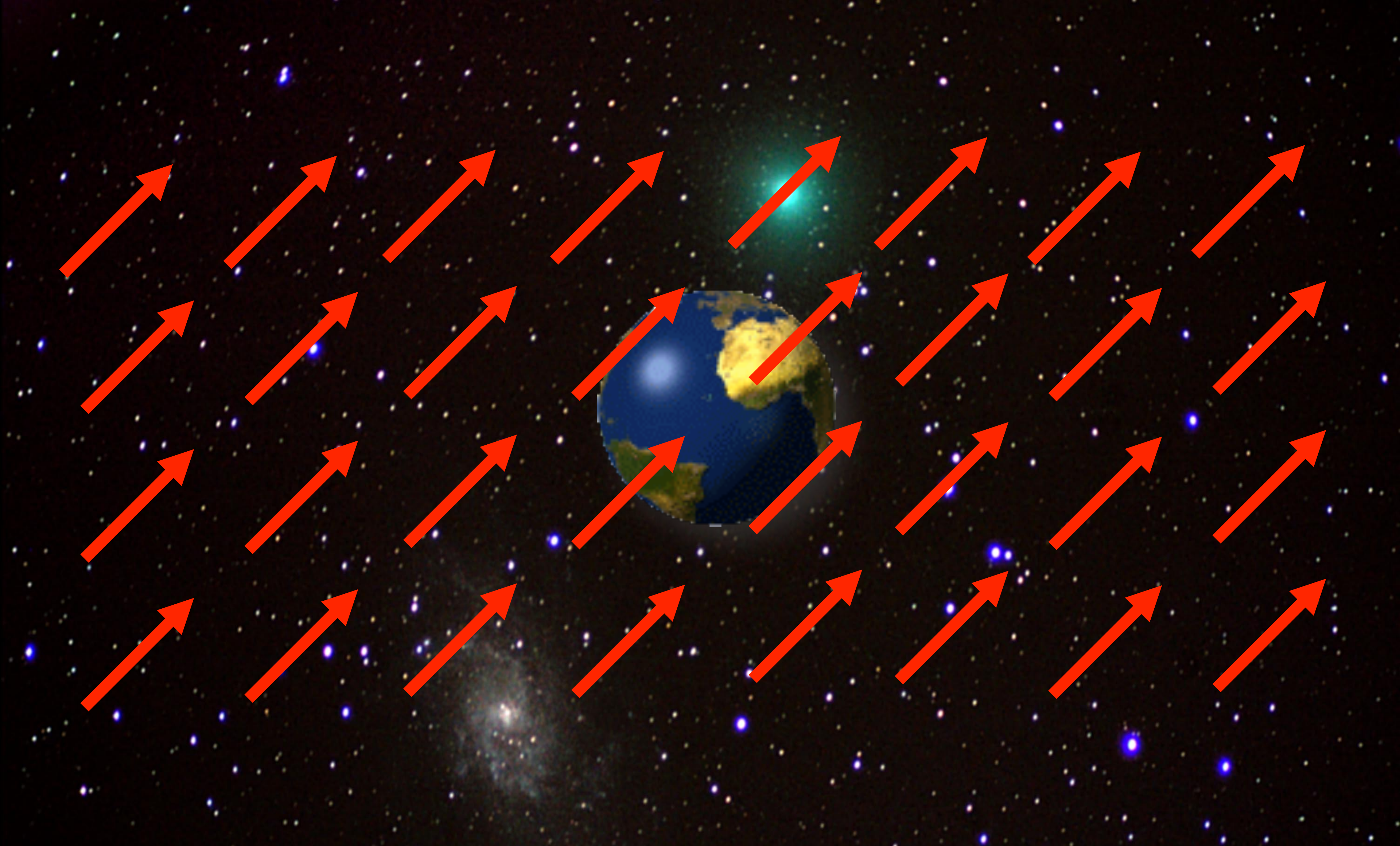

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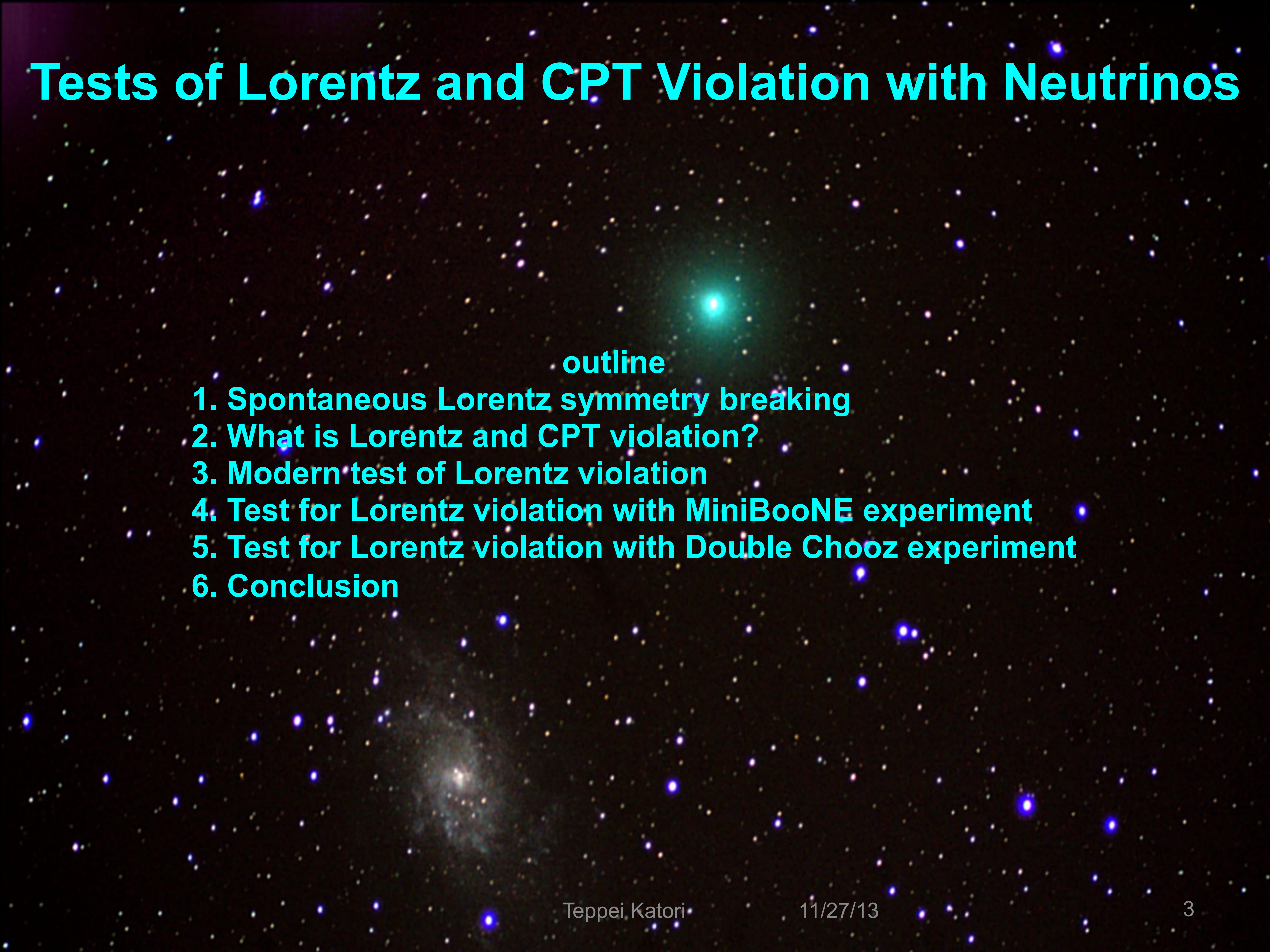

outline

1. Spontaneous Lorentz symmetry breaking
2. What is Lorentz and CPT violation?
3. Modern test of Lorentz violation
4. Test for Lorentz violation with MiniBooNE experiment
5. Test for Lorentz violation with Double Chooz experiment
6. Conclusion

1. Spontaneous Lorentz symmetry breaking
2. What is Lorentz and CPT violation?
3. Modern test of Lorentz violation
4. Test of Lorenz violation with MiniBooNE
5. Test of Lorentz violation with Double Chooz
6. Conclusion

1. Spontaneous Lorentz symmetry breaking (SLSB)

Every fundamental symmetry needs to be tested, including Lorentz symmetry.

After the recognition of theoretical processes that create Lorentz violation, testing Lorentz invariance becomes very exciting

Lorentz and CPT violation has been shown to occur in Planck scale theories, including:

- string theory
- noncommutative field theory
- quantum loop gravity
- extra dimensions
- etc

However, it is very difficult to build a self-consistent theory with Lorentz violation...

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Spontaneous Symmetry Breaking (SSB)!

Y. Nambu
(Nobel prize winner 2008),
picture taken from CPT04 at
Bloomington, IN

1. Spontaneous Lorentz symmetry breaking (SLSB)

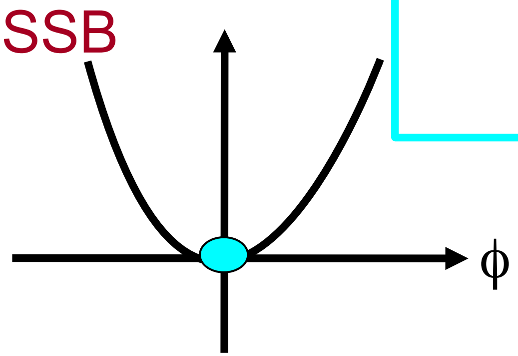
vacuum Lagrangian for fermion $L = i\bar{\Psi}\gamma_{\mu}\partial^{\mu}\Psi$

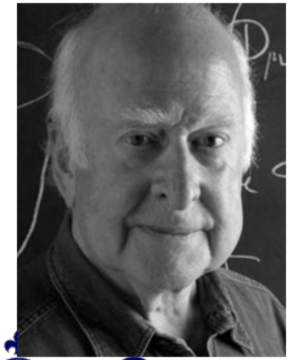
e.g.) SSB of scalar field in Standard Model (SM)

- If the scalar field has Mexican hat potential

$$L = \frac{1}{2}(\partial_{\mu}\varphi)^2 - \frac{1}{2}\mu^2(\varphi^*\varphi) - \frac{1}{4}\lambda(\varphi^*\varphi)^2$$

$$M(\varphi) = \mu^2 < 0$$





1. Spontaneous Lorentz symmetry breaking (SLSB)

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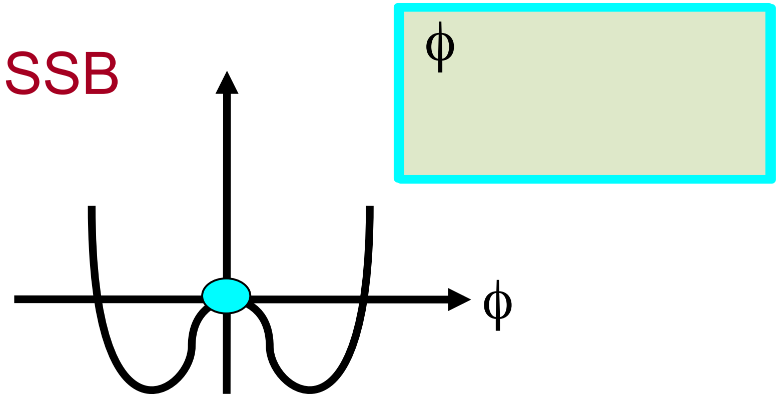

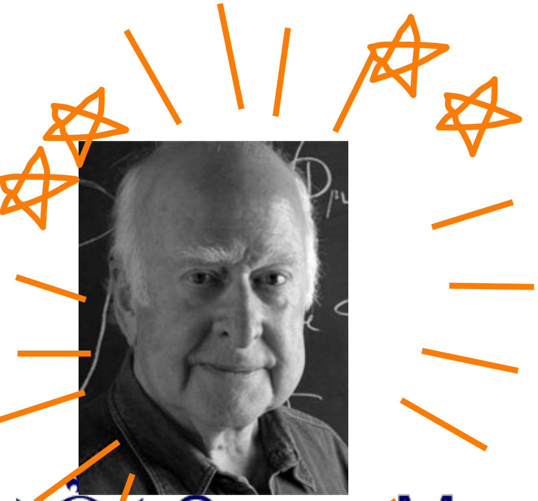

Particle acquires mass term!

Kostelecký and Samuel
PRD39(1989)683

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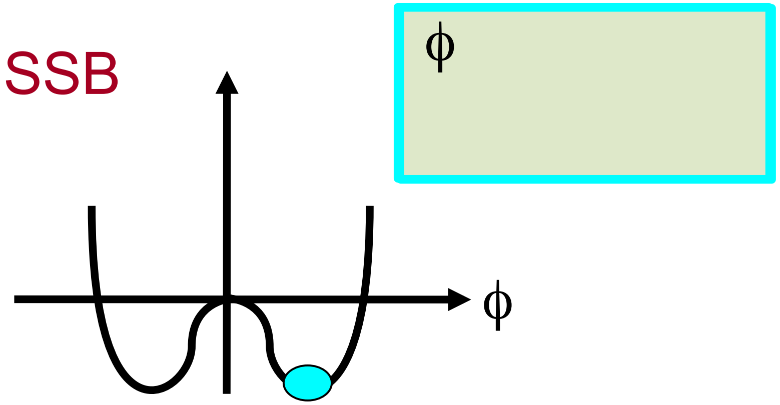
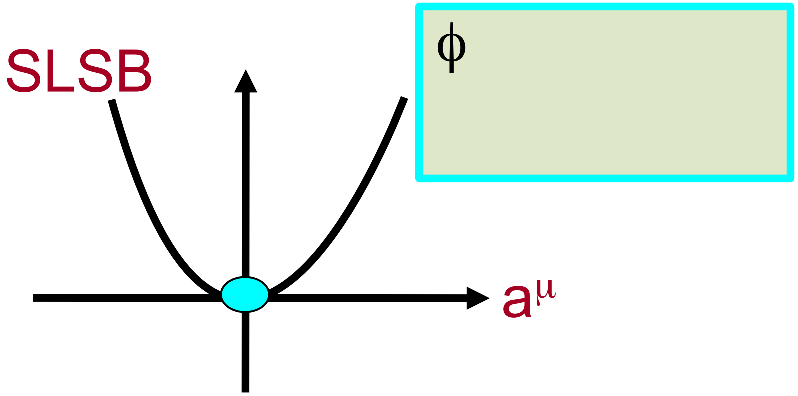
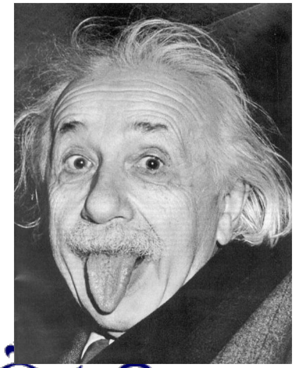
e.g.) SLSB in string field theory

- There are many Lorentz vector fields
- If any of vector field has Mexican hat potential

$$M(a^{\mu}) = \mu^2 < 0$$







Kostelecký and Samuel
PRD39(1989)683

1. Spontaneous Lorentz symmetry breaking (SLSB)

vacuum Lagrangian for fermion $L = i\bar{\Psi}\gamma_\mu\partial^\mu\Psi - m\bar{\Psi}\Psi + \bar{\Psi}\gamma_\mu a^\mu\Psi$

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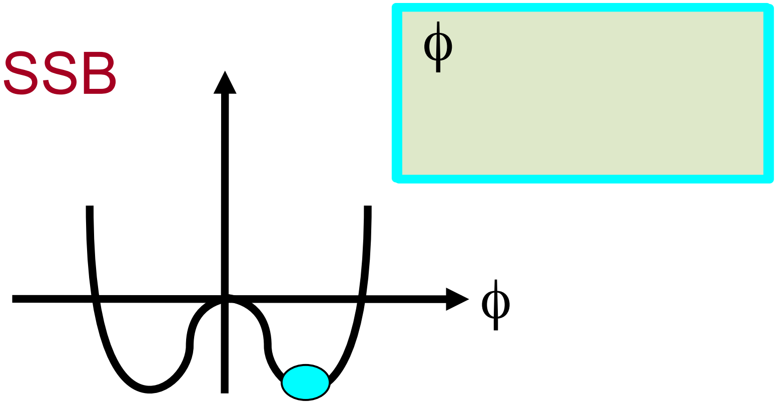

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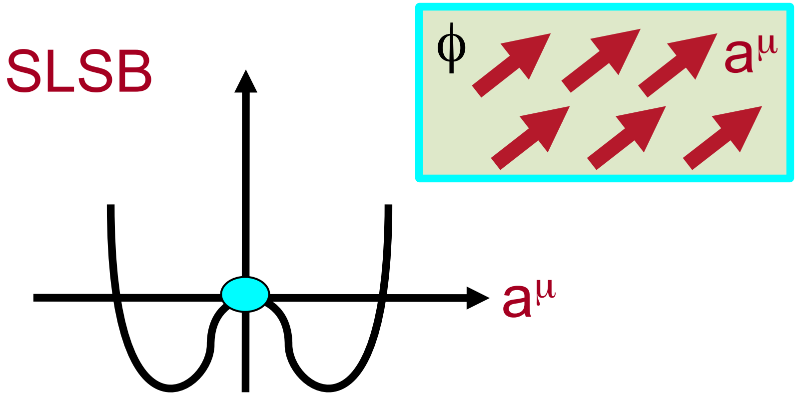
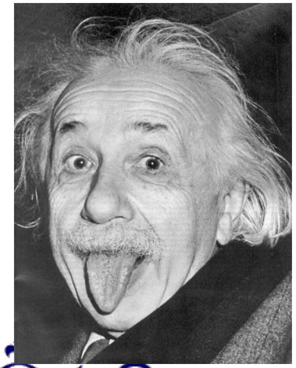




Lorentz symmetry is spontaneously broken!

1. Spontaneous Lorentz symmetry breaking

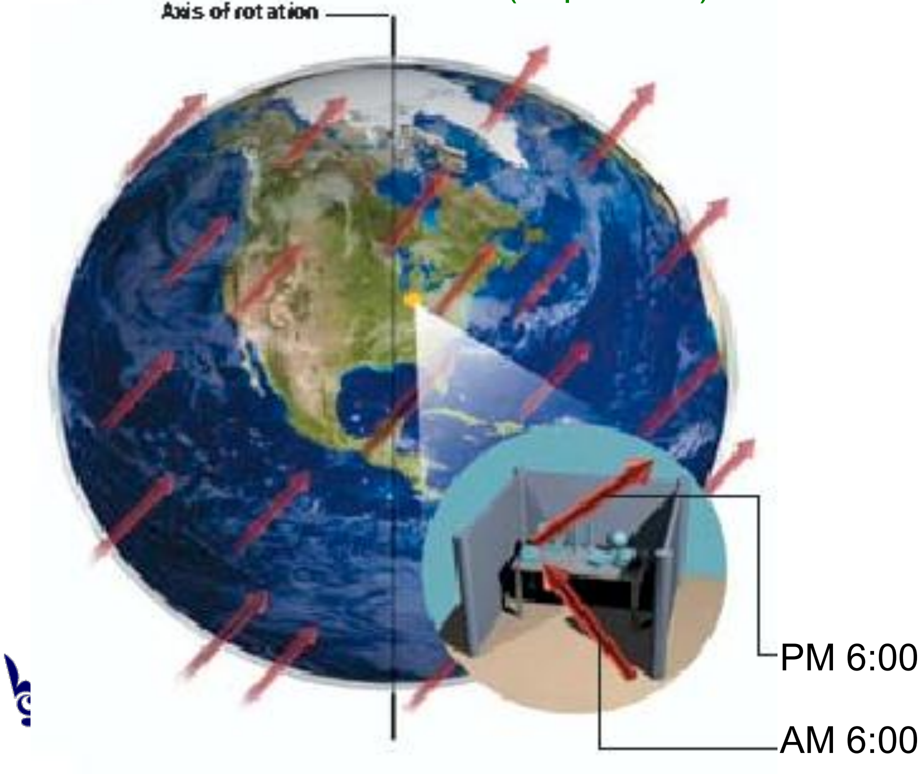
Test of Lorentz violation is to find the coupling of these background fields and ordinary fields (electrons, muons, neutrinos, etc); then the physical quantities may depend on the rotation of the earth (sidereal time dependence).

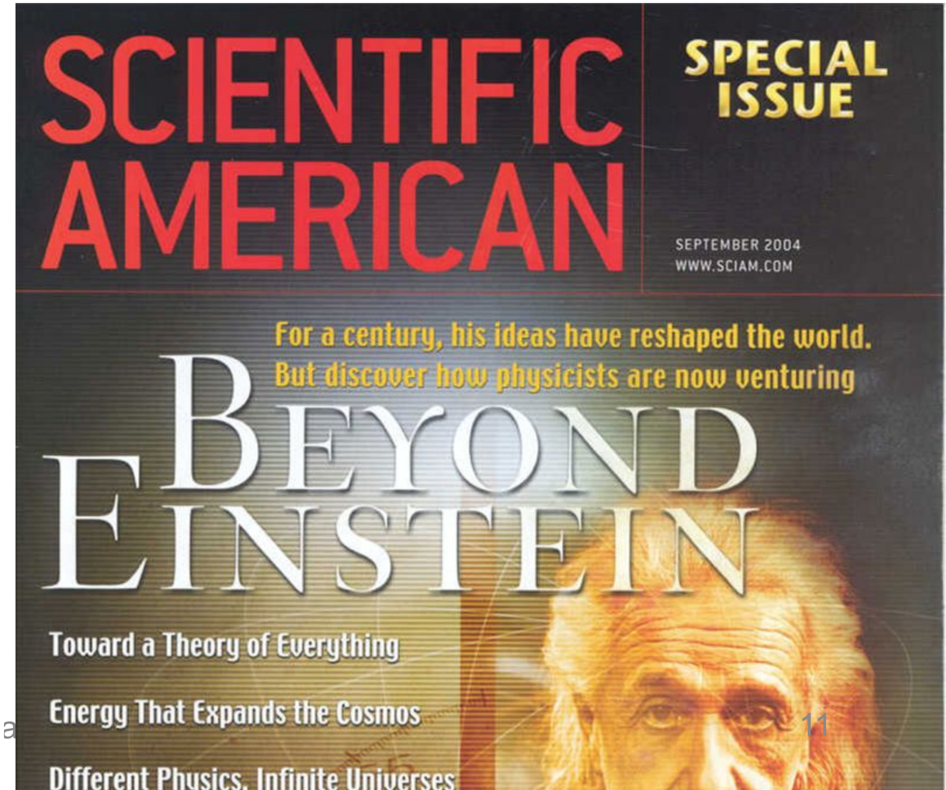
background fields of the universe

vacuum Lagrangian for fermion

$$L = i\bar{\Psi}\gamma_\mu\partial^\mu\Psi - m\bar{\Psi}\Psi + \bar{\Psi}\gamma_\mu\boxed{a^\mu}\Psi + \bar{\Psi}\gamma_\mu\boxed{c^{\mu\nu}}\partial_\nu\Psi\ldots$$

Scientific American (Sept. 2004)

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SPECIAL ISSUE

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Sidereal time dependence

The smoking gun of Lorentz violation is the sidereal time dependence of the observables.

Solar time: 24h 00m 00.0s
sidereal time: 23h 56m 04.1s

Sidereal time dependent physics is often smeared out in solar time distribution
→ Maybe we have some evidence of Lorentz violation but we just didn't notice?!

Target scale

Since it is Planck scale physics, either $>10^{19}$GeV or $<10^{-19}$GeV is the interesting region. $>10^{19}$GeV is not possible (LHC is 10^4GeV), but $<10^{-19}$GeV is possible.

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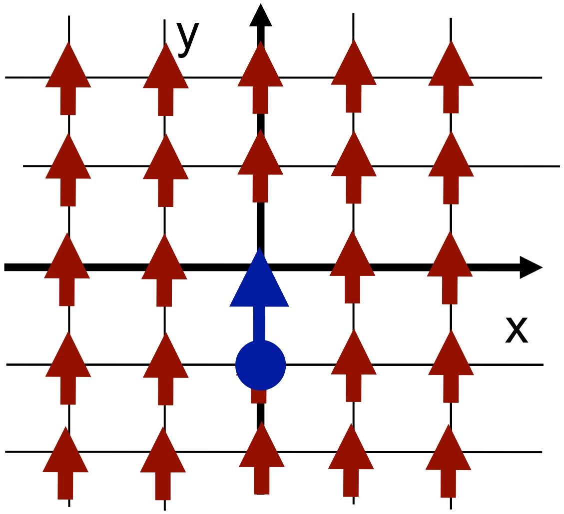
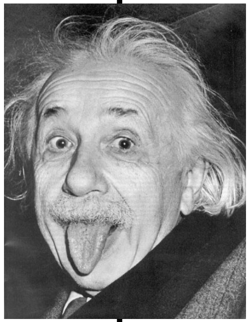
2. What is Lorentz violation?

$\overline{\Psi}(x)\gamma_\mu a^\mu \Psi(x)$



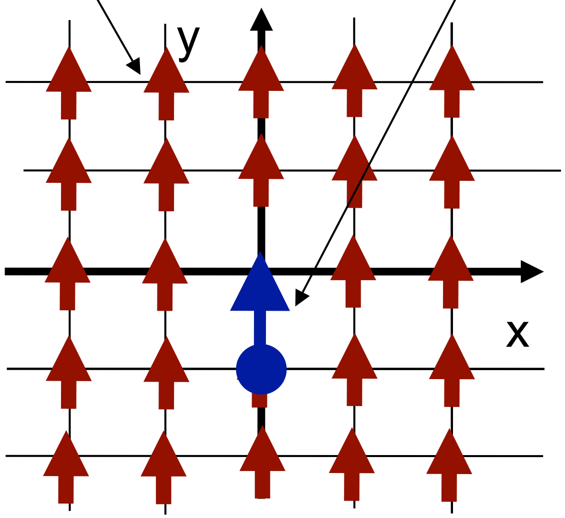


2. What is Lorentz violation?

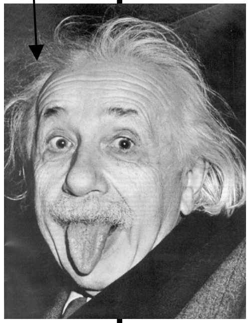
$$\overline{\Psi}(x)\gamma_{\mu}a^{\mu}\Psi(x)$$

hypothetical background vector field

moving particle



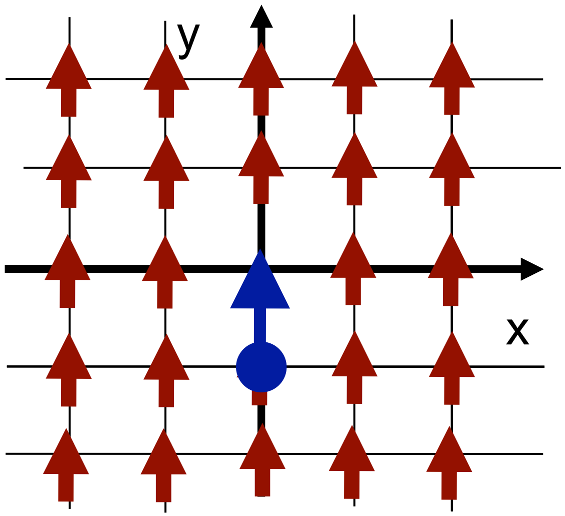
Einstein (observer)

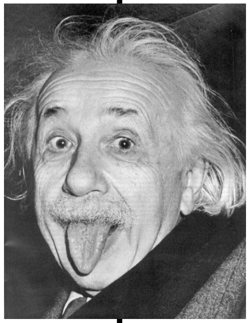


2. What is Lorentz violation?

Under the particle Lorentz transformation:

$$U\overline{\Psi}(x)\gamma_\mu a^\mu \Psi(x) U^{-1}$$





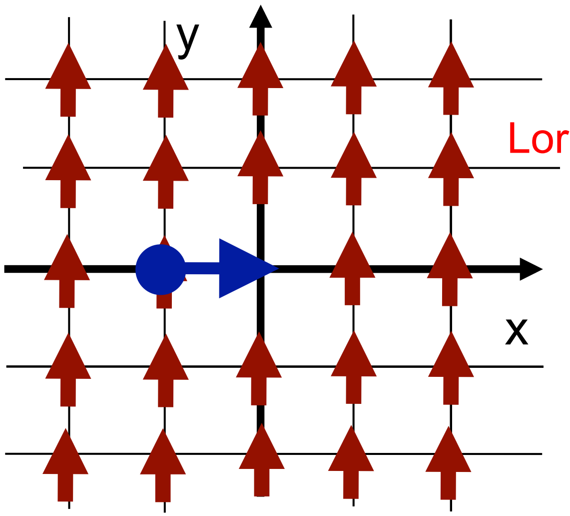
2. What is Lorentz violation?

Under the particle Lorentz transformation:

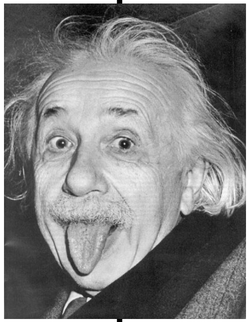
$$\overline{\Psi}(x)\gamma_\mu a^\mu \Psi(x) \rightarrow U[\overline{\Psi}(x)\gamma_\mu a^\mu \Psi(x)]U^{-1}$$

$$\neq \overline{\Psi}(\Lambda x)\gamma_\mu a^\mu \Psi(\Lambda x)$$

Lorentz violation is observable
when a particle is moving in the
fixed coordinate space



Lorentz violation!



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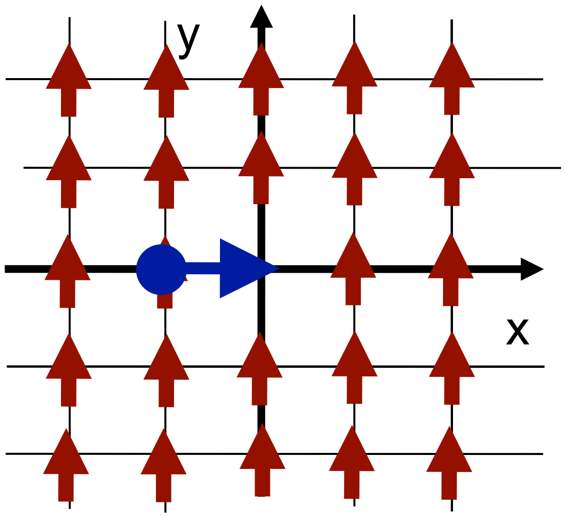
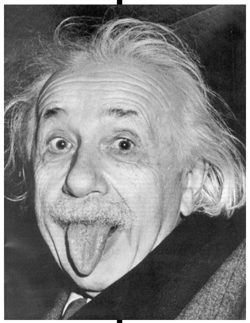
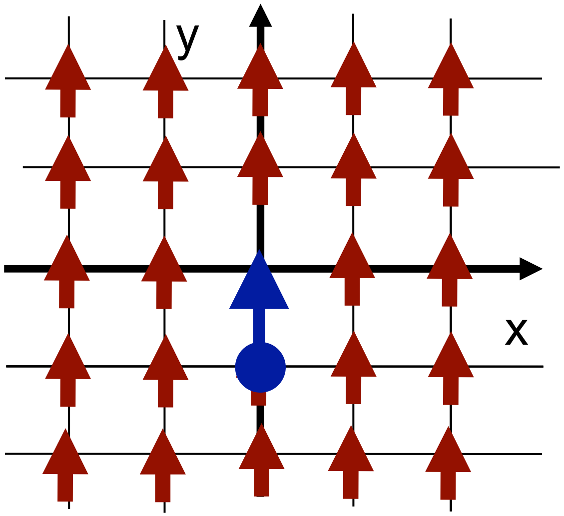




Under the observer Lorentz transformation:

$$\overline{\Psi}(x)\gamma_\mu a^\mu \Psi(x)$$



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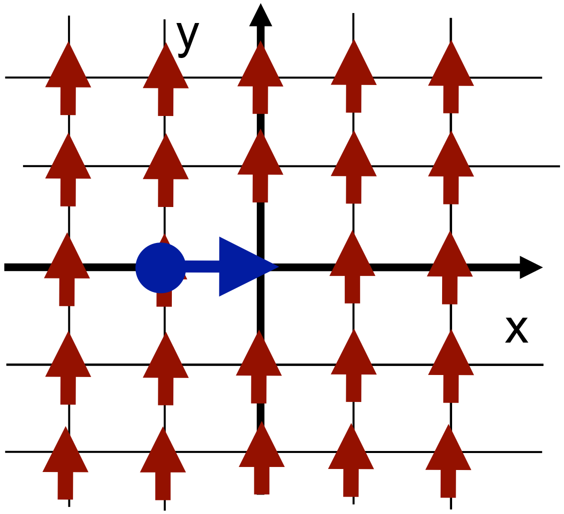
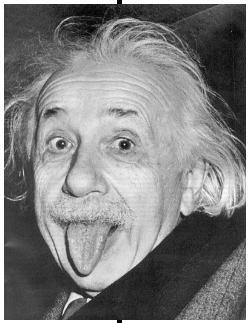
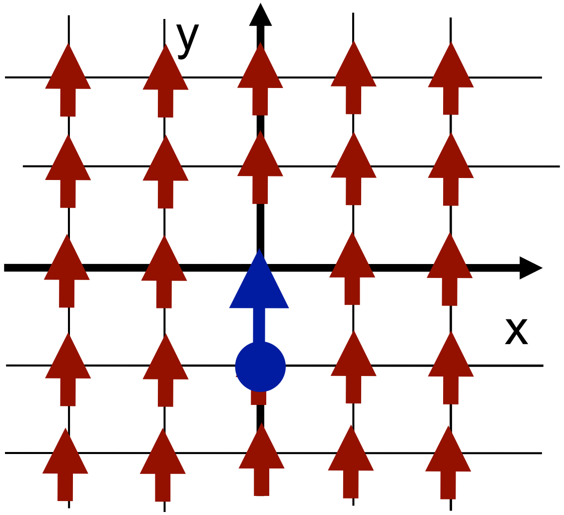


Under the observer Lorentz transformation:

$$\overline{\Psi}(x)\gamma_\mu a^\mu \Psi(x)$$

$$x \rightarrow \Lambda^{-1}x$$





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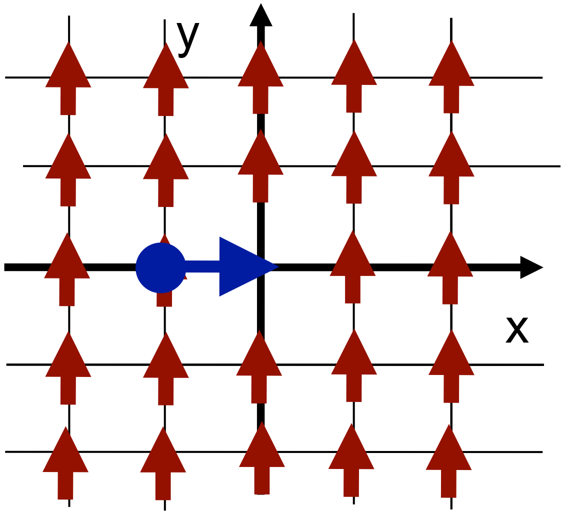
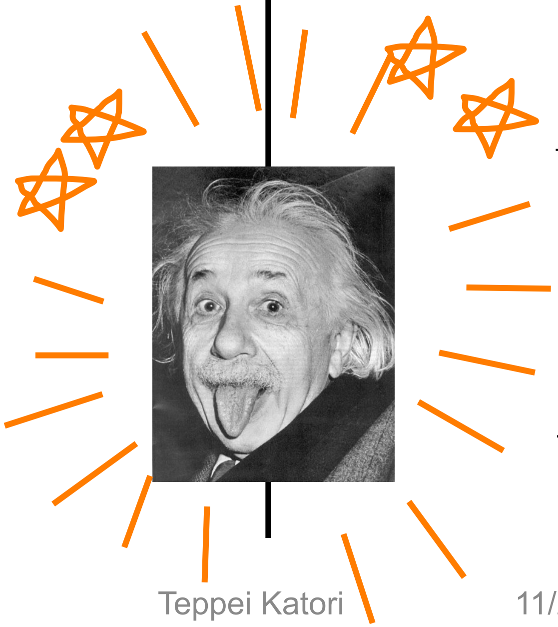


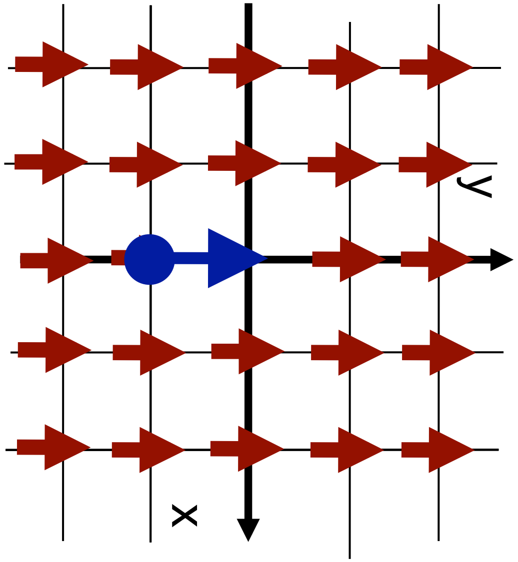
Under the observer Lorentz transformation:

$$\overline{\Psi}(x)\gamma_\mu a^\mu \Psi(x) \xrightarrow{\Lambda^{-1}} \overline{\Psi}(\Lambda^{-1}x)\gamma_\mu a^\mu \Psi(\Lambda^{-1}x)$$

Lorentz violation cannot be generated by observers motion (coordinate transformation is unbroken)

all observers agree for all observations



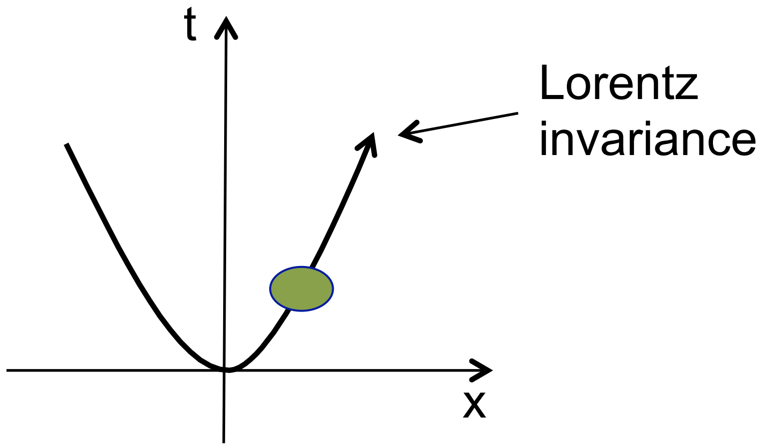


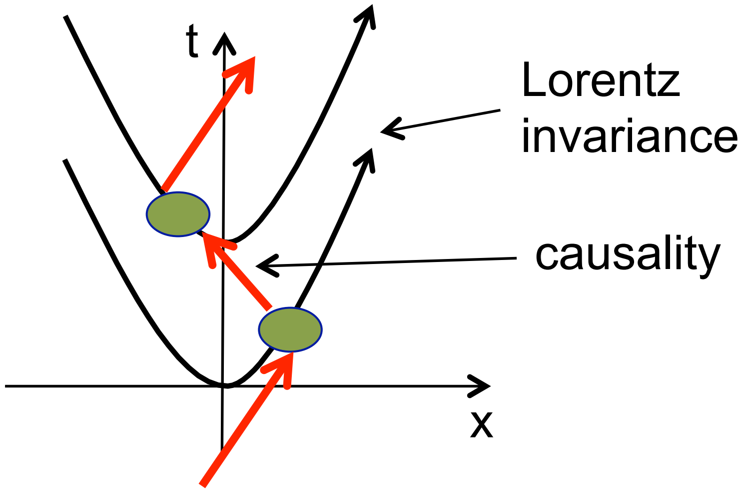
Greenberg, PRL(2002)231602

2. CPT violation implies Lorentz violation

Lorentz invariance → CPT → Lorentz invariance of quantum field theory

CPT violation implies Lorentz violation in interactive quantum field theory.





3. Test of Lorentz violation

Lorentz violation is realized as a coupling of particle fields and background fields, so the basic strategy to find Lorentz violation is:

(1) choose the coordinate system
(2) write down the Lagrangian, including Lorentz-violating terms under the formalism
(3) write down the observables using this Lagrangian

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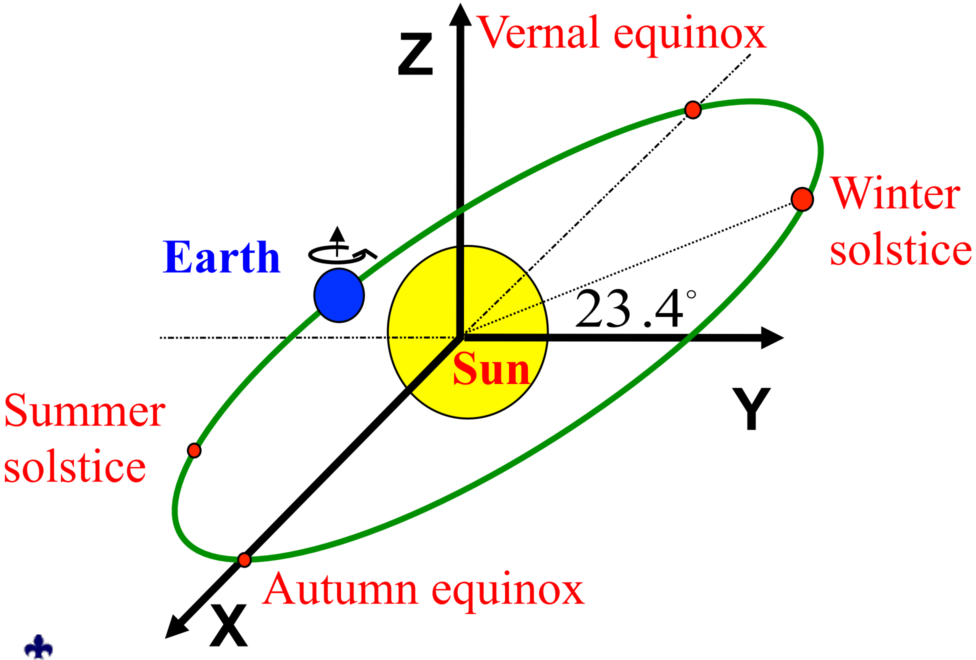
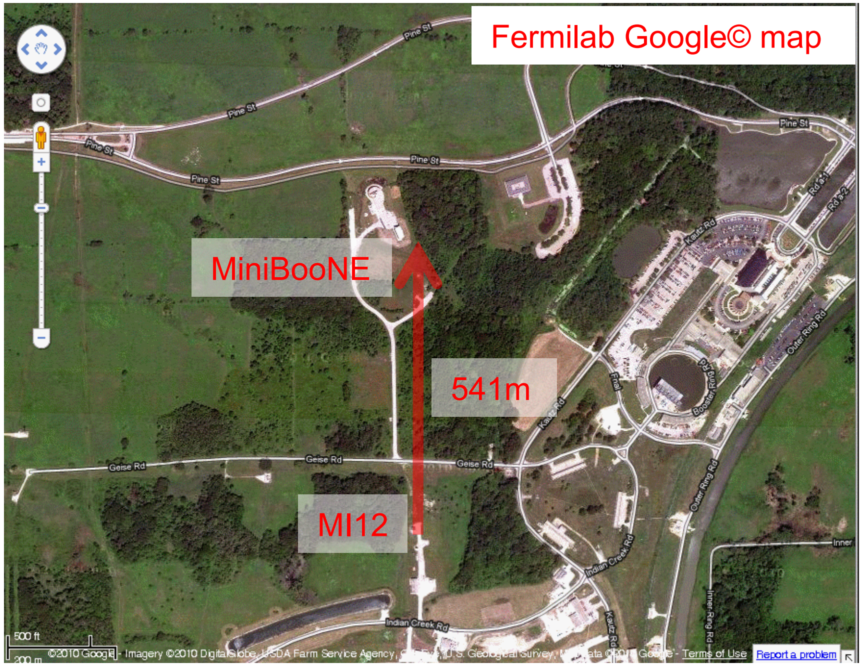
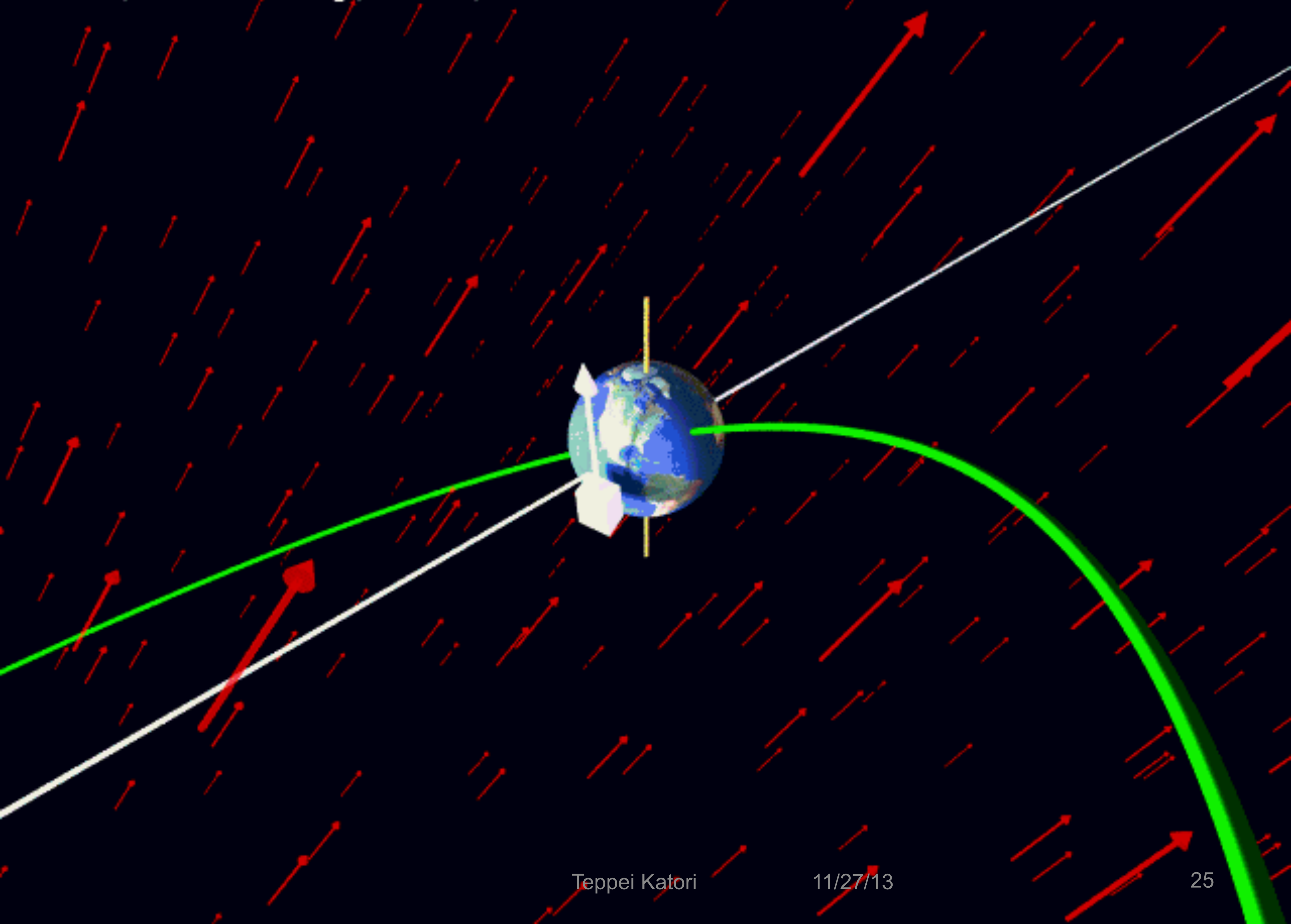
- Neutrino beamline is described in Sun-centred coordinates





MiniBooNE beamline

Bluhm, Kostelecky, Lane, Russell PRL 2002



Kostelecký and Mewes
PRD69(2004)016005

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Standard Model Extension (SME) is the standard formalism for the general search for Lorentz violation. SME is a minimum extension of QFT with Particle Lorentz violation

SME Lagrangian in neutrino sector

$$L = \frac{1}{2} i \bar{\psi}_A \Gamma^{\nu}_{AB} \partial_\nu \psi_B - M_{AB} \bar{\psi}_A \psi_B + h.c.$$

SME coefficients

$$\Gamma^{\nu}_{AB} = \gamma^{\nu} \delta_{AB} + c^{\mu\nu}_{AB} \gamma_\mu + d^{\mu\nu}_{AB} \gamma_\mu \gamma_5 + e^{\nu}_{AB} + i f^{\nu}_{AB} \gamma_5 + \frac{1}{2} g^{\lambda\mu\nu}_{AB} \sigma_{\lambda\mu} \cdots$$

$$M_{AB} = m_{AB} + i m_{5AB} \gamma_5 + a^{\mu}_{AB} \gamma_\mu + b^{\mu}_{AB} \gamma_5 \gamma_\mu + \frac{1}{2} H^{\mu\nu}_{AB} \sigma_{\mu\nu} \cdots$$

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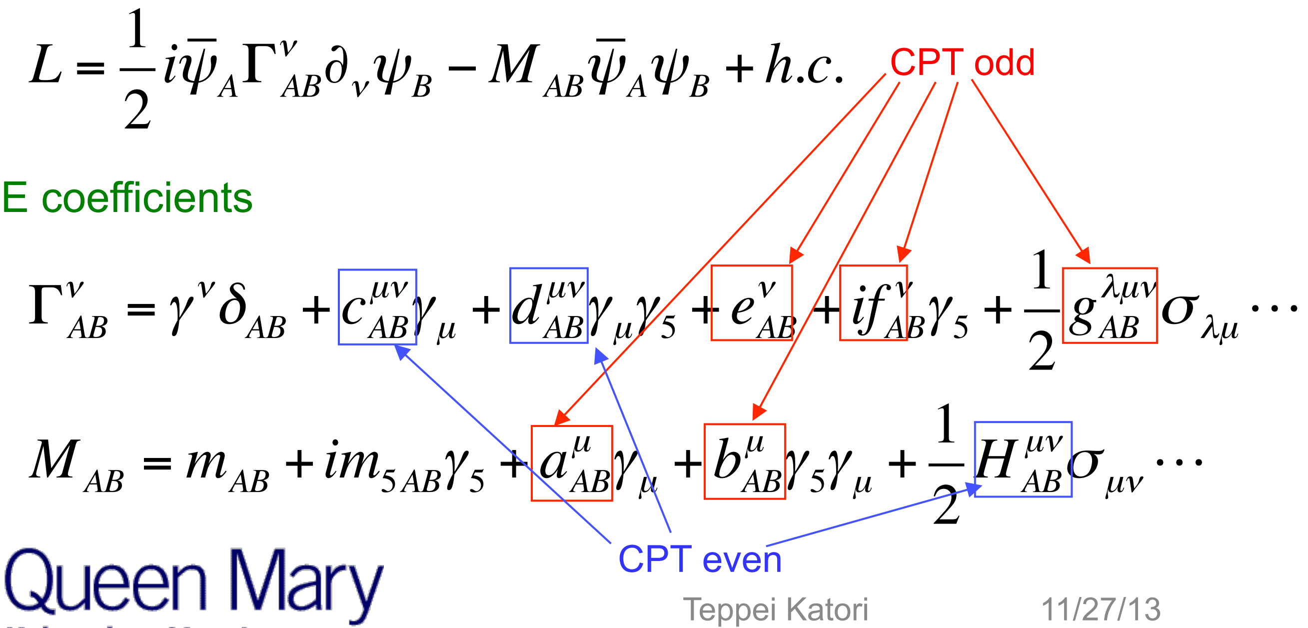

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CPT odd

SME coefficients

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CPT even

Kostelecký and Mewes
PRD70(2004)076002

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(3) write down the observables using this Lagrangian

Various physics are predicted under SME, but among them, the smoking gun of Lorentz violation is the sidereal time dependence of the observables

solar time: 24h 00m 00.0s
sidereal time: 23h 56m 04.1s

sidereal frequency $\omega_\oplus = \dfrac{2\pi}{23h56m4.1s}$

sidereal time $T_\oplus$

Lorentz-violating neutrino oscillation probability for short-baseline experiments

$$P_{\nu_\mu\to\nu_e} = \left(\frac{L}{\hbar c}\right)^2 \left|(C)_{e\mu} + (A_s)_{e\mu}\sin\omega_\oplus T_\oplus + (A_c)_{e\mu}\cos\omega_\oplus T_\oplus + (B_s)_{e\mu}\sin 2\omega_\oplus T_\oplus + (B_c)_{e\mu}\cos 2\omega_\oplus T_\oplus\right|^2$$

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PRD70(2004)076002

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Lorentz-violating neutrino oscillation probability for short-baseline experiments

time independent amplitude

sidereal time dependent amplitude

$$P_{\nu_\mu\to\nu_e} = \left(\frac{L}{\hbar c}\right)^2 \left|(C)_{e\mu} + (A_s)_{e\mu}\sin\omega_\oplus T_\oplus + (A_c)_{e\mu}\cos\omega_\oplus T_\oplus + (B_s)_{e\mu}\sin 2\omega_\oplus T_\oplus + (B_c)_{e\mu}\cos 2\omega_\oplus T_\oplus\right|^2$$

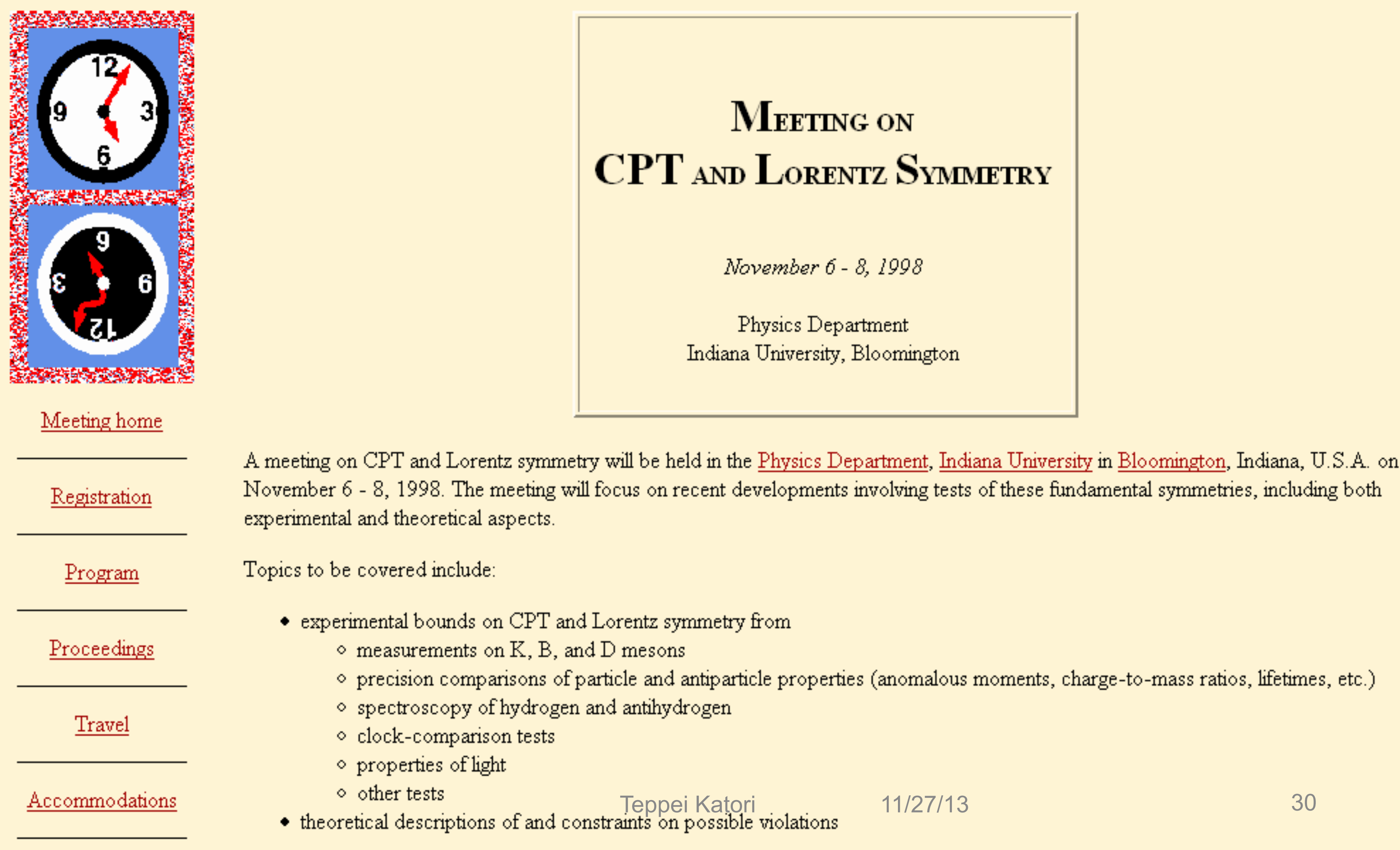
Sidereal variation analysis for short baseline neutrino oscillation is 5-parameter fitting problem

Queen Mary
University of London

3. Modern tests of Lorentz violation

Dedicated group of people formed a meeting since 1998.

http://www.physics.indiana.edu/~kostelec/faq.html



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MEETING ON
CPT AND LORENTZ SYMMETRY

November 6 - 8, 1998

Physics Department
Indiana University, Bloomington

A meeting on CPT and Lorentz symmetry will be held in the Physics Department, Indiana University in Bloomington, Indiana, U.S.A. on November 6 - 8, 1998. The meeting will focus on recent developments involving tests of these fundamental symmetries, including both experimental and theoretical aspects.

Topics to be covered include:

- experimental bounds on CPT and Lorentz symmetry from
    - measurements on K, B, and D mesons
    - precision comparisons of particle and antiparticle properties (anomalous moments, charge-to-mass ratios, lifetimes, etc.)
    - spectroscopy of hydrogen and antihydrogen
    - clock-comparison tests
    - properties of light
    - other tests
- theoretical descriptions of and constraints on possible violations

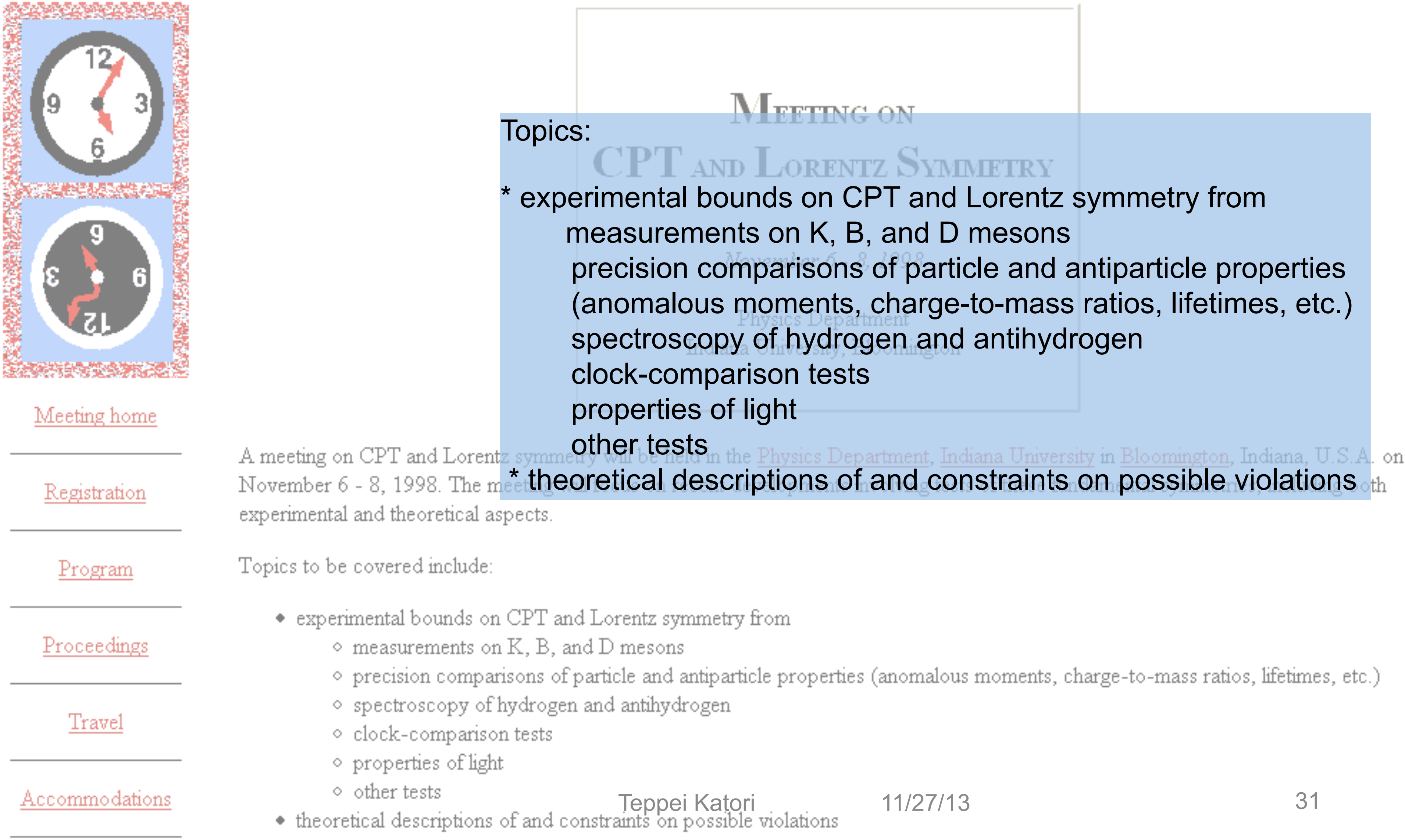
3. Modern tests of Lorentz violation

http://www.physics.indiana.edu/~kostelec/faq.html

Topics:

* experimental bounds on CPT and Lorentz symmetry from
  measurements on K, B, and D mesons
  precision comparisons of particle and antiparticle properties (anomalous moments, charge-to-mass ratios, lifetimes, etc.)
  spectroscopy of hydrogen and antihydrogen
  clock-comparison tests
  properties of light
  other tests

* theoretical descriptions of and constraints on possible violations



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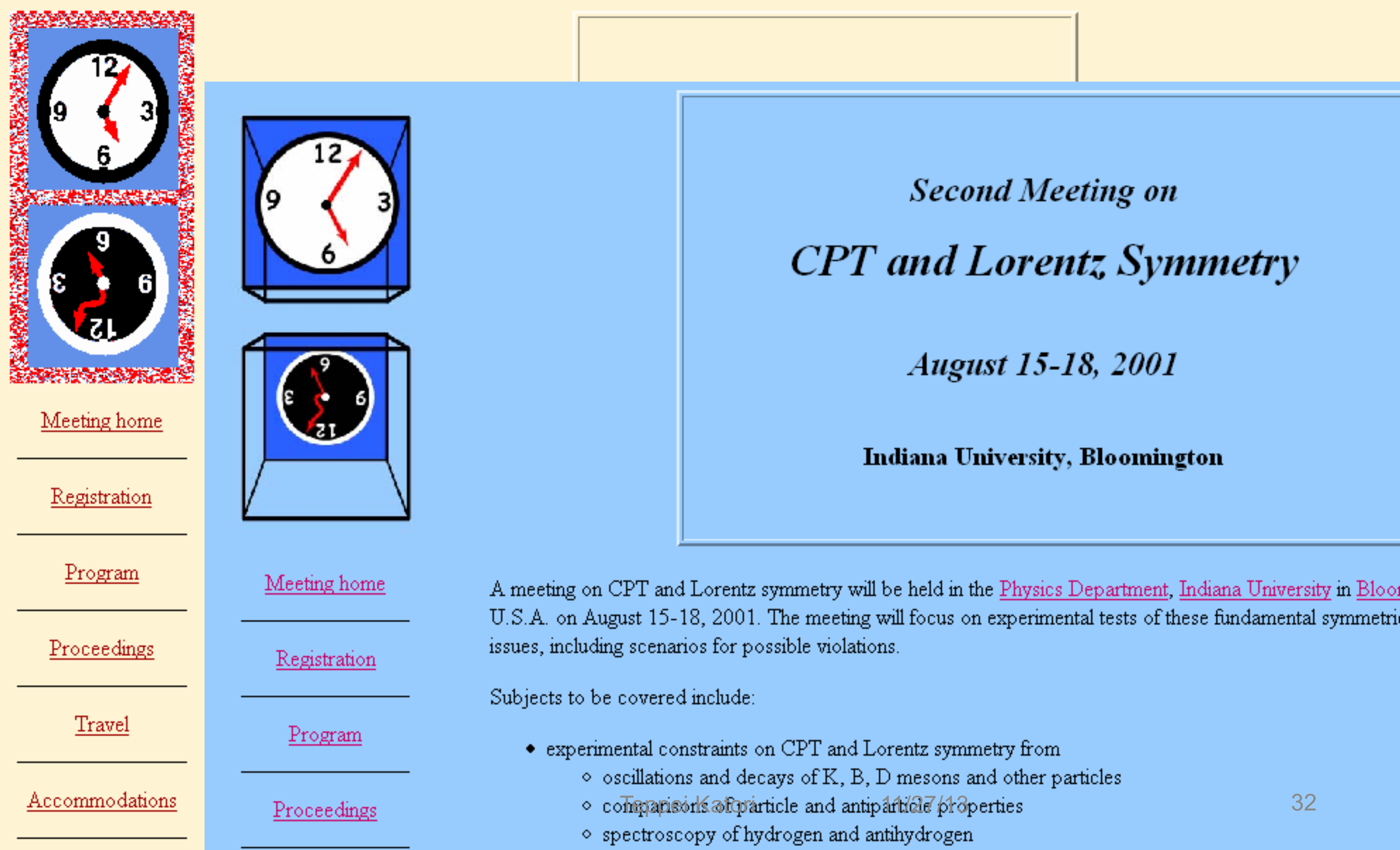
  - clock-comparison tests
  - properties of light
  - other tests
- theoretical descriptions of and constraints on possible violations

3. Modern tests of Lorentz violation

The second meeting was in 2001.

http://www.physics.indiana.edu/~kostelec/faq.html



Second Meeting on

CPT and Lorentz Symmetry

August 15-18, 2001

Indiana University, Bloomington

A meeting on CPT and Lorentz symmetry will be held in the Physics Department, Indiana University in Bloo
U.S.A. on August 15-18, 2001. The meeting will focus on experimental tests of these fundamental symmetri
issues, including scenarios for possible violations.

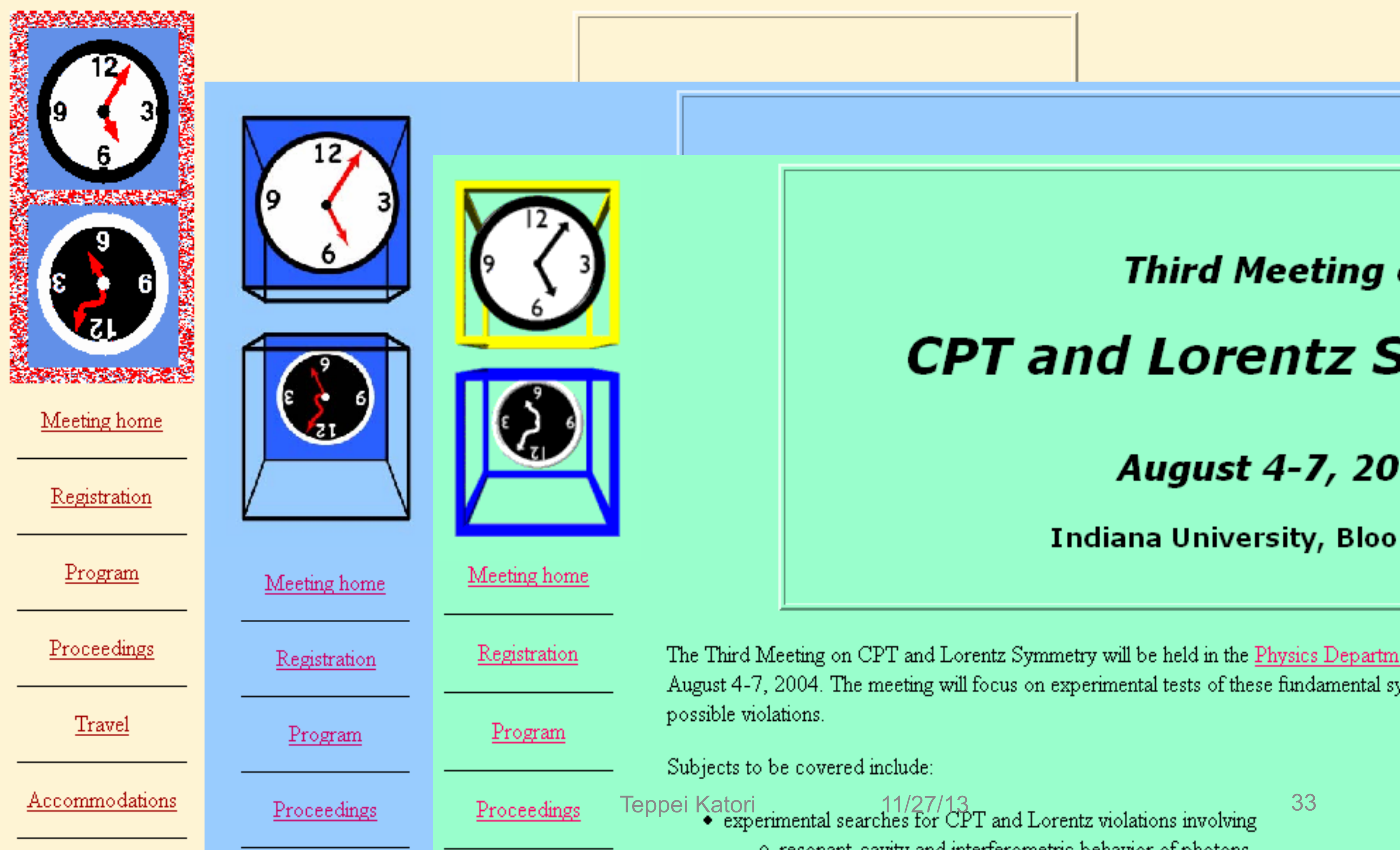
Subjects to be covered include:

- experimental constraints on CPT and Lorentz symmetry from
  - oscillations and decays of K, B, D mesons and other particles
  - comparisons of particle and antiparticle properties
  - spectroscopy of hydrogen and antihydrogen

3. Modern tests of Lorentz violation

The third meeting was in 2004.

http://www.physics.indiana.edu/~kostelec/faq.html



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Third Meeting

CPT and Lorentz S

August 4-7, 20

Indiana University, Bloo

The Third Meeting on CPT and Lorentz Symmetry will be held in the Physics Departm August 4-7, 2004. The meeting will focus on experimental tests of these fundamental sy possible violations.

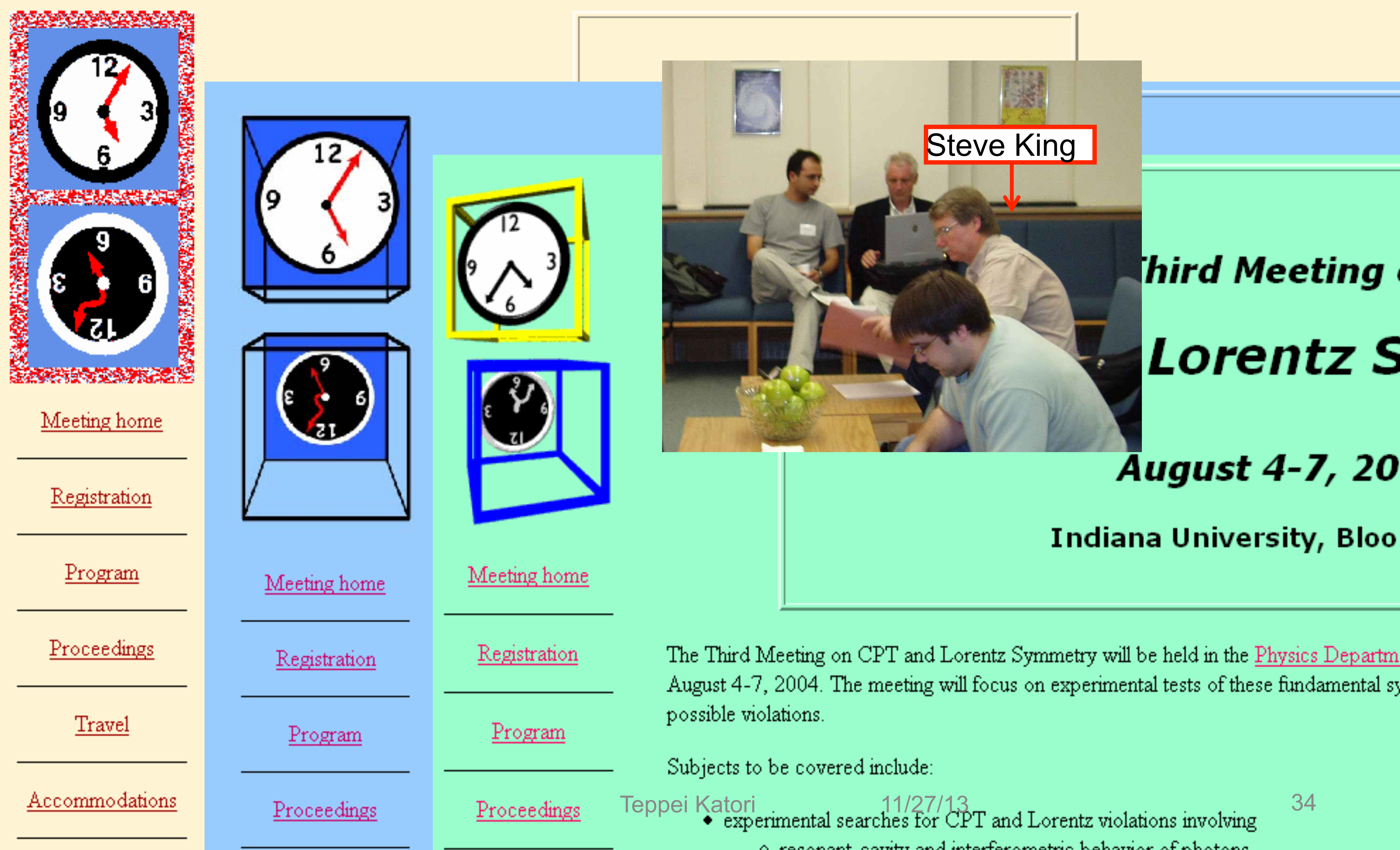
Subjects to be covered include:

- experimental searches for CPT and Lorentz violations involving
  - resonant cavity and interferometric behavior of photons

3. Modern tests of Lorentz violation

The third meeting was in 2004.

http://www.physics.indiana.edu/~kostelec/faq.html



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Lorentz S

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Subjects to be covered include:

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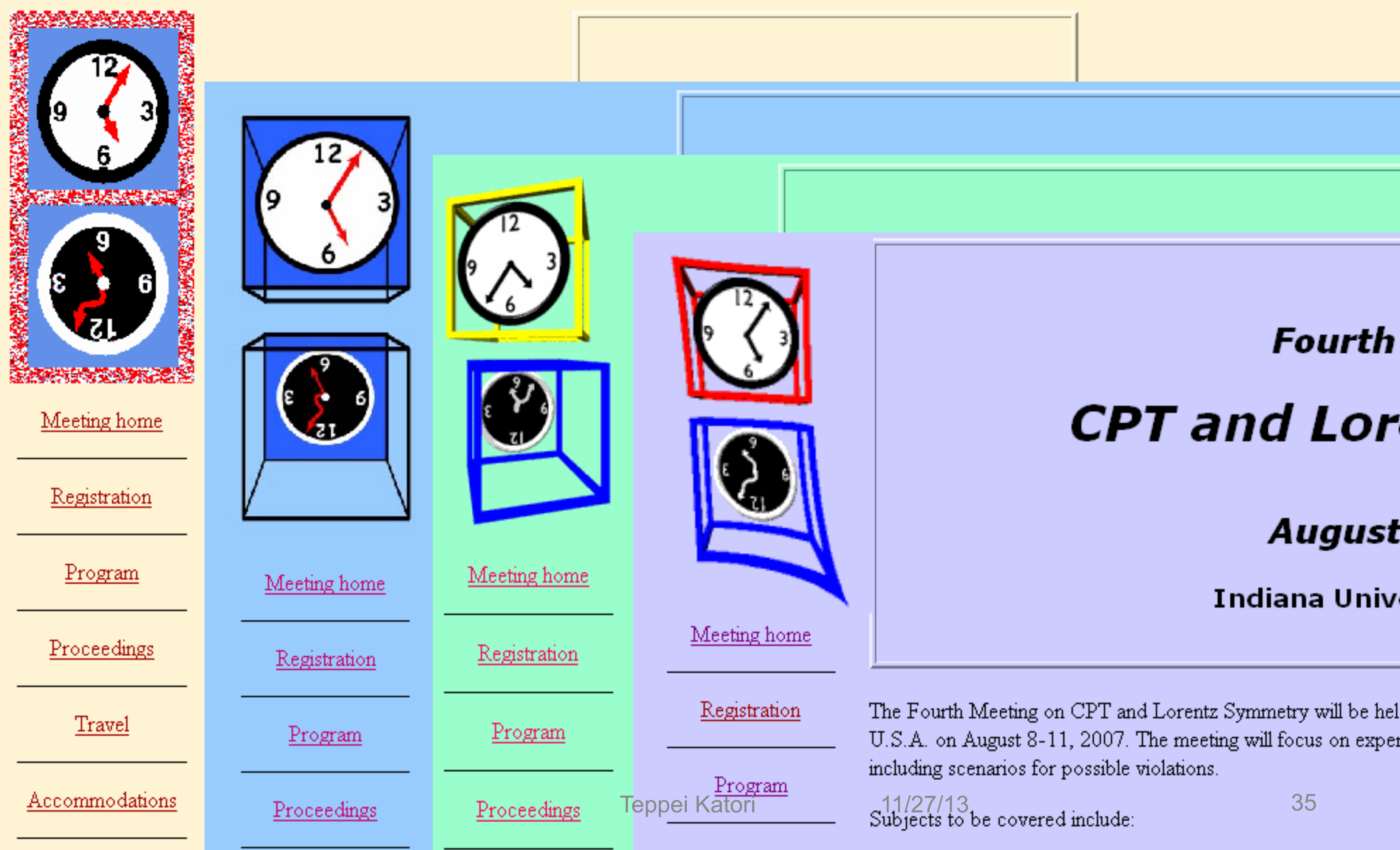
  - resonant cavity and interferometric behavior of photons

3. Modern tests of Lorentz violation

The fourth
meeting was
in 2007.

http://www.physics.indiana.edu/~kostelec/faq.html



Fourth

CPT and Lor

August

Indiana Univ

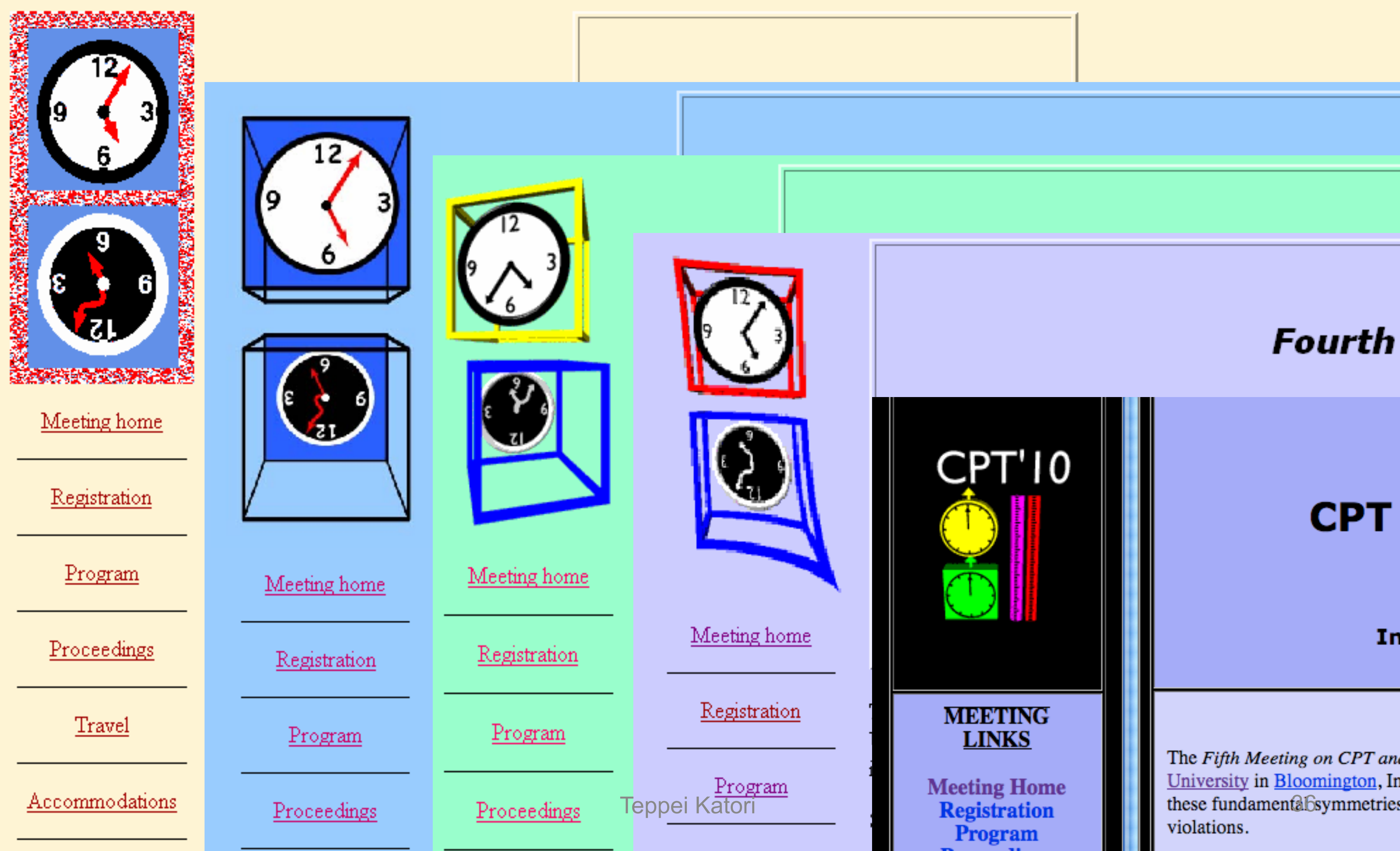
The Fourth Meeting on CPT and Lorentz Symmetry will be hel
U.S.A. on August 8-11, 2007. The meeting will focus on exper
including scenarios for possible violations.

Subjects to be covered include:

3. Modern tests of Lorentz violation

The fifth meeting was in 2010.

http://www.physics.indiana.edu/~kostelec/faq.html

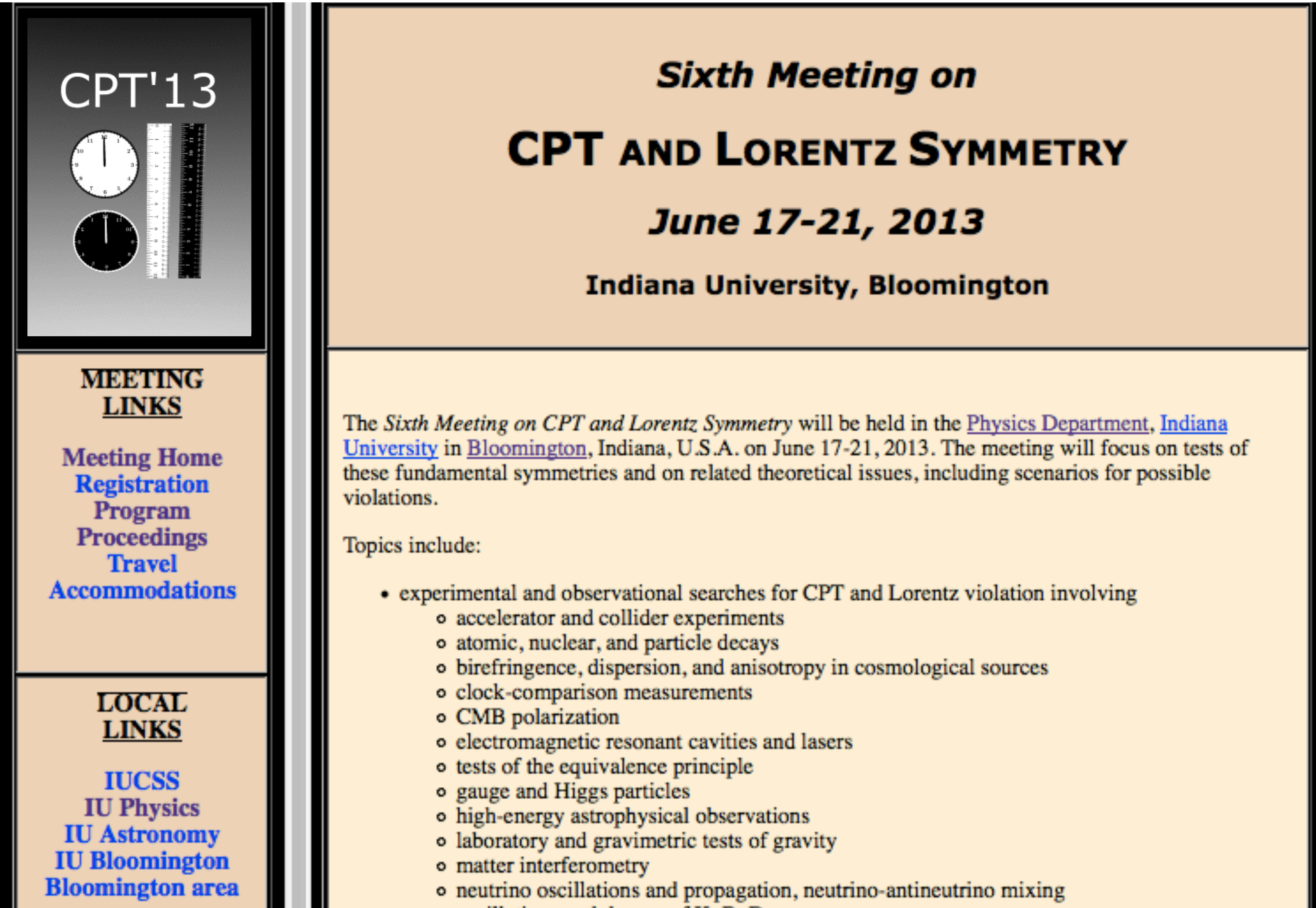


The *Fifth Meeting on CPT an*
University in Bloomington, In
these fundamental symmetries
violations.

3. Modern tests of Lorentz violation

The latest meeting was in June 2013

http://www.physics.indiana.edu/~kostelec/faq.html



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LOCAL LINKS

IUCSS
IU Physics
IU Astronomy
IU Bloomington
Bloomington area

Sixth Meeting on

CPT AND LORENTZ SYMMETRY

June 17-21, 2013

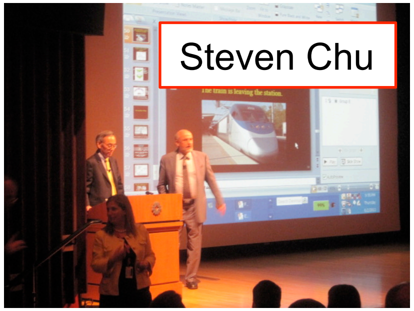
Indiana University, Bloomington

The *Sixth Meeting on CPT and Lorentz Symmetry* will be held in the Physics Department, Indiana University in Bloomington, Indiana, U.S.A. on June 17-21, 2013. The meeting will focus on tests of these fundamental symmetries and on related theoretical issues, including scenarios for possible violations.

Topics include:

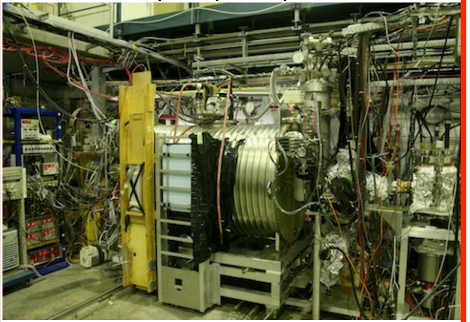
- experimental and observational searches for CPT and Lorentz violation involving
    - accelerator and collider experiments
    - atomic, nuclear, and particle decays
    - birefringence, dispersion, and anisotropy in cosmological sources
    - clock-comparison measurements
    - CMB polarization
    - electromagnetic resonant cavities and lasers
    - tests of the equivalence principle
    - gauge and Higgs particles
    - high-energy astrophysical observations
    - laboratory and gravimetric tests of gravity
    - matter interferometry
    - neutrino oscillations and propagation, neutrino-antineutrino mixing

Atomic Interferometer
$(a,c)^{n,p,e} < 10^{-6}$



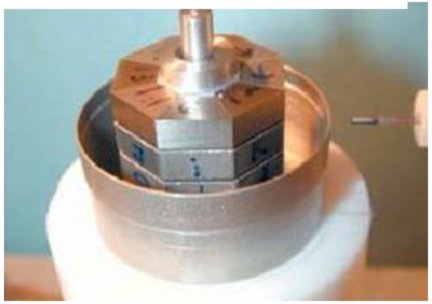
PRL106(2011)151102

CERN Antiproton Decelerat
$(M_p-\overline{M}_p)/M_p<10^{-8}$



Nature419(2002)456

Spin torsion pendulum
$b_e<10^{-30}$ GeV



PRL97(2006)

Tevatron and LEP
$-5.8\times10^{-12}<\kappa_{tr}-4/3c_e^{00}<1.2\times10^{-11}$

02(2009)170402

- clock-comparison measurements

CMB polarization

- elec ers
- tests

gaug

high

labo

matter interferometry

- neutrino oscillations and propagation, neutrino-a
- oscillations and decays of K, B, D mesons

Limits from all these experiments >50 page tables!
Rev.Mod.Phys.83(2011)11
ArXiv:0801.0287v6

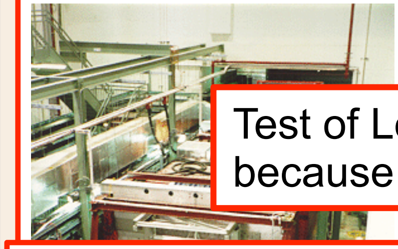
Neutrino TOF
$(v-c)/c<10^{-5}$

PRD76(2007)072005
JHEP11(2012)049

acuum birefringence
$\kappa_{e+}, \kappa_{o-}<10^{-37}$

2006)140401

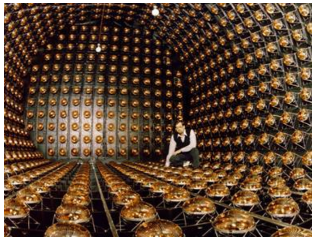
KTeV/KLOE (strange)
$\Delta a_K<10^{-22}$ GeV
FOCUS (charm)
$\Delta a_D<10^{-16}$ GeV
BaBar/Belle (bottom)
$\Delta m_B/m_B<10^{-14}$



uble gas maser
$\text{tation})<10^{-33}$GeV
$\text{oost})<10^{-27}$GeV

Test of Lorentz invariance with neutrino oscillation is very interesting, because neutrinos are the least known standard model particles!

LSND

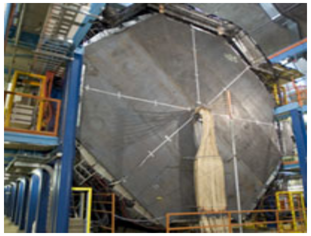


PRD72(2005)076004

MINOS ND

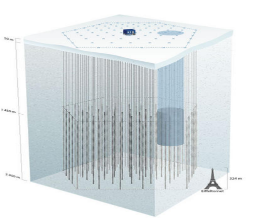
PRL101(2008)151601

MINOS FD



PRL105(2010)151601

IceCube

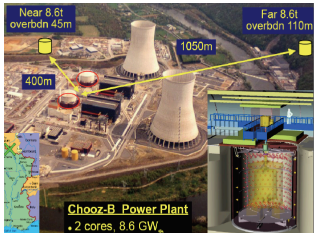


PRD82(2010)112003

MiniBooNE

PLB718(2013)1303

Double Chooz



PRD86(2013)112009

PRL100(2008)131802

mathematical foundations, Fin

PRL99(2007)050401

PRL107(2010)171604

1. Spontaneous Lorentz symmetry breaking
2. What is Lorentz and CPT violation?
3. Modern test of Lorentz violation
4. **Test of Lorenz violation with MiniBooNE**
5. Test of Lorentz violation with Double Chooz
6. Conclusion

MiniBooNE collaboration,
PRD79(2009)072002
NIM.A599(2009)28

4. MiniBooNE experiment

MiniBooNE is a short-baseline neutrino oscillation experiment at Fermilab.

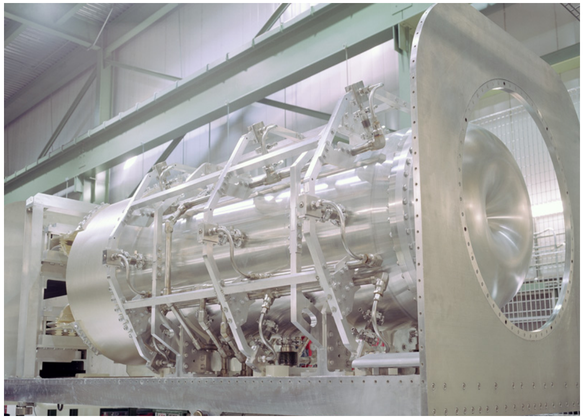
$$\nu_\mu \xrightarrow{oscillation} \nu_e + n \rightarrow e^- + p$$

$$\bar{\nu}_\mu \xrightarrow{oscillation} \bar{\nu}_e + p \rightarrow e^+ + n$$

Booster Neutrino Beamline (BNB) creates ~800(600)MeV neutrino(anti-neutrino) by pion decay-in-flight. Cherenkov radiation from the charged leptons are observed by MiniBooNE Cherenkov detector to reconstruct neutrino energy.

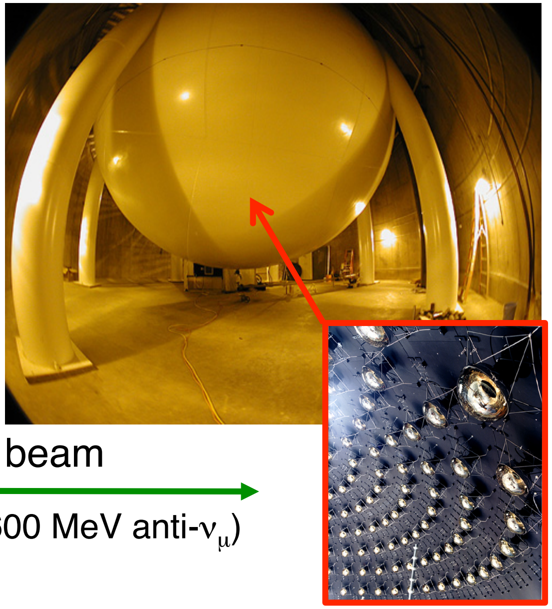
FNAL Booster

Magnetic focusing horn



~520m →

MiniBooNE detector



1280 of 8” PMT

primary beam

(8 GeV protons)

secondary beam

(1-2 GeV pions)

tertiary beam

(800 MeV ν_μ , 600 MeV anti-ν_μ)

MiniBooNE collaboration,
PRL102(2009)101802,
PRL105(2010)181801

4. MiniBooNE experiment

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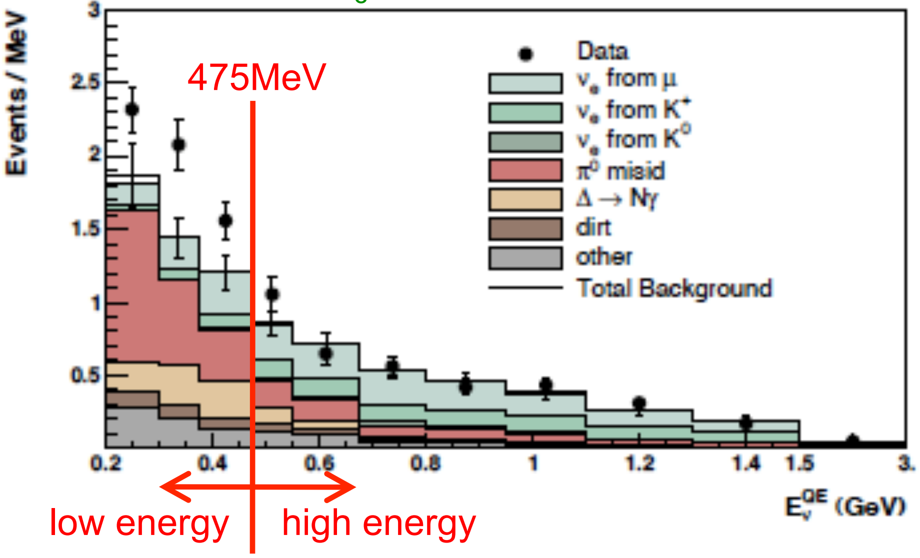
$$\nu_\mu \xrightarrow{oscillation} \nu_e + n \rightarrow e^- + p$$

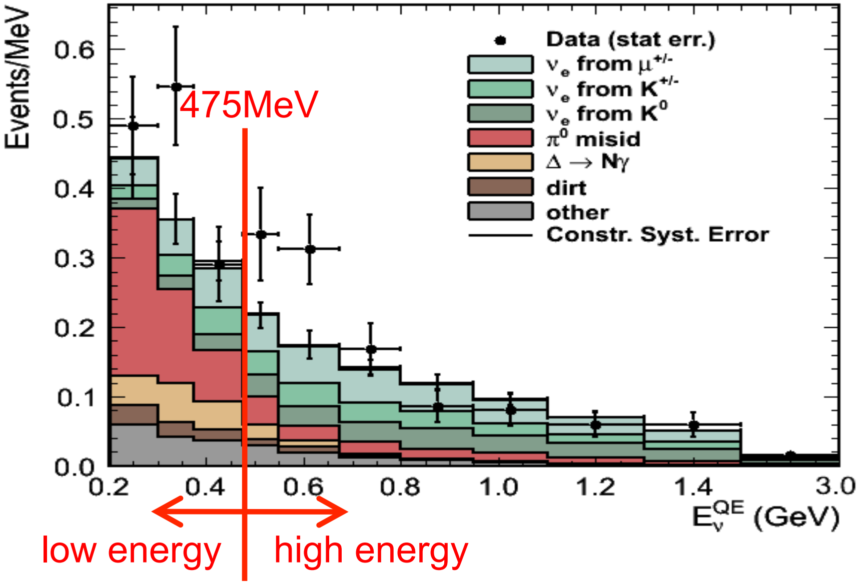
$$\bar{\nu}_\mu \xrightarrow{oscillation} \bar{\nu}_e + p \rightarrow e^+ + n$$

Neutrino mode analysis: MiniBooNE saw the 3.0σ excess at low energy region
Antineutrino mode analysis: MiniBooNE saw the 1.4σ excess at low and high energy region

MiniBooNE low E ν_e excess



MiniBooNE anti-ν_e excess



MiniBooNE collaboration,
PLB718(2013)1303

4. Lorentz violation with MiniBooNE neutrino data

MiniBooNE is a short-baseline neutrino oscillation experiment at Fermilab.

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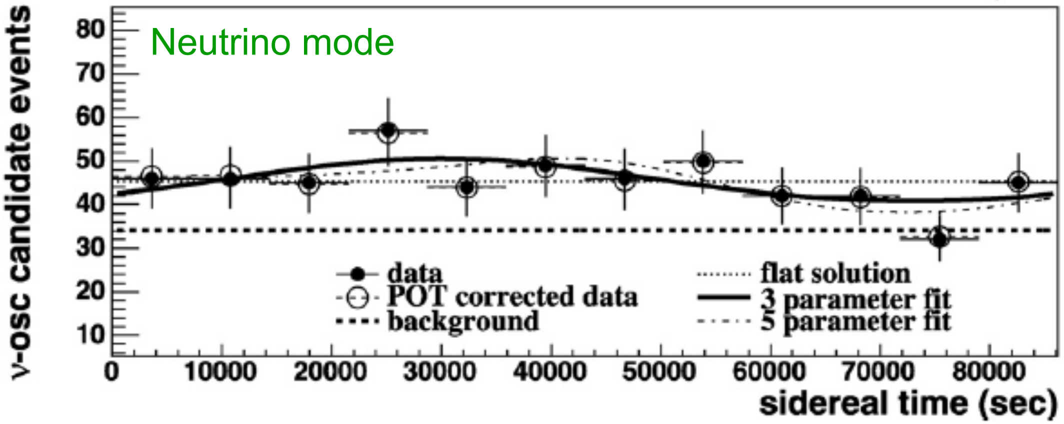
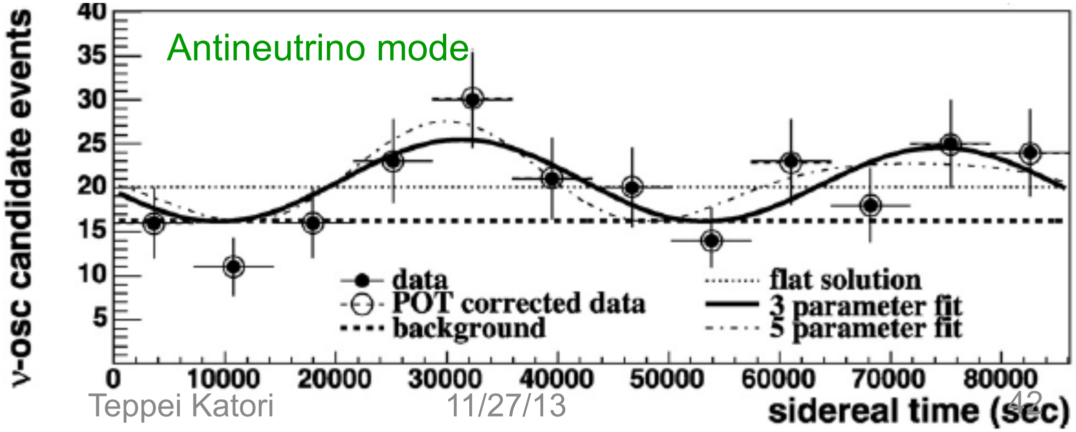
Neutrino mode analysis: MiniBooNE saw the 3.0σ excess at low energy region
Antineutrino mode analysis: MiniBooNE saw the 1.4σ excess at low and high energy region

Electron neutrino candidate data prefer sidereal time independent solution (flat)



Electron antineutrino candidate data prefer sidereal time dependent solution, however statistical significance is marginal

We find no evidence of Lorentz violation



MiniBooNE collaboration,
PLB718(2013)1303

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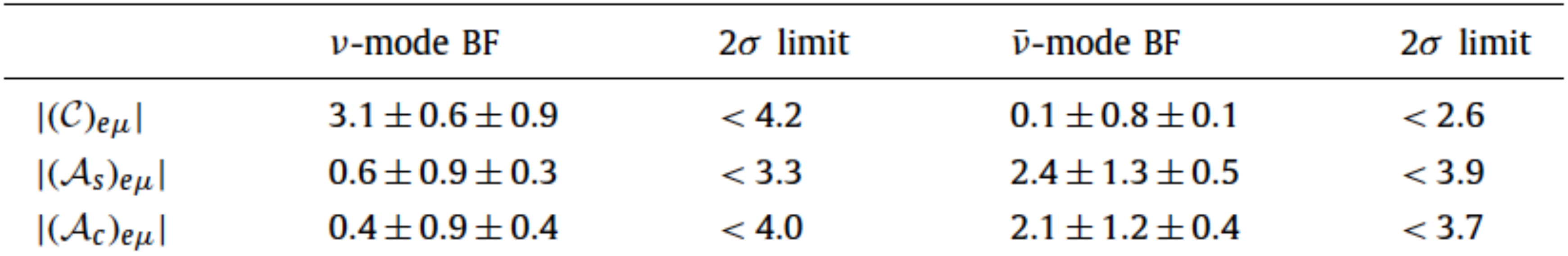
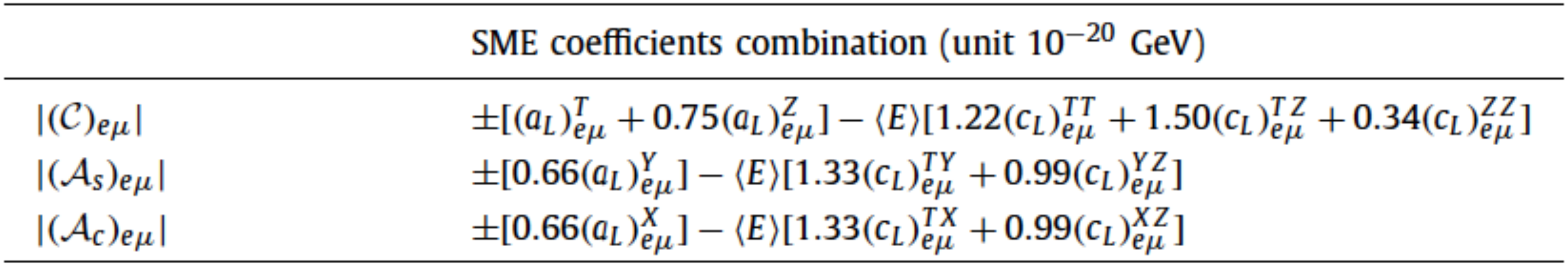
Neutrino mode analysis: MiniBooNE saw the 3.0σ excess at low energy region
Antineutrino mode analysis: MiniBooNE saw the 1.4σ excess at low and high energy region

Since we find no evidence of Lorentz violation, we set limits on the combination SME coefficients.



	ν-mode BF	2σ limit	$\bar{\nu}$-mode BF	2σ limit
$\lvert(\mathcal{C})_{e\mu}\rvert$	$3.1 \pm 0.6 \pm 0.9$	< 4.2	$0.1 \pm 0.8 \pm 0.1$	< 2.6
$\lvert(\mathcal{A}_s)_{e\mu}\rvert$	$0.6 \pm 0.9 \pm 0.3$	< 3.3	$2.4 \pm 1.3 \pm 0.5$	< 3.9
$\lvert(\mathcal{A}_c)_{e\mu}\rvert$	$0.4 \pm 0.9 \pm 0.4$	< 4.0	$2.1 \pm 1.2 \pm 0.4$	< 3.7



	SME coefficients combination (unit 10^{-20} GeV)
$\lvert(\mathcal{C})_{e\mu}\rvert$	$\pm[(a_L)^T_{e\mu} + 0.75(a_L)^Z_{e\mu}] - \langle E\rangle[1.22(c_L)^{TT}_{e\mu} + 1.50(c_L)^{TZ}_{e\mu} + 0.34(c_L)^{ZZ}_{e\mu}]$
$\lvert(\mathcal{A}_s)_{e\mu}\rvert$	$\pm[0.66(a_L)^Y_{e\mu}] - \langle E\rangle[1.33(c_L)^{TY}_{e\mu} + 0.99(c_L)^{YZ}_{e\mu}]$
$\lvert(\mathcal{A}_c)_{e\mu}\rvert$	$\pm[0.66(a_L)^X_{e\mu}] - \langle E\rangle[1.33(c_L)^{TX}_{e\mu} + 0.99(c_L)^{XZ}_{e\mu}]$

LSND collaboration,
PRD72(2005)076004

4. Summary of results

LSND experiment

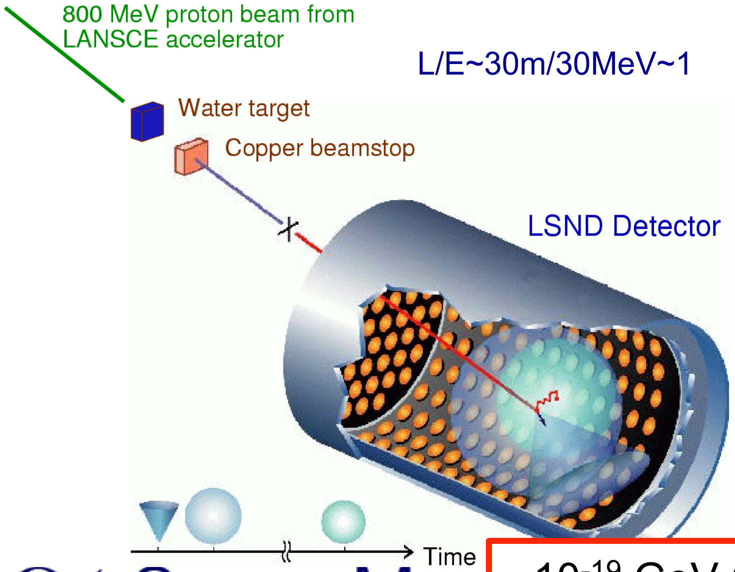
LSND is a short-baseline neutrino oscillation experiment at Los Alamos.

$$\bar{\nu}_\mu \xrightarrow{oscillation} \bar{\nu}_e + p \rightarrow e^+ + n$$

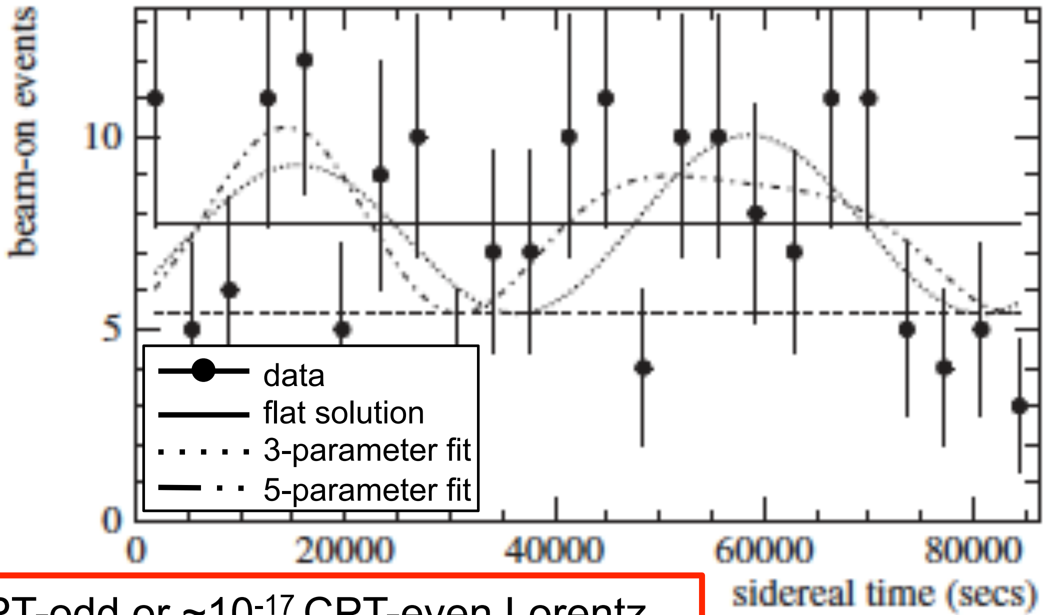
$$n + p \rightarrow d + \gamma$$

LSND saw the 3.8σ excess of electron antineutrinos from muon antineutrino beam; since this excess is not understood by neutrino Standard Model, it might be new physics

Data is consistent with flat solution, but sidereal time solution is not excluded.



LSND oscillation candidate sidereal time distribution



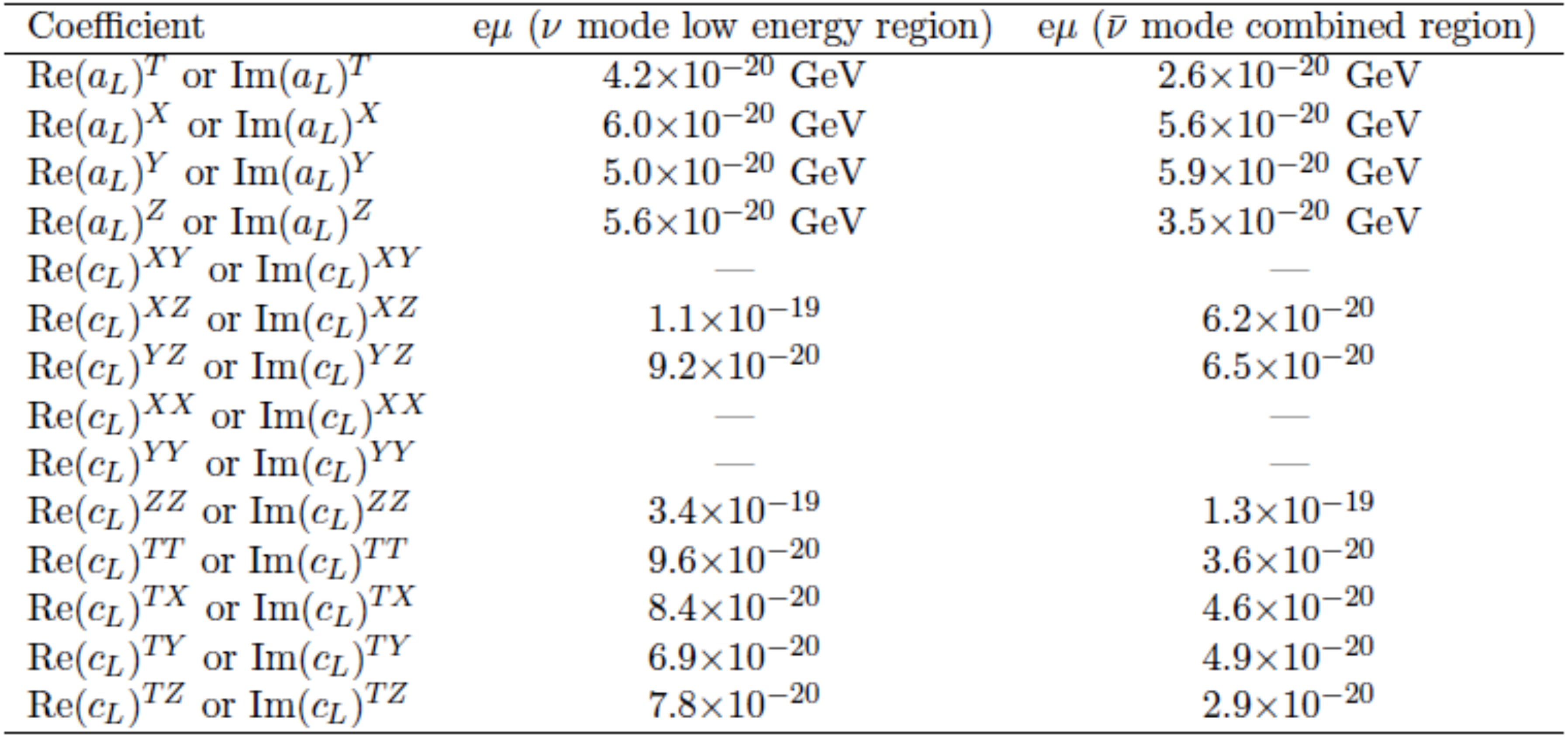
~10^{-19} GeV CPT-odd or ~10^{-17} CPT-even Lorentz violation could be the solution of LSND excess

TK, MPLA27(2012)1230024

4. Summary of results

Since we find no evidence of Lorentz violation from MiniBooNE analysis, we set limits on the SME coefficients.

These limits exclude SME values to explain LSND data, therefore there is no simple Lorentz violation motivated scenario to accommodate LSND and MiniBooNE results simultaneously



Coefficient	$e\mu$ (ν mode low energy region)	$e\mu$ ($\bar{\nu}$ mode combined region)
$\mathrm{Re}(a_L)^T$ or $\mathrm{Im}(a_L)^T$	4.2×10^{-20} GeV	2.6×10^{-20} GeV
$\mathrm{Re}(a_L)^X$ or $\mathrm{Im}(a_L)^X$	6.0×10^{-20} GeV	5.6×10^{-20} GeV
$\mathrm{Re}(a_L)^Y$ or $\mathrm{Im}(a_L)^Y$	5.0×10^{-20} GeV	5.9×10^{-20} GeV
$\mathrm{Re}(a_L)^Z$ or $\mathrm{Im}(a_L)^Z$	5.6×10^{-20} GeV	3.5×10^{-20} GeV
$\mathrm{Re}(c_L)^{XY}$ or $\mathrm{Im}(c_L)^{XY}$	—	—
$\mathrm{Re}(c_L)^{XZ}$ or $\mathrm{Im}(c_L)^{XZ}$	1.1×10^{-19}	6.2×10^{-20}
$\mathrm{Re}(c_L)^{YZ}$ or $\mathrm{Im}(c_L)^{YZ}$	9.2×10^{-20}	6.5×10^{-20}
$\mathrm{Re}(c_L)^{XX}$ or $\mathrm{Im}(c_L)^{XX}$	—	—
$\mathrm{Re}(c_L)^{YY}$ or $\mathrm{Im}(c_L)^{YY}$	—	—
$\mathrm{Re}(c_L)^{ZZ}$ or $\mathrm{Im}(c_L)^{ZZ}$	3.4×10^{-19}	1.3×10^{-19}
$\mathrm{Re}(c_L)^{TT}$ or $\mathrm{Im}(c_L)^{TT}$	9.6×10^{-20}	3.6×10^{-20}
$\mathrm{Re}(c_L)^{TX}$ or $\mathrm{Im}(c_L)^{TX}$	8.4×10^{-20}	4.6×10^{-20}
$\mathrm{Re}(c_L)^{TY}$ or $\mathrm{Im}(c_L)^{TY}$	6.9×10^{-20}	4.9×10^{-20}
$\mathrm{Re}(c_L)^{TZ}$ or $\mathrm{Im}(c_L)^{TZ}$	7.8×10^{-20}	2.9×10^{-20}

1. Spontaneous Lorentz symmetry breaking

2. What is Lorentz and CPT violation?

3. Modern test of Lorentz violation

4. Test of Lorenz violation with MiniBooNE

5. **Test of Lorentz violation with Double Chooz**

6. Conclusion

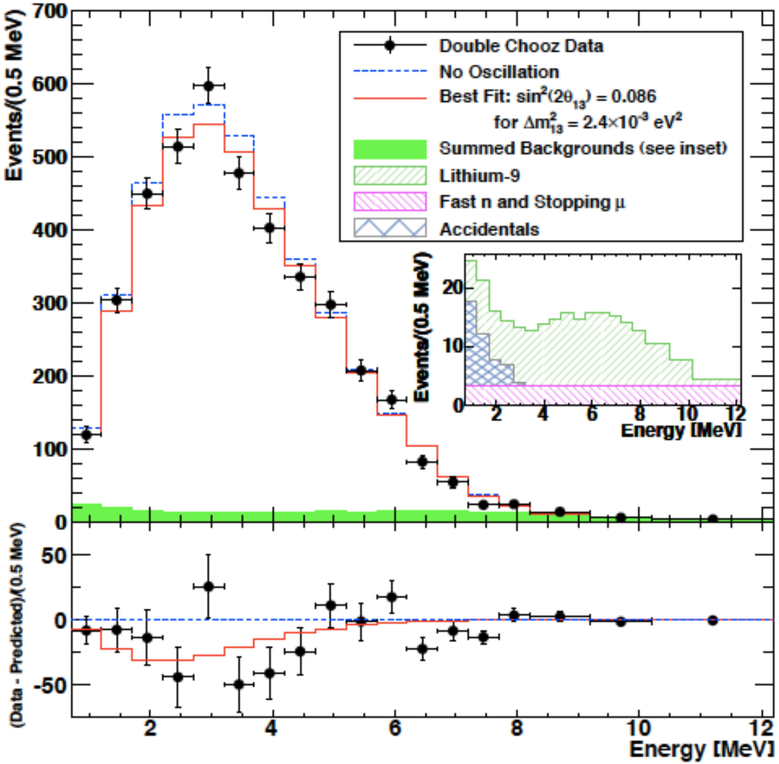
Double Chooz collaboration
PRL108(2012)131801

5. Double Chooz experiment

Reactor electron antineutrino disappearance experiment

- The first result shows small anti-ν_e disappearance!

Double Chooz reactor neutrino candidate



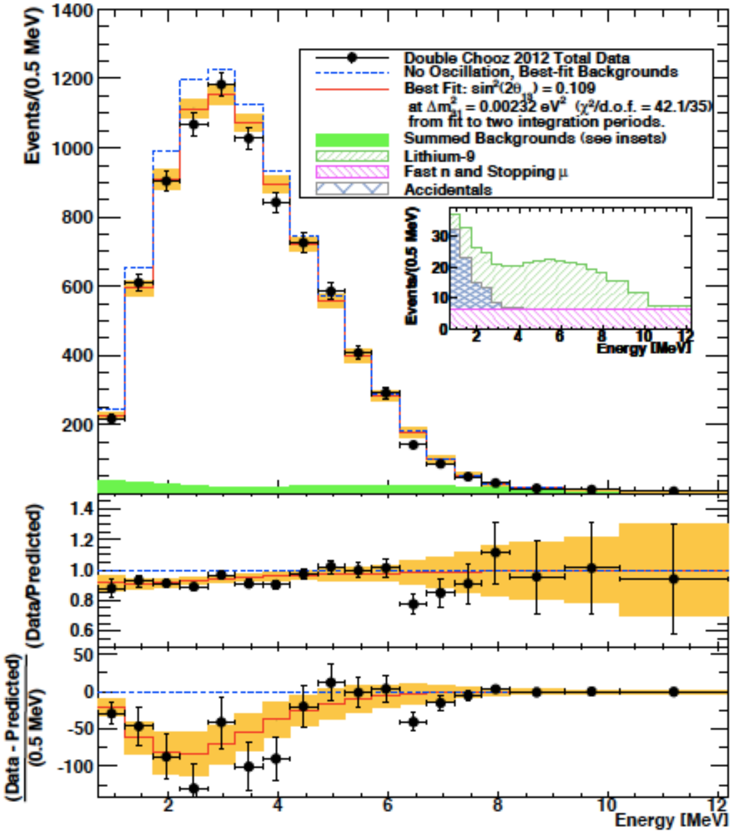
Double Chooz collaboration
PRL108(2012)131801
PRD86(2012)052008
DayaBay collaboration
PRL108(2012)171803
RENO collaboration
PRL108(2012)191802

5. Double Chooz experiment

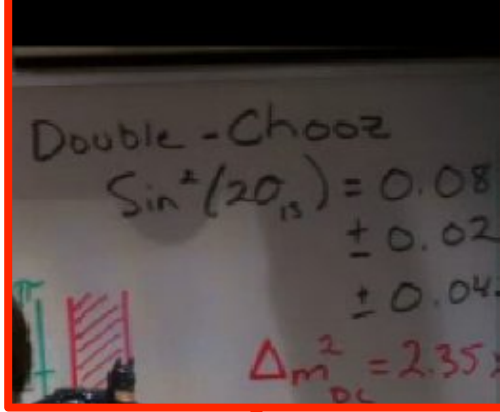
Reactor electron antineutrino disappearance experiment

- The first result shows small anti-ν_e disappearance!
- The second result reaches 3.1σ signal
- DayaBay and RENO experiments saw disappearance signals, too

Double Chooz reactor neutrino candidate



5. Double Chooz experiment



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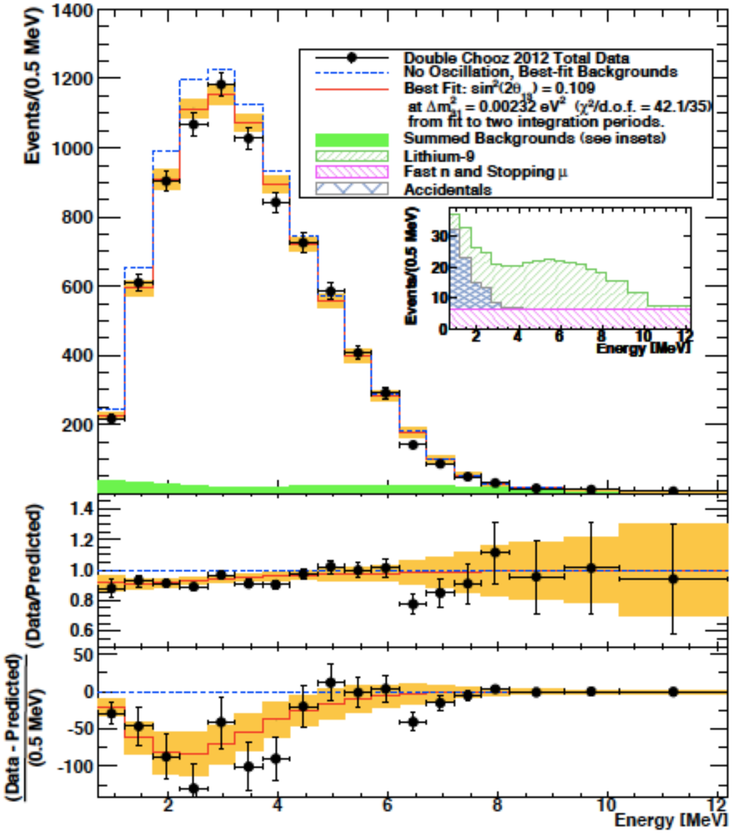
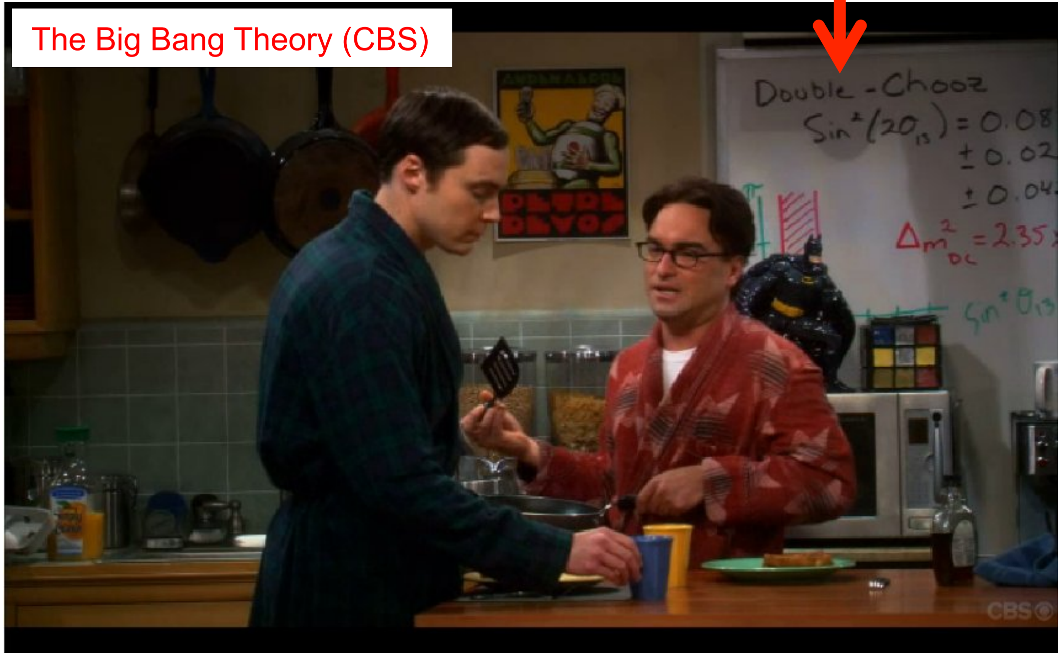
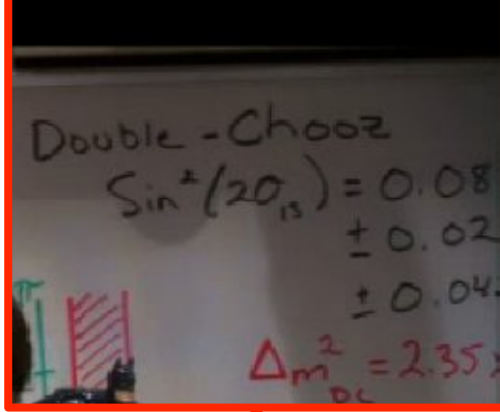
Double Chooz reactor neutrino candidate




The Big Bang Theory (CBS)

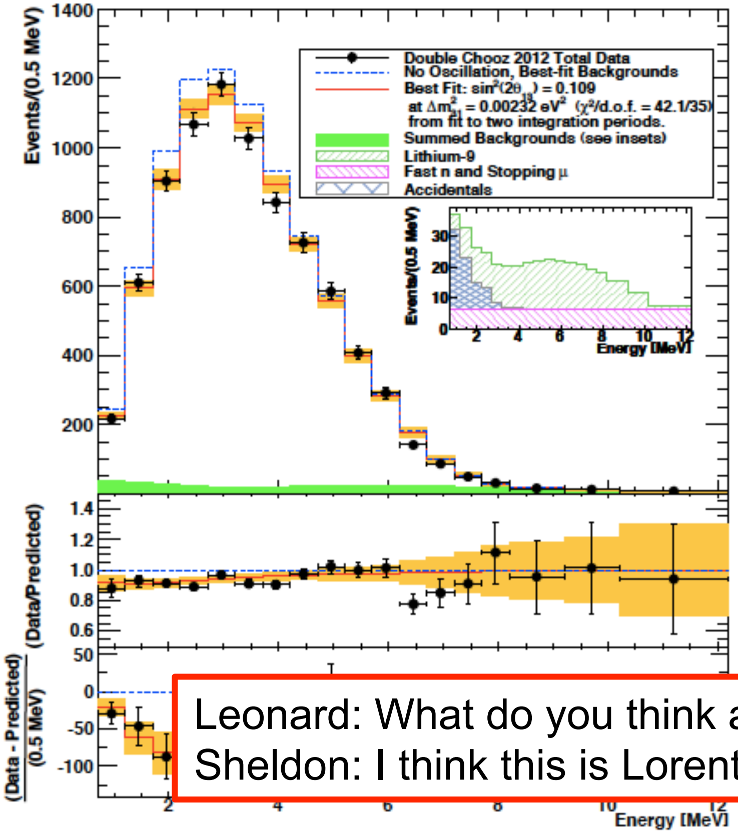
5. Double Chooz experiment

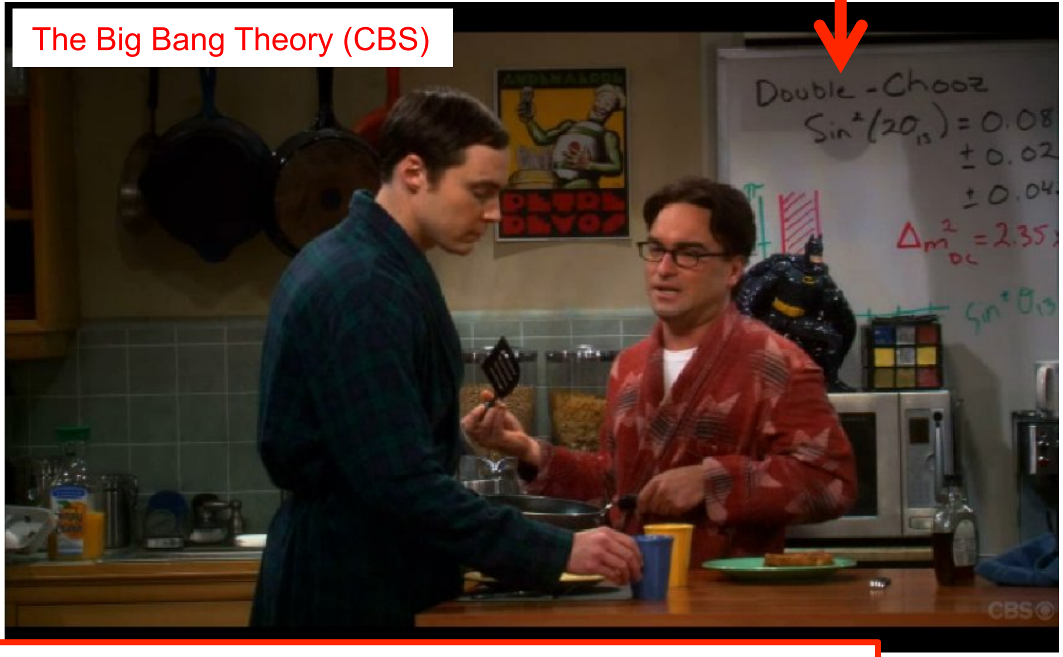


Reactor electron antineutrino disappearance experiment

- The first result shows small anti-ν_e disappearance!
- The second result reaches 3.1σ signal
- DayaBay and RENO experiments saw disappearance signals, too
- This small disappearance may have sidereal time dependence

Double Chooz reactor neutrino candidate

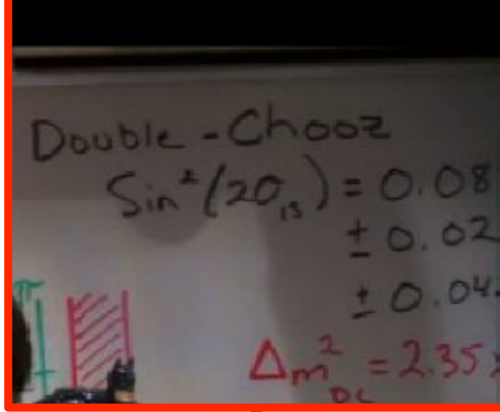




The Big Bang Theory (CBS)

Leonard: What do you think about the latest Double Chooz result?
Sheldon: I think this is Lorentz violation..., check sidereal time dependence

5. Double Chooz experiment



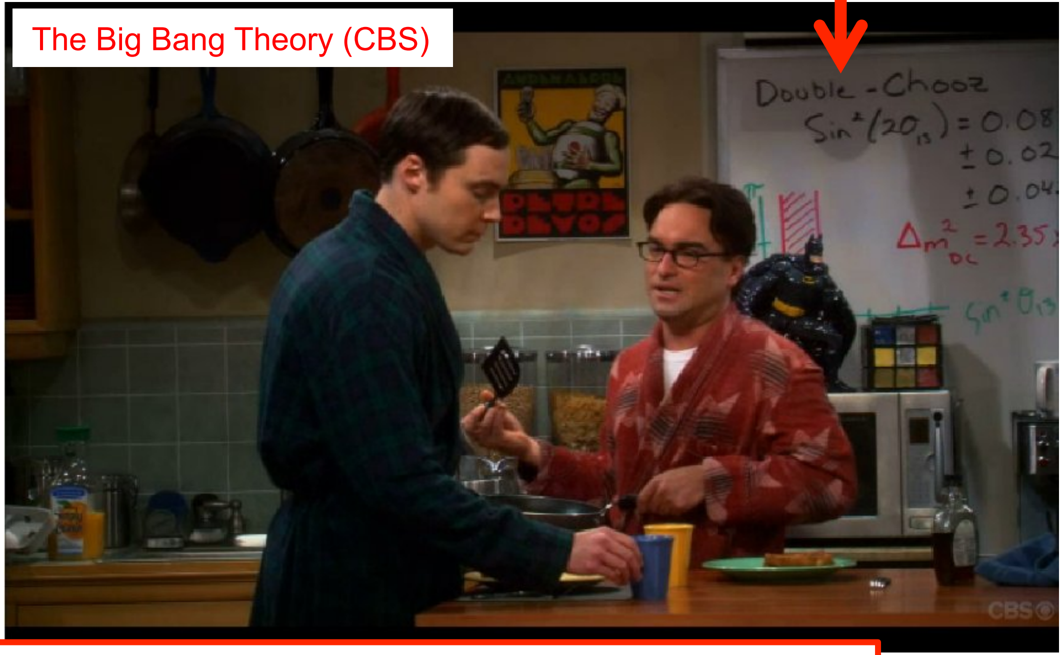
So far, we have set limits on

1. $\nu_e \leftrightarrow \nu_\mu$ channel: LSND, MiniBooNE, MINOS ($<10^{-20}$ GeV)
2. $\nu_\mu \leftrightarrow \nu_\tau$ channel: MINOS, IceCube ($<10^{-23}$ GeV)

The last untested channel is $\nu_e \leftrightarrow \nu_\tau$

It is possible to limit $\nu_e \leftrightarrow \nu_\tau$ channel from reactor ν_e disappearance experiment

$$P(\nu_e \leftrightarrow \nu_e) = 1 - P(\nu_e \leftrightarrow \nu_\mu) - P(\nu_e \leftrightarrow \nu_\tau) \sim 1 - P(\nu_e \leftrightarrow \nu_\tau)$$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Double Chooz collaboration, PRD86(2012)112009

5. Double Chooz experiment

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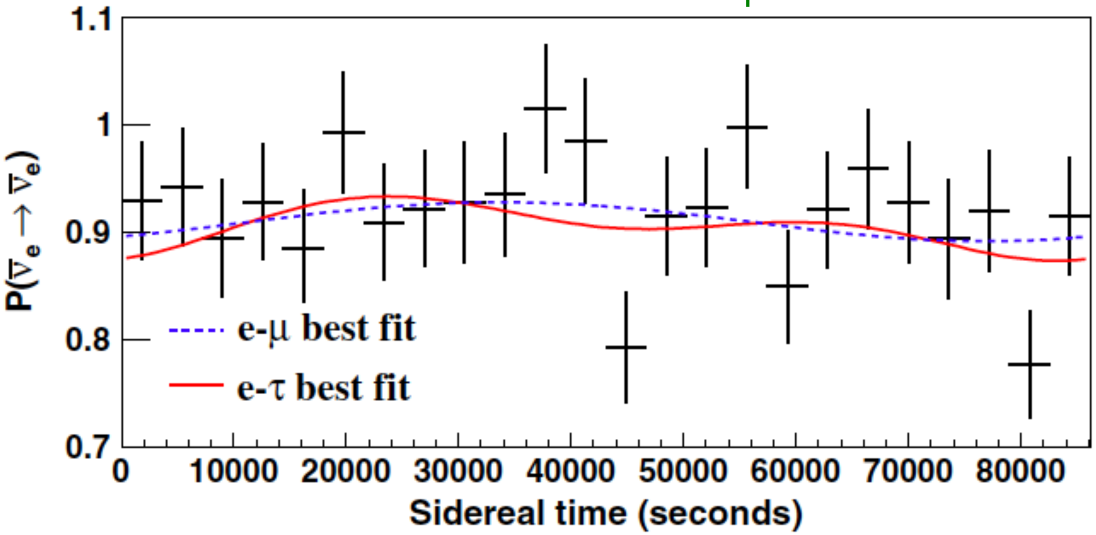
$$P(\nu_e \leftrightarrow \nu_e) = 1 - P(\nu_e \leftrightarrow \nu_\mu) - P(\nu_e \leftrightarrow \nu_\tau) \sim 1 - P(\nu_e \leftrightarrow \nu_\tau)$$

Small disappearance signal prefers sidereal time independent solution (flat)

We set limits in the e-τ sector for the first time; $\nu_e \leftrightarrow \nu_\tau$ ($<10^{-20}$ GeV)

Double Chooz reactor neutrino data/prediction ratio

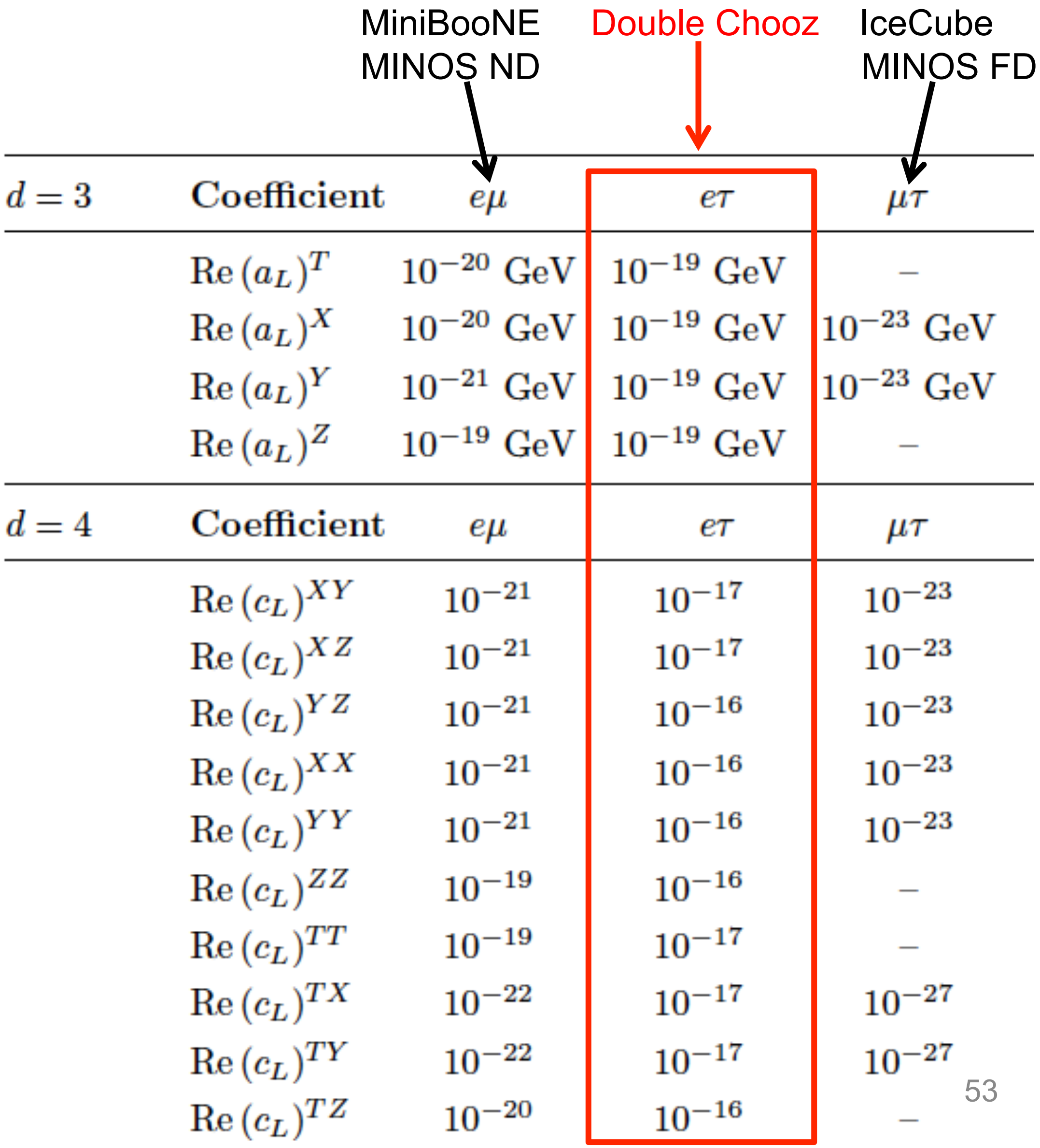


5. Double Chooz experiment

Kostelecký and Russel
Rev.Mod.Phys.83(2011)11
ArXiv:0801.0287v6

By this work, Lorentz violation is tested with all neutrino channels

Chance to see the Lorentz violation in terrestrial neutrino experiments will be very small


MiniBooNE
MINOS ND

Double Chooz

IceCube
MINOS FD

$d=3$	**Coefficient**	$e\mu$	$e\tau$	$\mu\tau$
	$\mathrm{Re}\,(a_L)^T$	10^{-20} GeV	10^{-19} GeV	–
	$\mathrm{Re}\,(a_L)^X$	10^{-20} GeV	10^{-19} GeV	10^{-23} GeV
	$\mathrm{Re}\,(a_L)^Y$	10^{-21} GeV	10^{-19} GeV	10^{-23} GeV
	$\mathrm{Re}\,(a_L)^Z$	10^{-19} GeV	10^{-19} GeV	–
$d=4$	**Coefficient**	$e\mu$	$e\tau$	$\mu\tau$
	$\mathrm{Re}\,(c_L)^{XY}$	10^{-21}	10^{-17}	10^{-23}
	$\mathrm{Re}\,(c_L)^{XZ}$	10^{-21}	10^{-17}	10^{-23}
	$\mathrm{Re}\,(c_L)^{YZ}$	10^{-21}	10^{-16}	10^{-23}
	$\mathrm{Re}\,(c_L)^{XX}$	10^{-21}	10^{-16}	10^{-23}
	$\mathrm{Re}\,(c_L)^{YY}$	10^{-21}	10^{-16}	10^{-23}
	$\mathrm{Re}\,(c_L)^{ZZ}$	10^{-19}	10^{-16}	–
	$\mathrm{Re}\,(c_L)^{TT}$	10^{-19}	10^{-17}	–
	$\mathrm{Re}\,(c_L)^{TX}$	10^{-22}	10^{-17}	10^{-27}
	$\mathrm{Re}\,(c_L)^{TY}$	10^{-22}	10^{-17}	10^{-27}
	$\mathrm{Re}\,(c_L)^{TZ}$	10^{-20}	10^{-16}	–

Díaz, Kostelecký, Mewes
PRD80(2009)076007

5. Anomalous energy spectrum

Sidereal variation is one of many predicted phenomena of Lorentz violating neutrino oscillations.

Lorentz violation predicts unexpected energy dependence of neutrino oscillations from standard neutrino mass oscillations.

massive neutrino oscillation

Lorentz violating neutrino oscillation

Effective Hamiltonian for neutrino oscillation

$$h_{eff} = \frac{m^2}{2E} + a + cE + \cdots$$

This is very useful to differentiate 2 effects:

- massive neutrino oscillation
- sidereal time independent Lorentz violating neutrino oscillation

Double Chooz released its energy spectrum (with full error matrix). We use this to test time independent Lorentz violating neutrino oscillation.

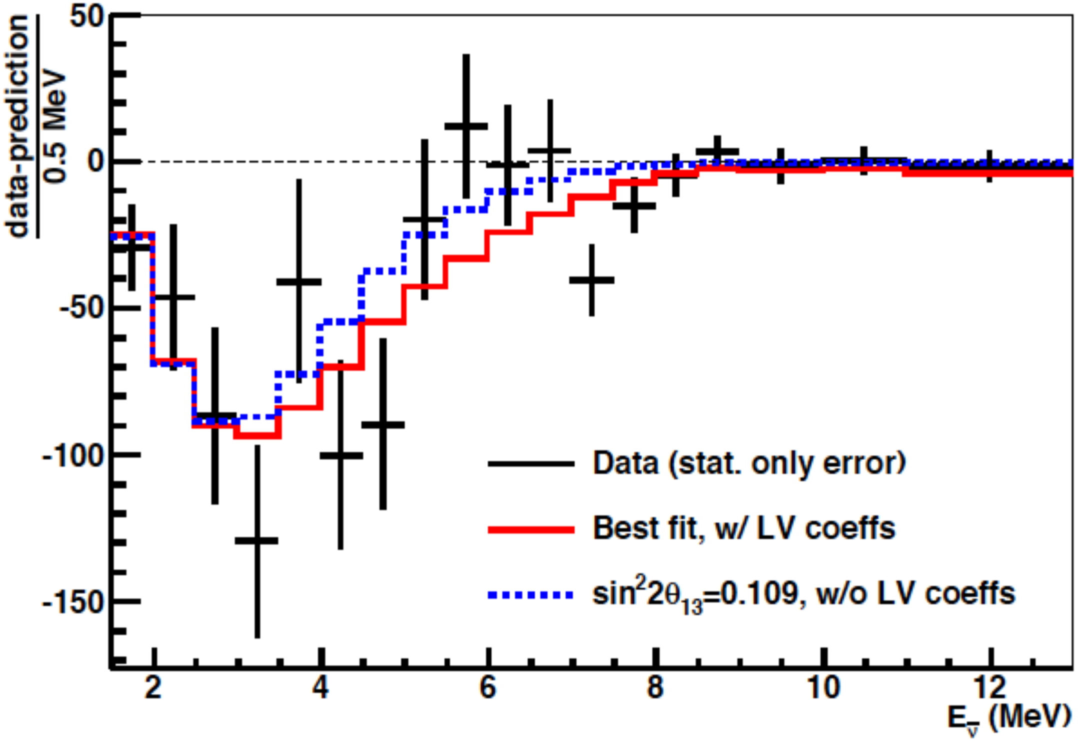
Díaz, TK, Spitz, Conrad
PLB727(2013)412

5. Double Chooz spectrum fit

Neutrino-Antineutrino oscillation

- Most of neutrino-neutrino oscillation channels are constraint from past analyses
- Here, we focus to test neutrino-antineutrino oscillation (conservation of angular momentum)

ex) anti-$\nu_e \rightarrow \nu_e$ oscillation fit with Double Chooz data



These fits provide first limits on neutrino-antineutrino time independent Lorentz violating coefficients

Conclusion

Lorentz and CPT violation has been shown to occur in Planck-scale theories.

There is a world wide effort to test Lorentz violation with various state-of-the-art technologies.

MiniBooNE sets limits on Lorentz violation on $\nu_\mu \rightarrow \nu_e$ oscillation coefficients. These limits together with MINOS exclude simple Lorentz violation motivated scenario to explain LSND anomaly.

MiniBooNE, LSND, MINOS, IceCube, and Double Chooz set stringent limits on Lorentz violation in neutrino sector in terrestrial level.

Thank you for your attention!

backup

2. Comment: Is there preferred frame?

As we see, all observers are related with observer's Lorentz transformation, so there is no special "preferred" frame (all observer's are consistent)

But there is a frame where universe looks isotropic even with a Lorentz violating vector field. You may call that is the "preferred frame", and people often speculate the frame where CMB looks isotropic is such a frame (called "CMB frame").

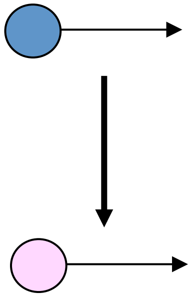
However, we are not on CMB frame (e.g., dipole term of WMAP is nonzero), so we expect anisotropy by lab experiments even CMB frame is the preferred frame.

2. What is CPT violation?

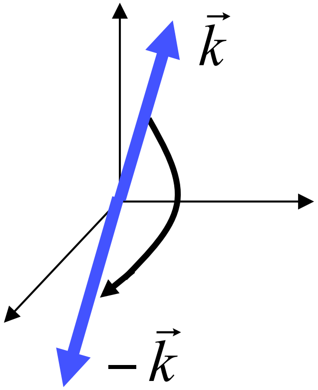
CPT symmetry is the invariance under the CPT transformation

$$L \xrightarrow{CPT} \Theta L \Theta^{-1} = L' = L, \qquad \Theta = CPT$$

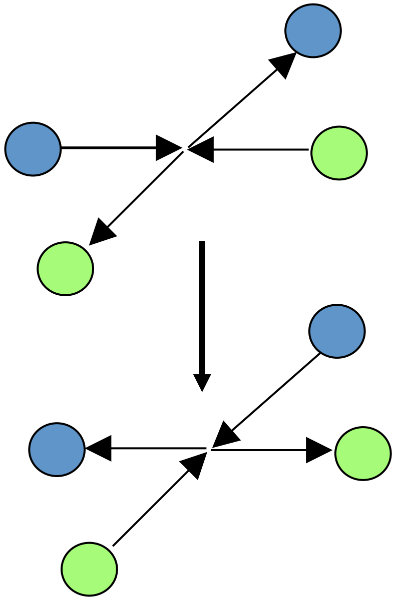
C: charge conjugation



P: parity transformation



T: time reversal



Jost, Helv.Phys.Acta.30(1957)409

2. What is CPT violation?

CPT symmetry is the invariance under the CPT transformation

$$\mathrm{L} \xrightarrow{\mathrm{CPT}} \Theta \mathrm{L} \Theta^{-1} = \mathrm{L}' = \mathrm{L}, \qquad \Theta = \mathrm{CPT}$$

CPT is the perfect symmetry of the Standard Model, due to CPT theorem

CPT theorem
If the relativistic transformation law and the weak microcausality holds in a real neighbourhood of a Jost point, the CPT condition holds everywhere.

number of Lorentz indices
→ always even number

$$\text{CPT phase} = (-1)^{n}$$

2. What is CPT violation?

CPT symmetry is the invariance under the CPT transformation

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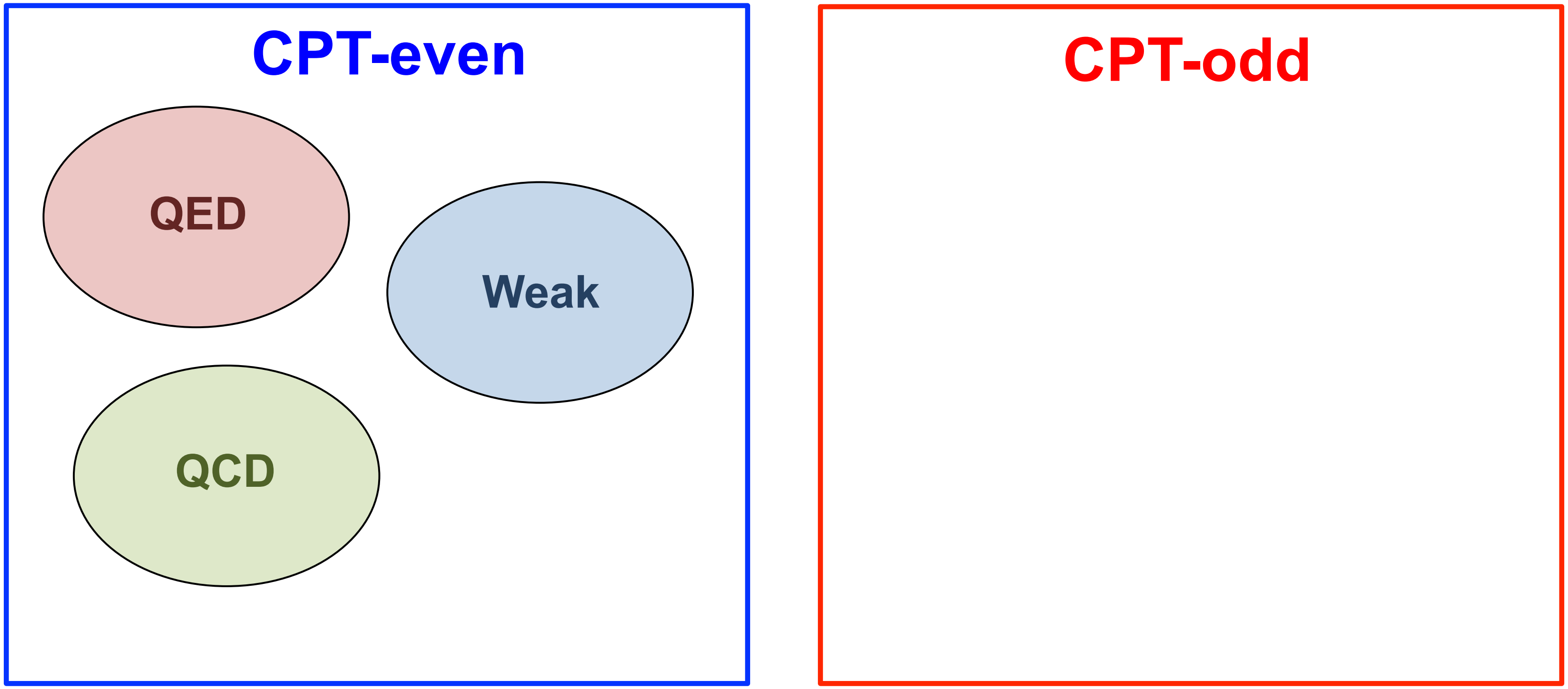

CPT-even

QED

Weak

QCD

CPT-odd

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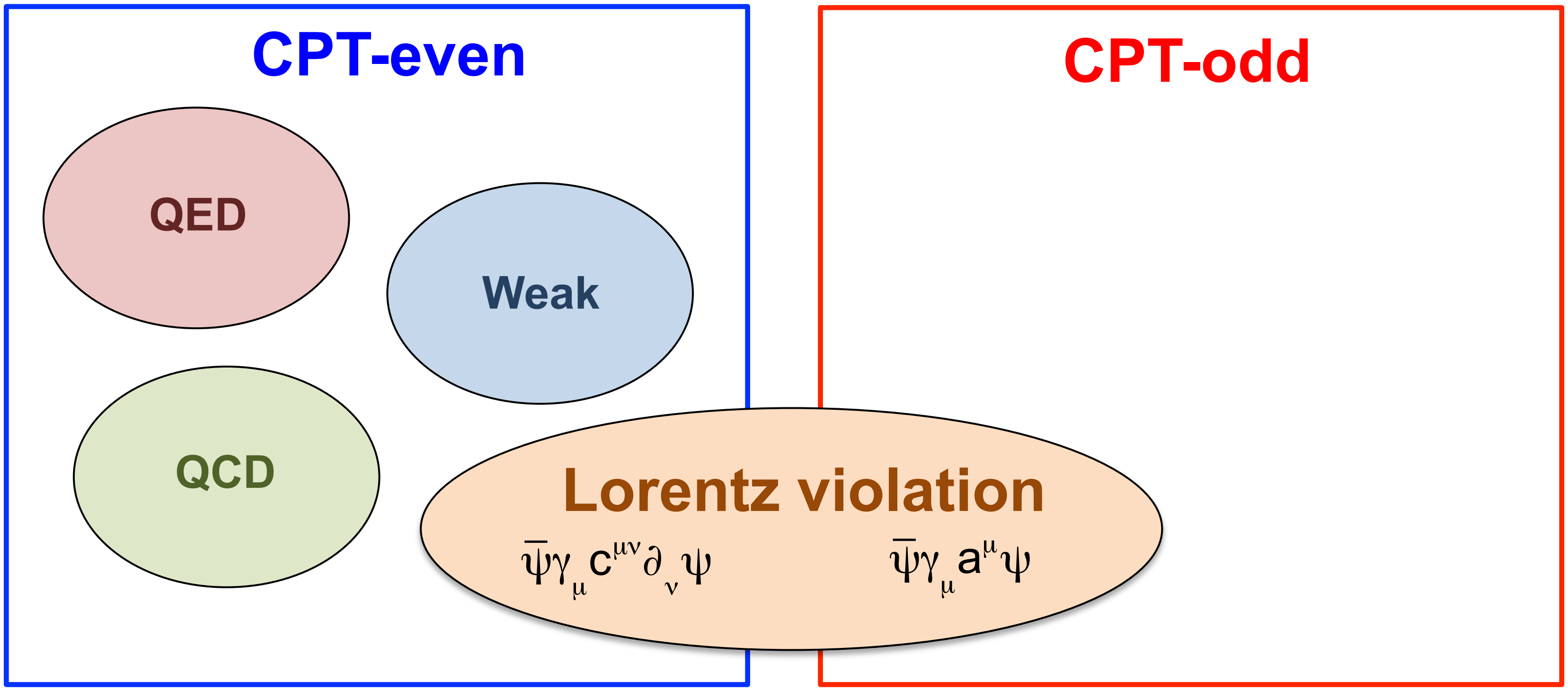
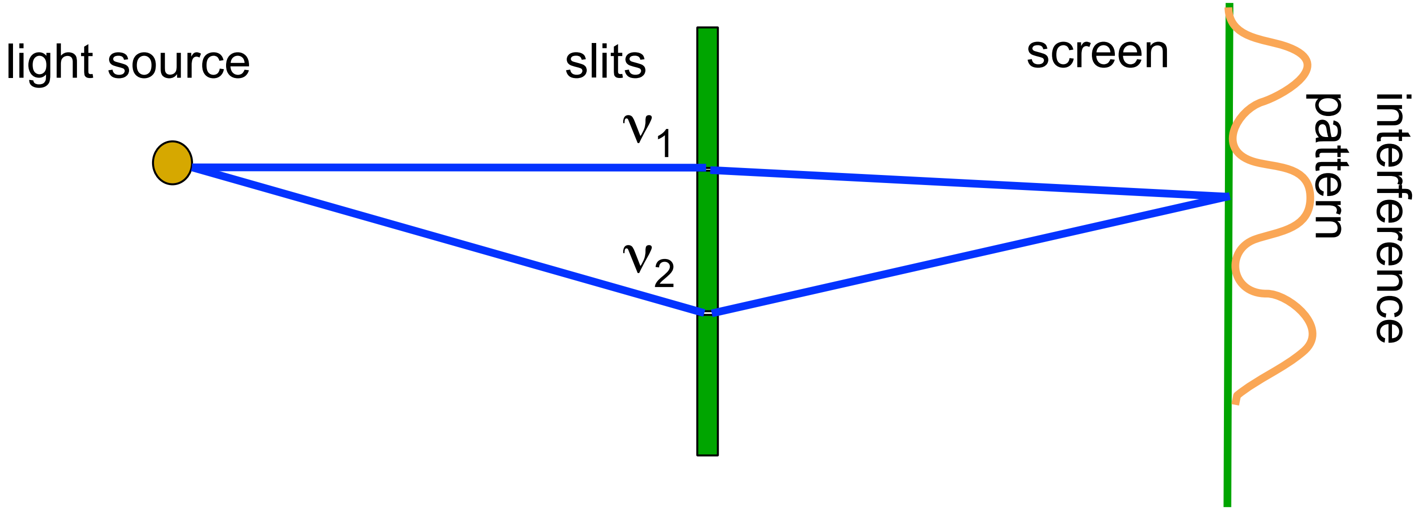


CPT-odd Lorentz violating coefficients (odd number Lorentz indices, e.g., a^{μ} , $g^{\lambda\mu\nu}$)

CPT-even Lorentz violating coefficients (even number Lorentz indices, e.g., $c^{\mu\nu}$, $\kappa^{\alpha\beta\mu\nu}$)

4. Neutrino oscillations, natural interferometers

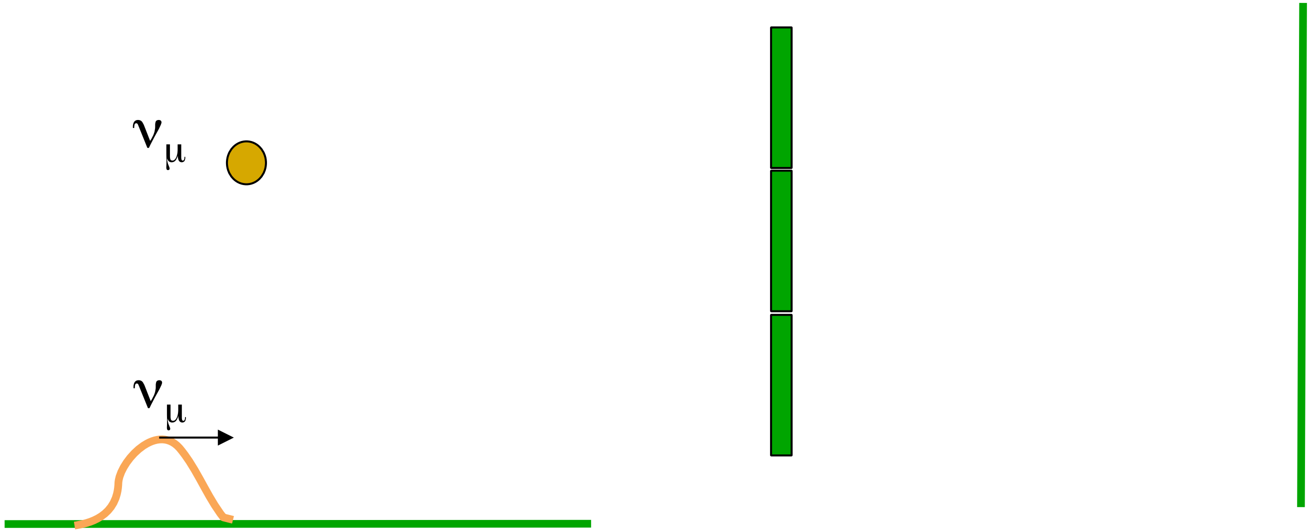
Neutrino oscillation is an interference experiment (cf. double slit experiment)



For double slit experiment, if path ν_1 and path ν_2 have different length, they have different phase rotations and it causes interference.

4. Neutrino oscillations, natural interferometers

Neutrino oscillation is an interference experiment (cf. double slit experiment)



If 2 neutrino Hamiltonian eigenstates, ν_1 and ν_2, have different phase rotation, they cause quantum interference.

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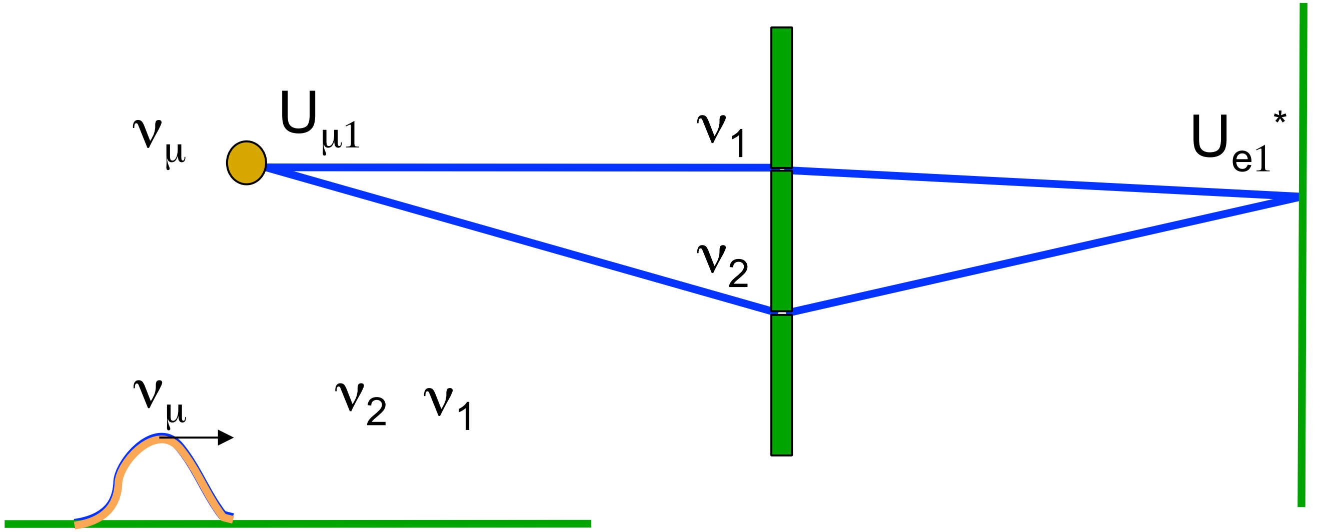

If 2 neutrino Hamiltonian eigenstates, ν_1 and ν_2, have different phase rotation, they cause quantum interference.

If ν_1 and ν_2, have different mass, they have different velocity, so thus different phase rotation.

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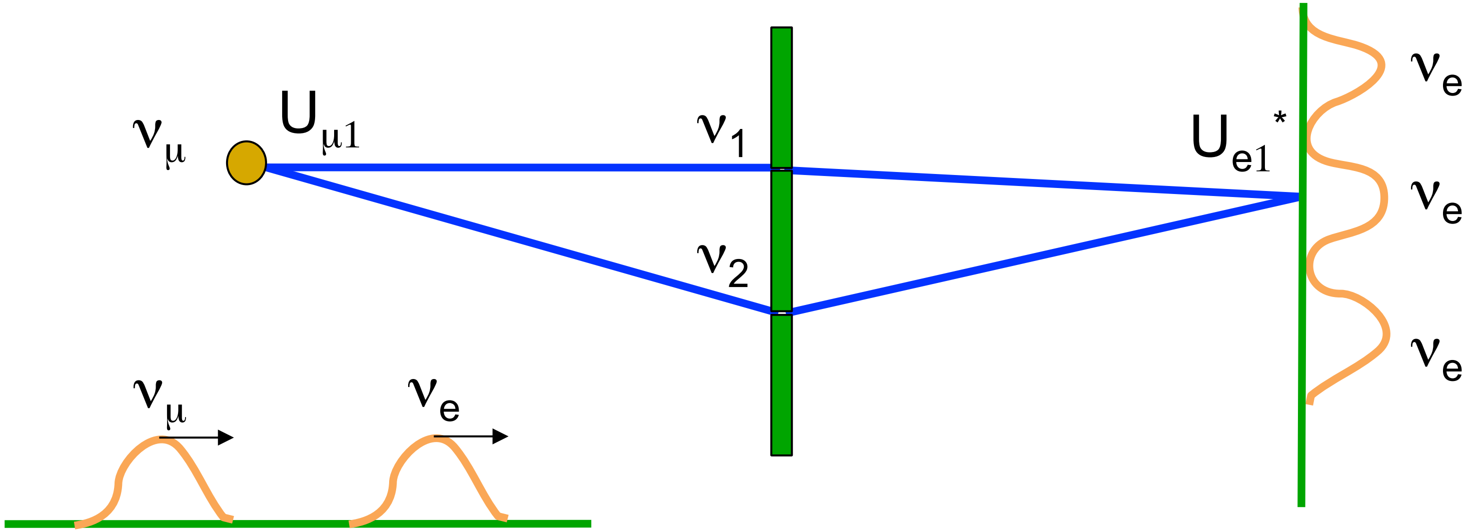


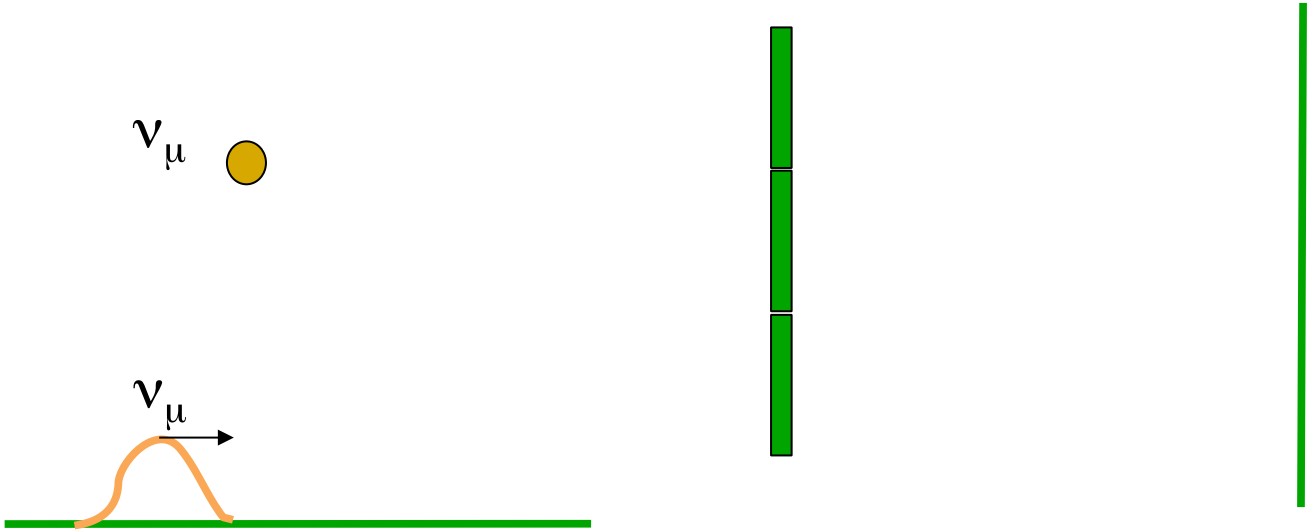
If 2 neutrino Hamiltonian eigenstates, ν_1 and ν_2, have different phase rotation, they cause quantum interference.

If ν_1 and ν_2, have different mass, they have different velocity, so thus different phase rotation.

The detection may be different flavor (neutrino oscillations).

4. Lorentz violation with neutrino oscillation

Neutrino oscillation is an interference experiment (cf. double slit experiment)



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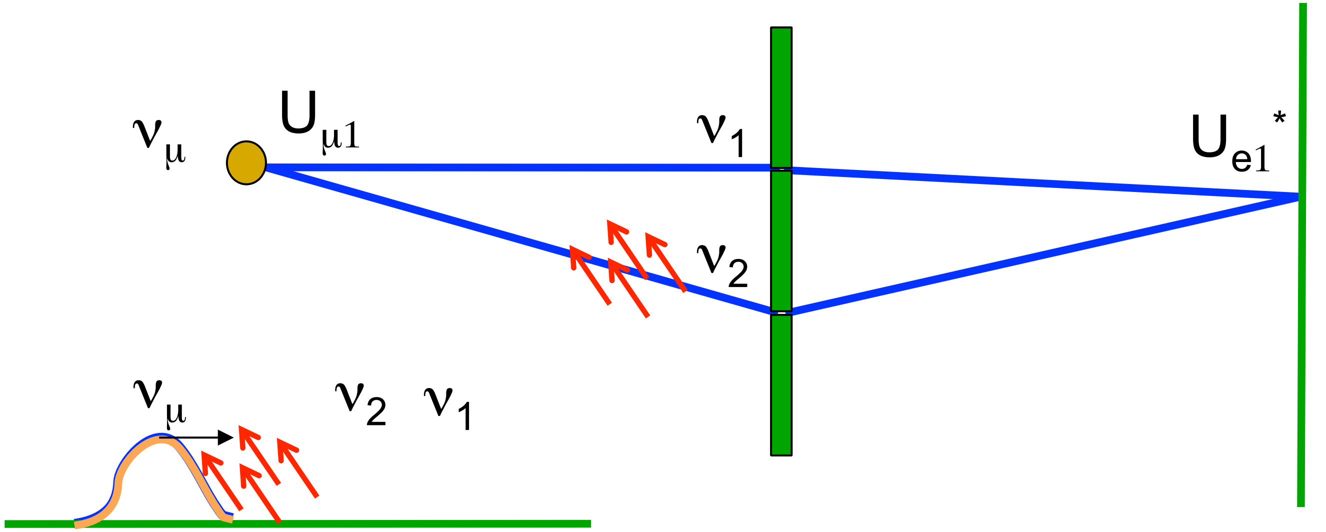

If 2 neutrino Hamiltonian eigenstates, ν_1 and ν_2, have different phase rotation, they cause quantum interference.

If ν_1 and ν_2, have different coupling with Lorentz violating field, neutrinos also oscillate. The sensitivity of neutrino oscillation is comparable the target scale of Lorentz violation ($<10^{-19}$GeV).

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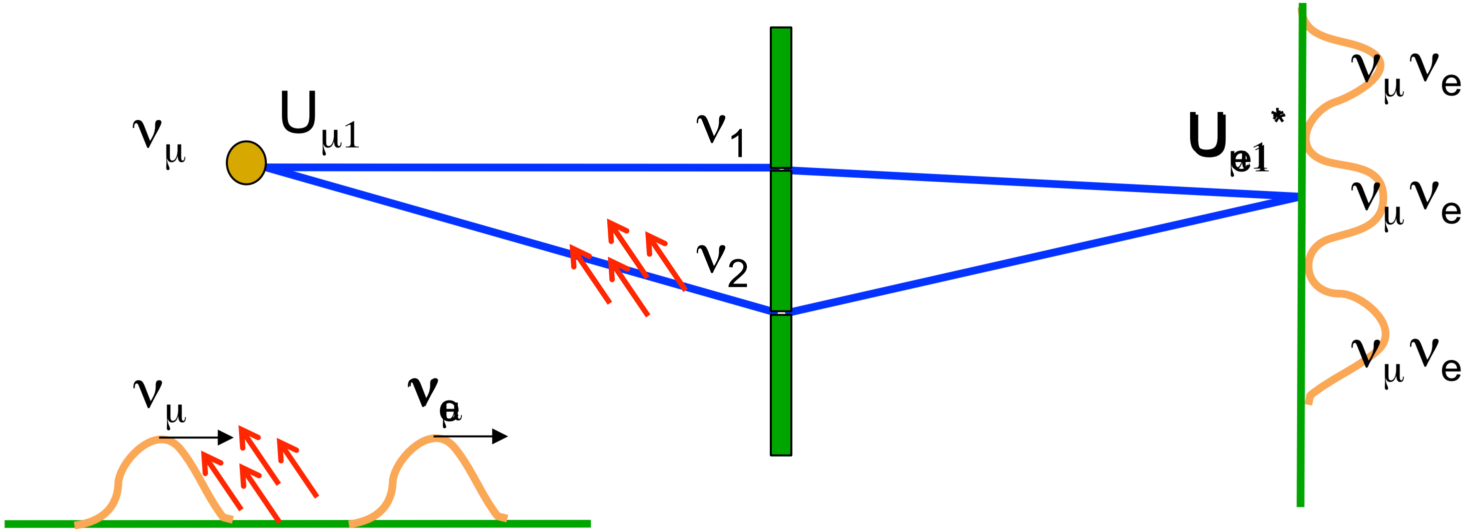


If 2 neutrino Hamiltonian eigenstates, ν_1 and ν_2, have different phase rotation, they cause quantum interference.

If ν_1 and ν_2, have different coupling with Lorentz violating field, neutrinos also oscillate. The sensitivity of neutrino oscillation is comparable the target scale of Lorentz violation (<10^{-19}GeV).

If neutrino oscillation is caused by Lorentz violation, interference pattern (oscillation probability) may have sidereal time dependence.

MiniBooNE collaboration,PRL110(2013)161801

2. MiniBooNE

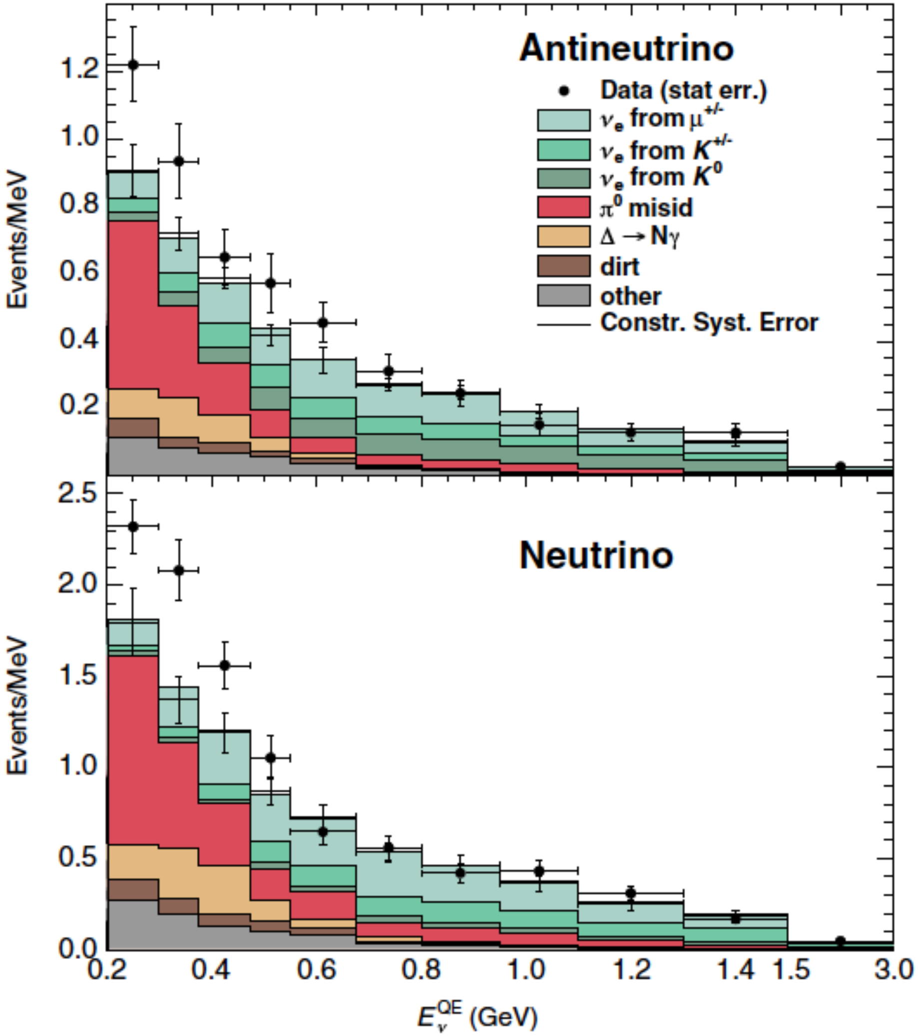
MiniBooNE observed event excesses in both mode

Neutrino mode

162.0 ± 28.1 ± 38.7 (3.4σ)

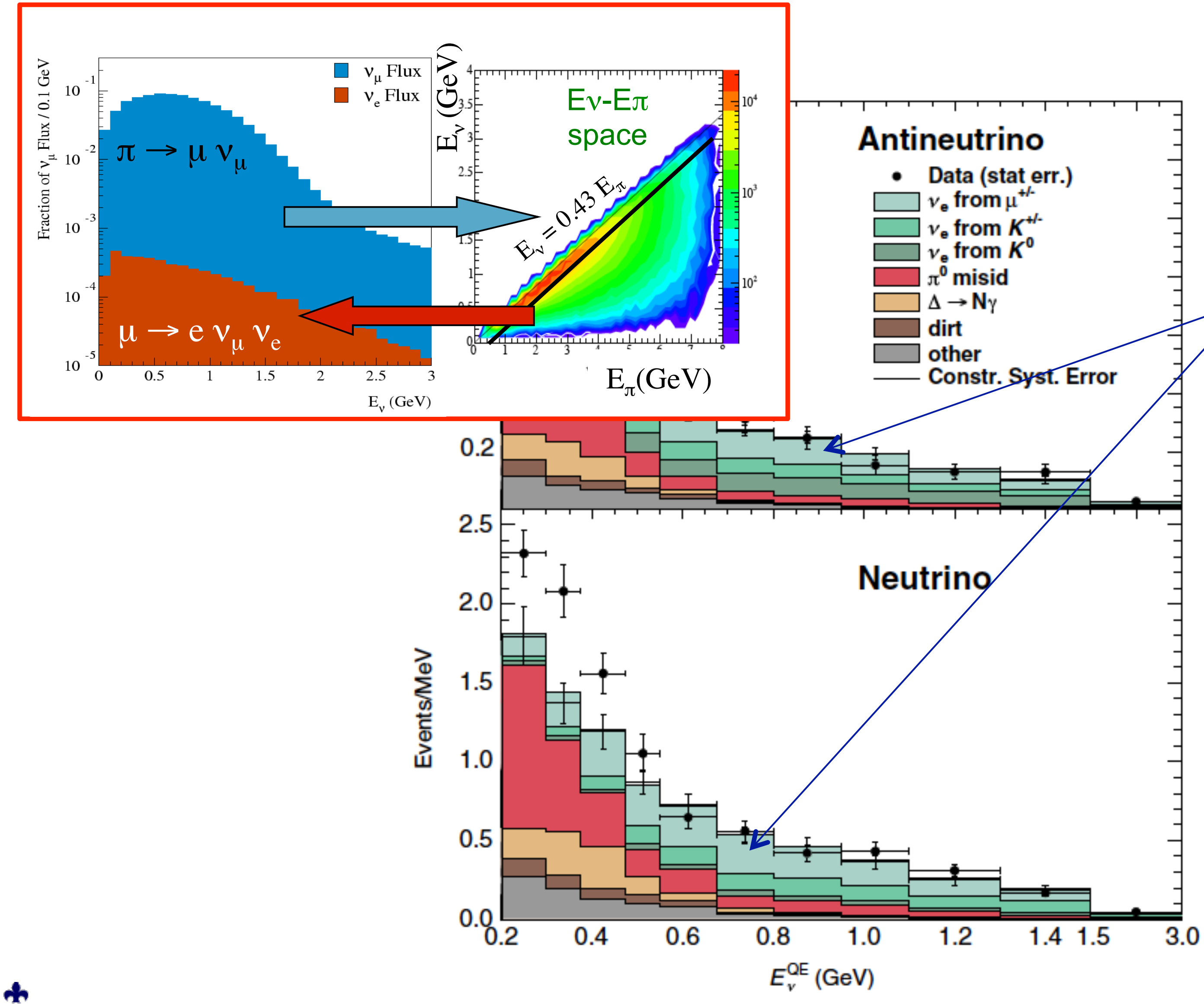
Antineutrino mode

78.9 ± 20.0 ± 20.3 (2.8σ)



MiniBooNE collaboration,PRL110(2013)161801

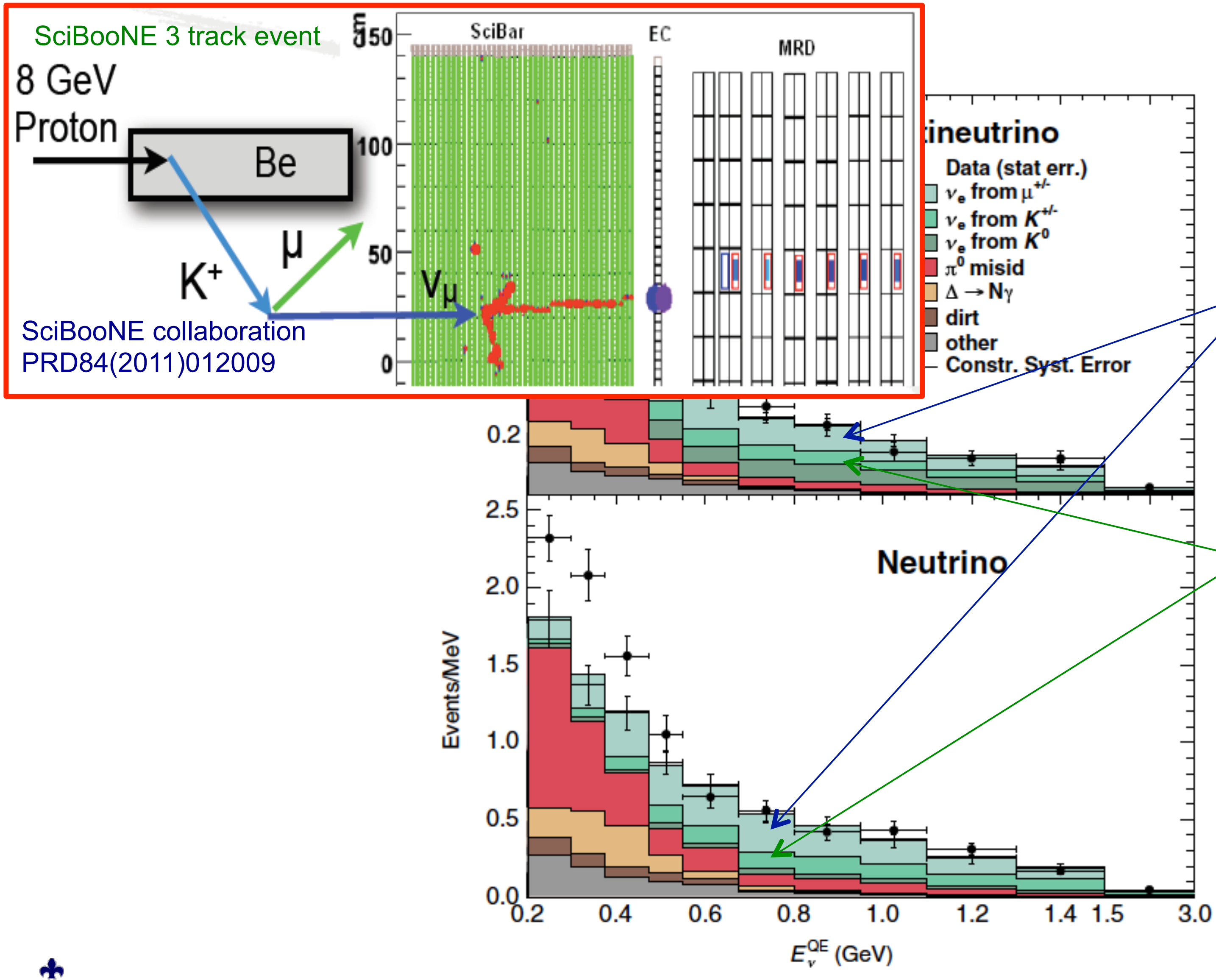
2. MiniBooNE



ν_e from μ decay is constrained from ν_μCCQE measurement

MiniBooNE collaboration,PRL110(2013)161801

2. MiniBooNE



SciBooNE collaboration PRD84(2011)012009

ν_e from μ decay is constrained from ν_μCCQE measurement

ν_e from K decay is constrained from high energy ν_μ event measurement in SciBooNE

MiniBooNE collaboration,PRL110(2013)161801

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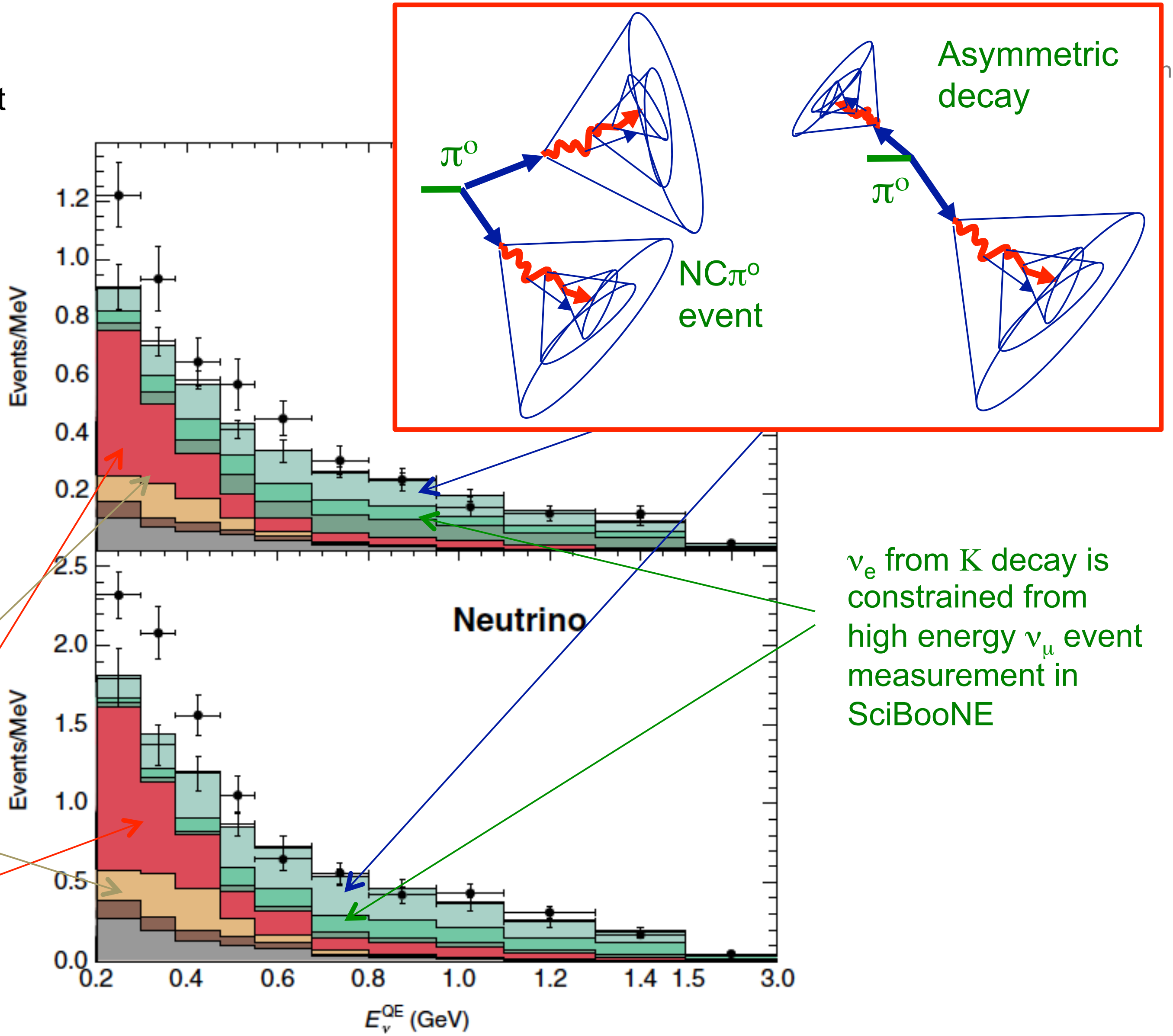


ν_e from K decay is constrained from high energy ν_μ event measurement in SciBooNE

Radiative Δ-decay (Δ→Nγ) rate is constrained from measured NCπ°

Asymmetric π° decay is constrained from measured CCπ° rate (π°→γ)

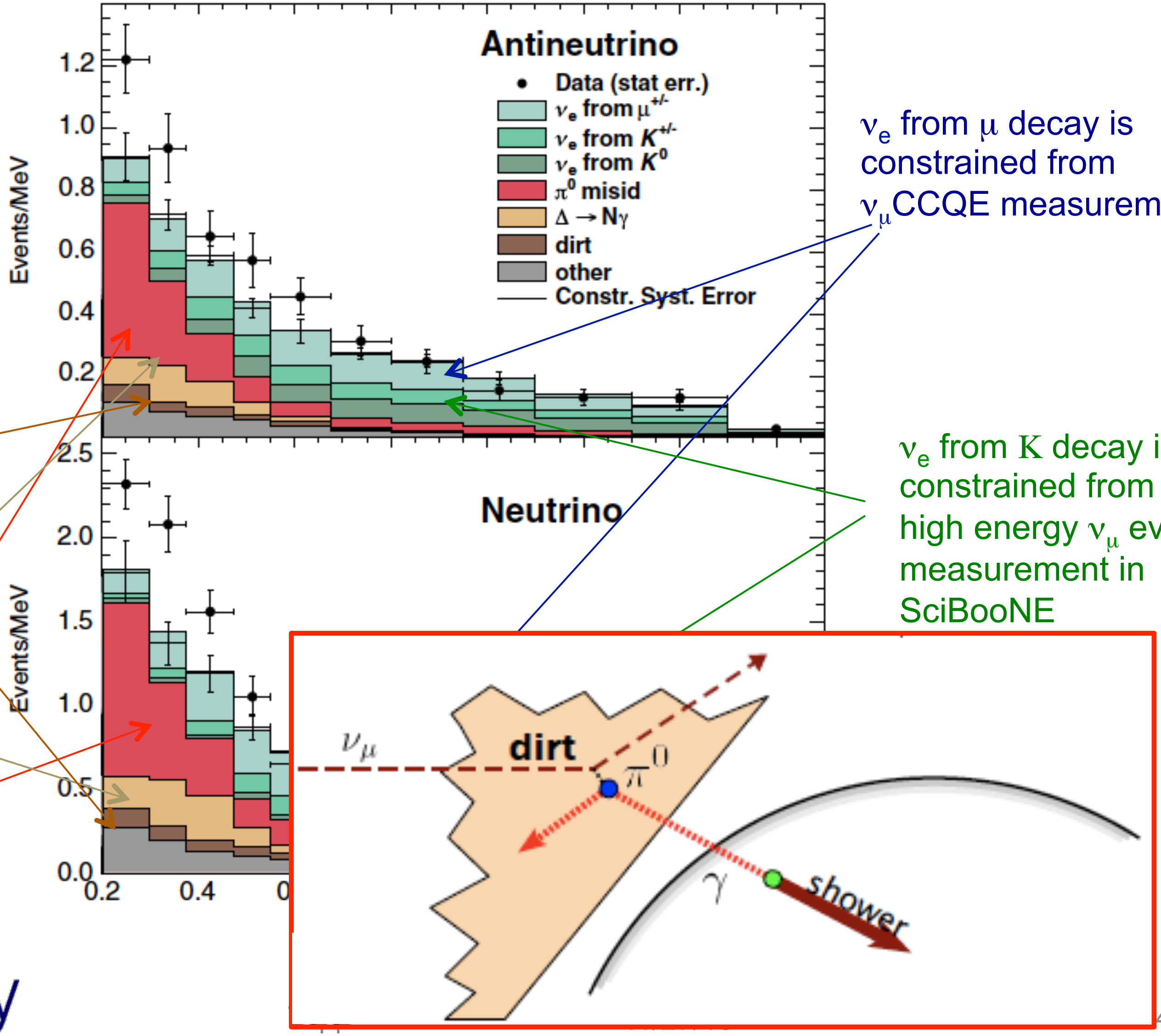
MiniBooNE collaboration,PRL110(2013)161801

2. MiniBooNE

MiniBooNE observed event excesses in both mode

Neutrino mode
162.0 ± 28.1 ± 38.7 (3.4σ)

Antineutrino mode
78.9 ± 20.0 ± 20.3 (2.8σ)



ν_e from μ decay is constrained from ν_μCCQE measurement

ν_e from K decay is constrained from high energy ν_μ event measurement in SciBooNE

dirt rate is measured from dirt enhanced data sample

Radiative Δ-decay ($\Delta \rightarrow N\gamma$) rate is constrained from measured NCπ^o

Asymmetric π^o decay is constrained from measured CCπ^o rate ($\pi^o \rightarrow \gamma$)

MiniBooNE collaboration,PRL110(2013)161801

2. MiniBooNE

MiniBooNE observed event excesses in both mode

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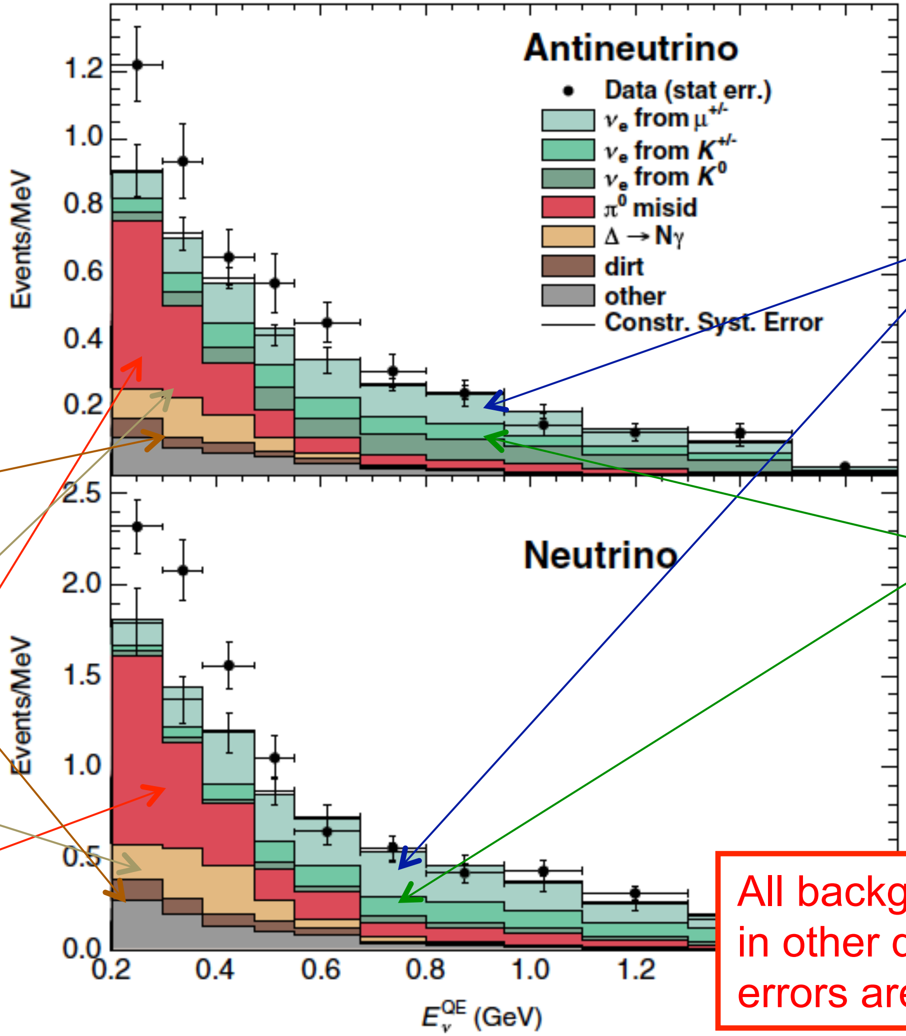

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Asymmetric π^o decay is constrained from measured CCπ^o rate ($\pi^o \rightarrow \gamma$)

All backgrounds are measured in other data sample and their errors are constrained!

Kostelecký and Mewes
PRD70(2004)076002

6. Lorentz violation with MiniBooNE

Sidereal variation of neutrino oscillation probability for MiniBooNE (5 parameters)

$$P_{\nu_e \to \nu_\mu} = \left(\frac{L}{\hbar c}\right)^2 \left| (C)_{e\mu} + (A_s)_{e\mu} \sin w_\oplus T_\oplus + (A_c)_{e\mu} \cos w_\oplus T_\oplus + (B_s)_{e\mu} \sin 2w_\oplus T_\oplus + (B_c)_{e\mu} \cos 2w_\oplus T_\oplus \right|^2$$

Expression of 5 observables (14 SME parameters)

$$(C)_{e\mu} = (a_L)^T_{e\mu} - N^Z (a_L)^Z_{e\mu} + E\left[-\frac{1}{2}(3 - N^Z N^Z)(c_L)^{TT}_{e\mu} + 2N^Z (c_L)^{TZ}_{e\mu} + \frac{1}{2}(1 - 3N^Z N^Z)(c_L)^{ZZ}_{e\mu} \right]$$

$$(A_s)_{e\mu} = N^Y (a_L)^X_{e\mu} - N^X (a_L)^Y_{e\mu} + E\left[-2N^Y (c_L)^{TX}_{e\mu} + 2N^X (c_L)^{TY}_{e\mu} + 2N^Y N^Z (c_L)^{XZ}_{e\mu} - 2N^X N^Z (c_L)^{YZ}_{e\mu} \right]$$

$$(A_c)_{e\mu} = -N^X (a_L)^X_{e\mu} - N^Y (a_L)^Y_{e\mu} + E\left[2N^X (c_L)^{TX}_{e\mu} + 2N^Y (c_L)^{TY}_{e\mu} - 2N^X N^Z (c_L)^{XZ}_{e\mu} - 2N^Y N^Z (c_L)^{YZ}_{e\mu} \right]$$

$$(B_s)_{e\mu} = E\left[N^X N^Y \left((c_L)^{XX}_{e\mu} - (c_L)^{YY}_{e\mu} \right) - (N^X N^X - N^Y N^Y)(c_L)^{XY}_{e\mu} \right]$$

$$(B_c)_{e\mu} = E\left[-\frac{1}{2}(N^X N^X - N^Y N^Y)\left((c_L)^{XX}_{e\mu} - (c_L)^{YY}_{e\mu} \right) - 2N^X N^Y (c_L)^{XY}_{e\mu} \right]$$

$$\begin{pmatrix} N^X \\ N^Y \\ N^Z \end{pmatrix} = \begin{pmatrix} \cos\chi \sin\theta \cos\phi - \sin\chi \cos\theta \\ \sin\theta \sin\phi \\ -\sin\chi \sin\theta \cos\phi - \cos\chi \cos\theta \end{pmatrix}$$

coordinate dependent direction vector (depends on the latitude of FNAL, location of BNB and MiniBooNE detector)