

Inclusion of multi-nucleon effects in RPA-based calculations for ν -nucleus scattering

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Outline

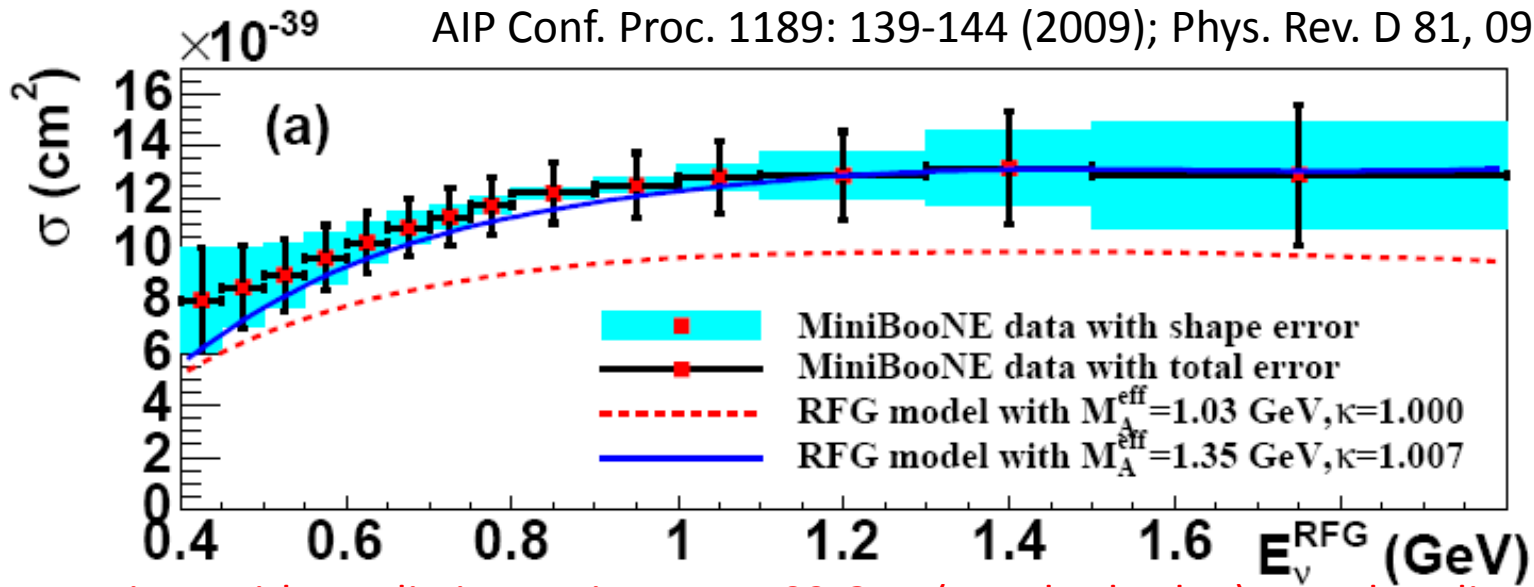
- Review of the main results obtained with our model and comparison with experimental data
- Description of our theoretical model
 - nuclear response functions in RPA
 - np-nh excitations
- Comparison among theoretical models

Phys. Rev. C 80 065501 (2009)
Phys. Rev. C 81 045502 (2010)
Phys. Rev. C 84 055502 (2011)
Phys. Rev. D 85 093012 (2012)
Phys. Rev. D 87 013009 (2013)
Phys. Rev. C 87 065501 (2013)

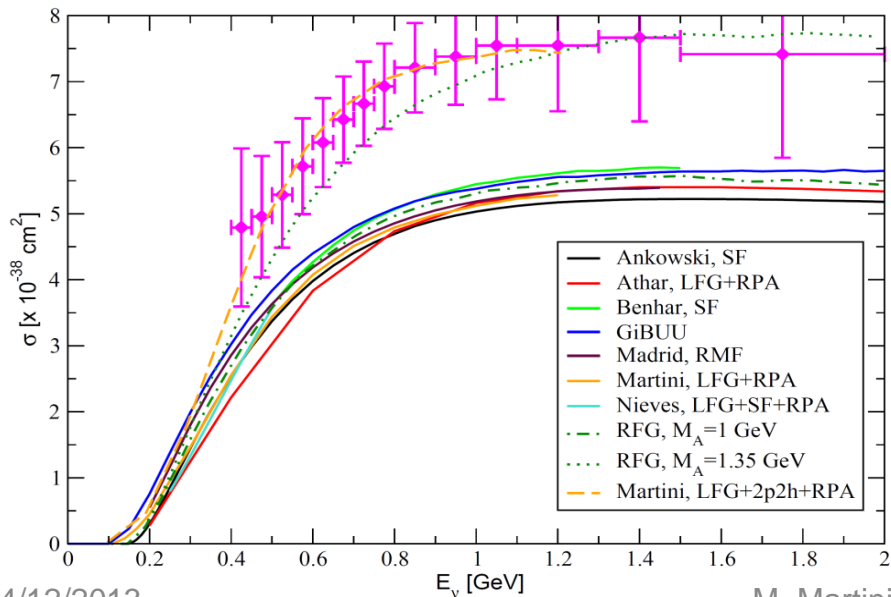
Review of our main results

MiniBooNE CC Quasielastic neutrino cross section on Carbon

AIP Conf. Proc. 1189: 139-144 (2009); Phys. Rev. D 81, 092005 (2010)



Comparison with predictions using $M_A=1.03$ GeV (standard value) reveals a discrepancy
 In the Relativistic Fermi Gas (RFG) model an axial mass of 1.35 GeV is needed to account for data

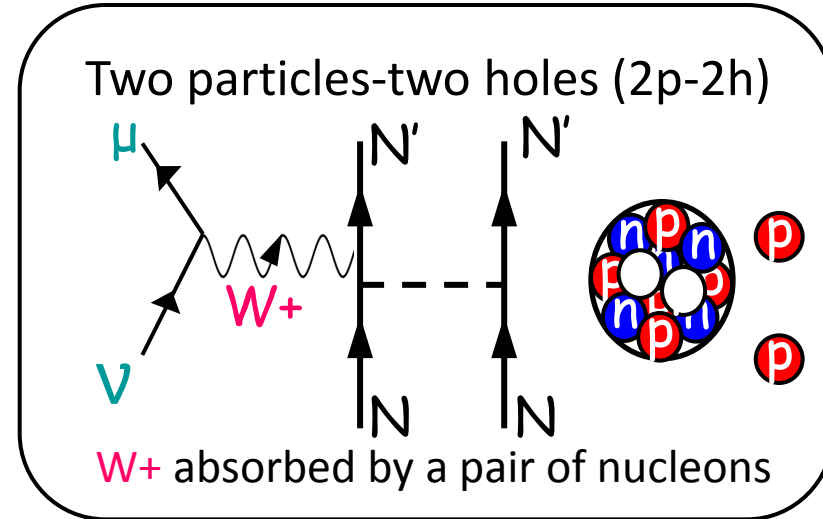
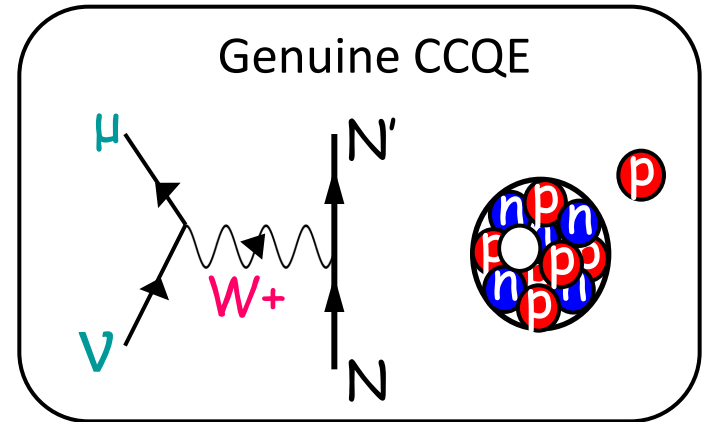
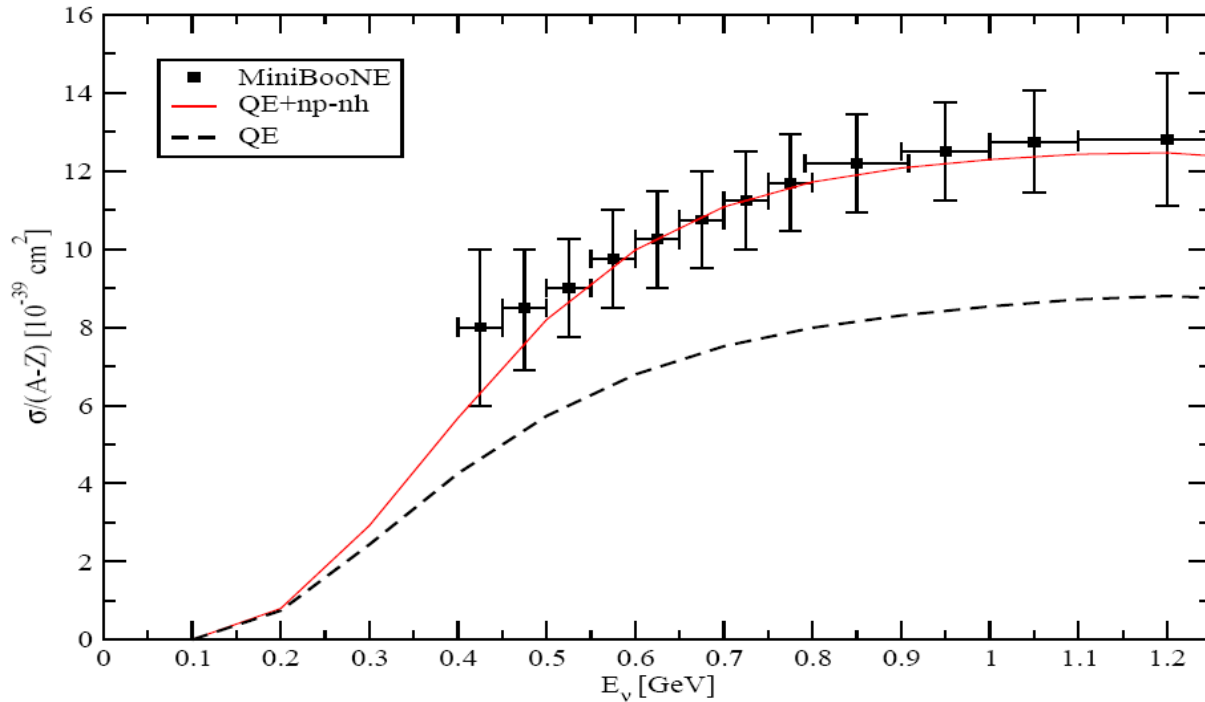


L. Alvarez-Ruso
 arXiv:1012.3871
 (Neutrino 2010)

puzzle??

An explanation of this puzzle

Inclusion of the multinucleon emission channel (np-nh)

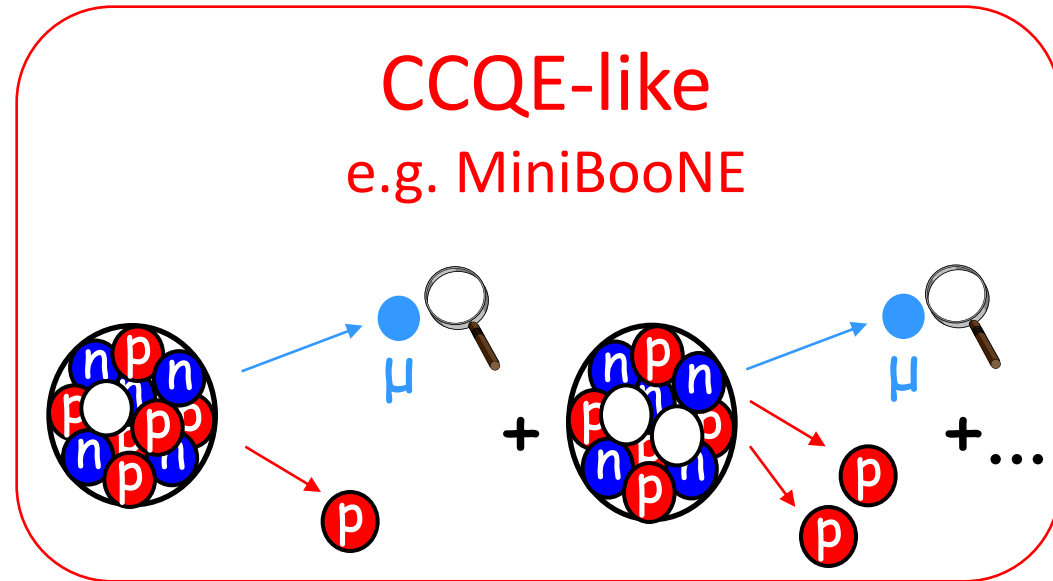
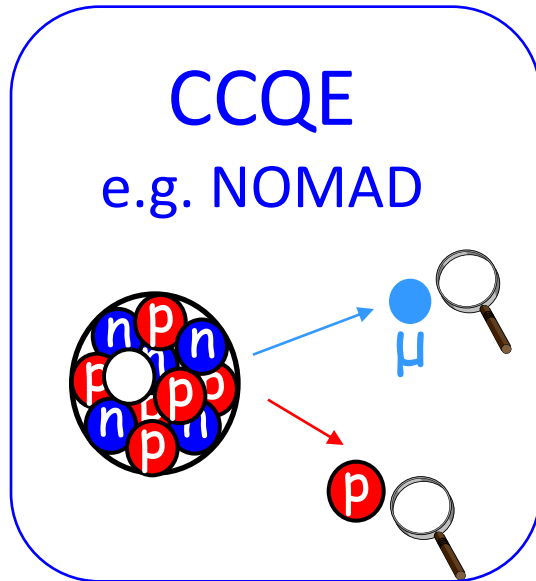
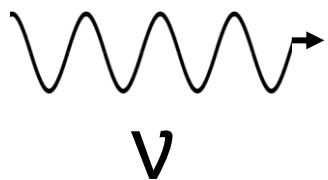
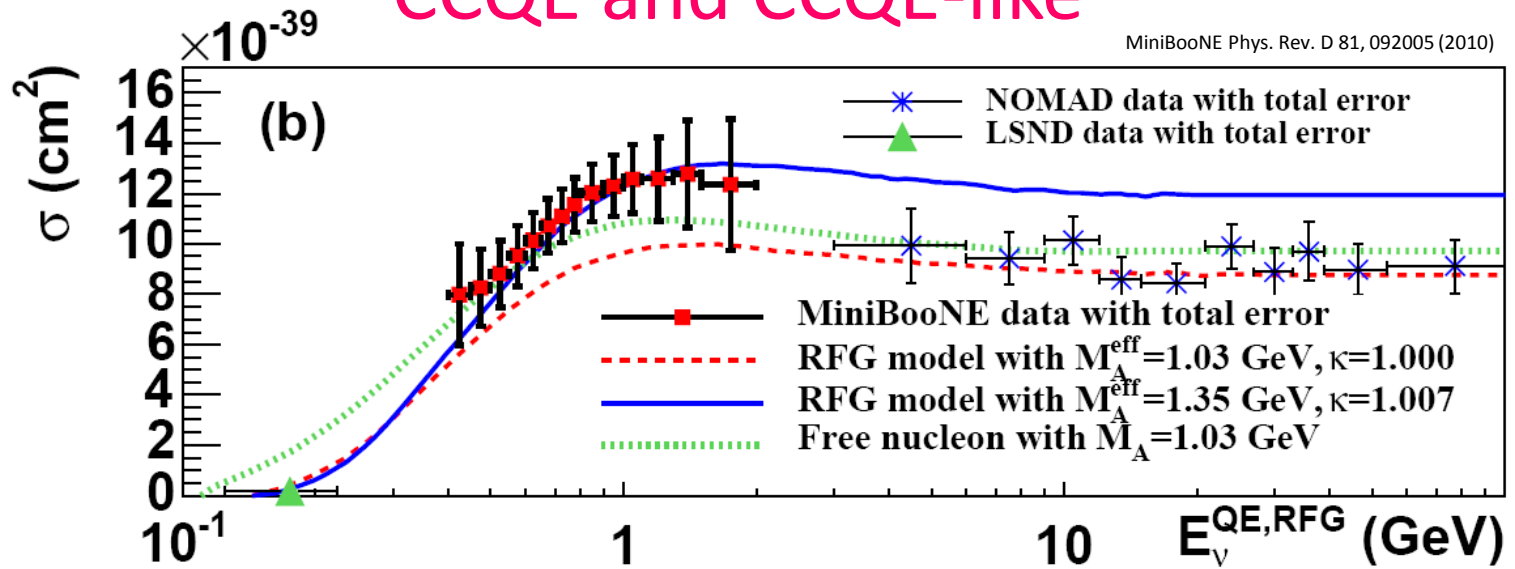


M. Martini, M. Ericson, G. Chanfray, J. Marteau *Phys. Rev. C* 80 065501 (2009)

Agreement with MiniBooNE without increasing M_A

CCQE and CCQE-like

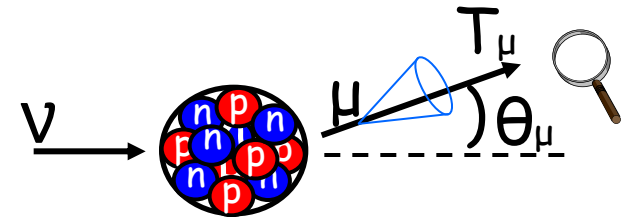
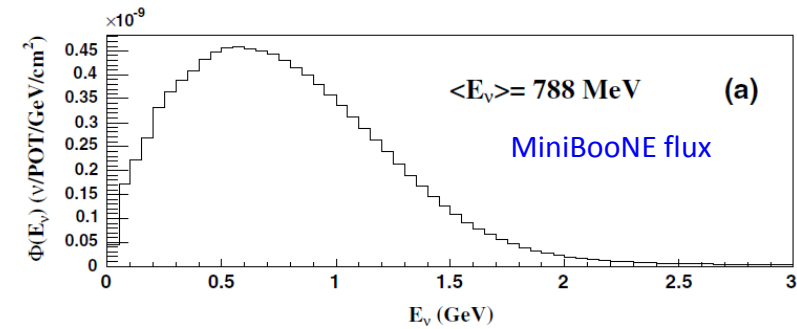
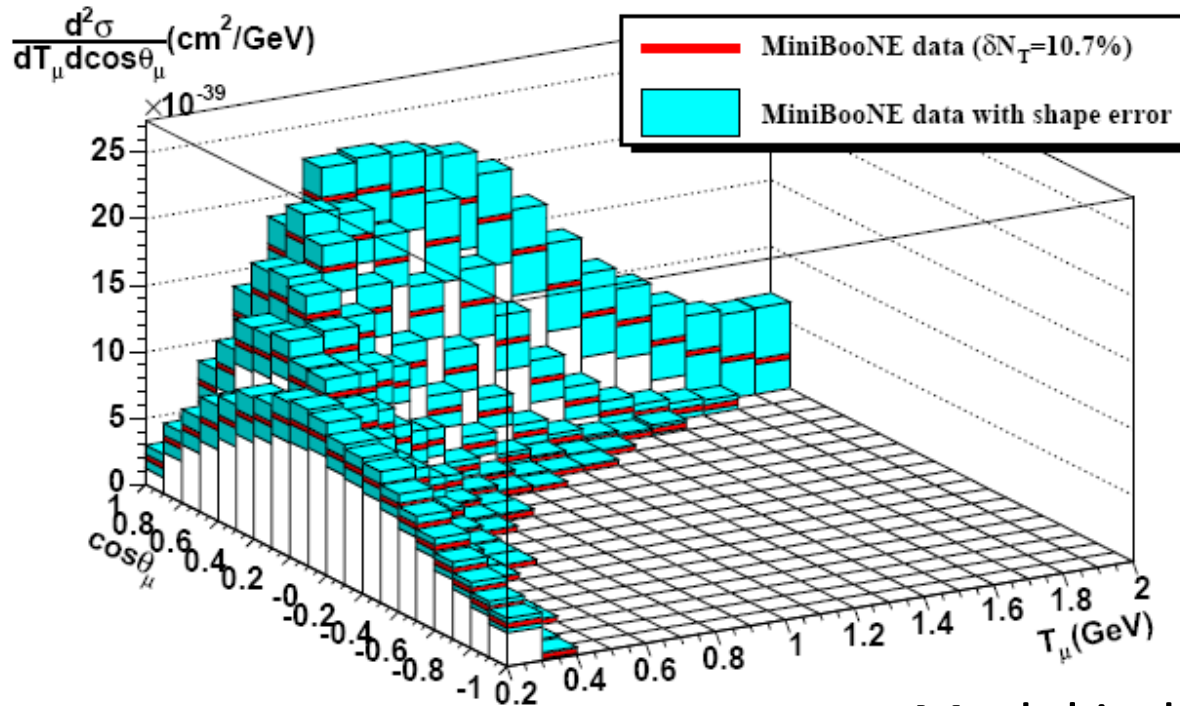
MiniBooNE Phys. Rev. D 81, 092005 (2010)



Cherenkov detectors measure CCQE-like which includes np-nh contributions

MiniBooNE CCQE-like flux-integrated double diff. X section

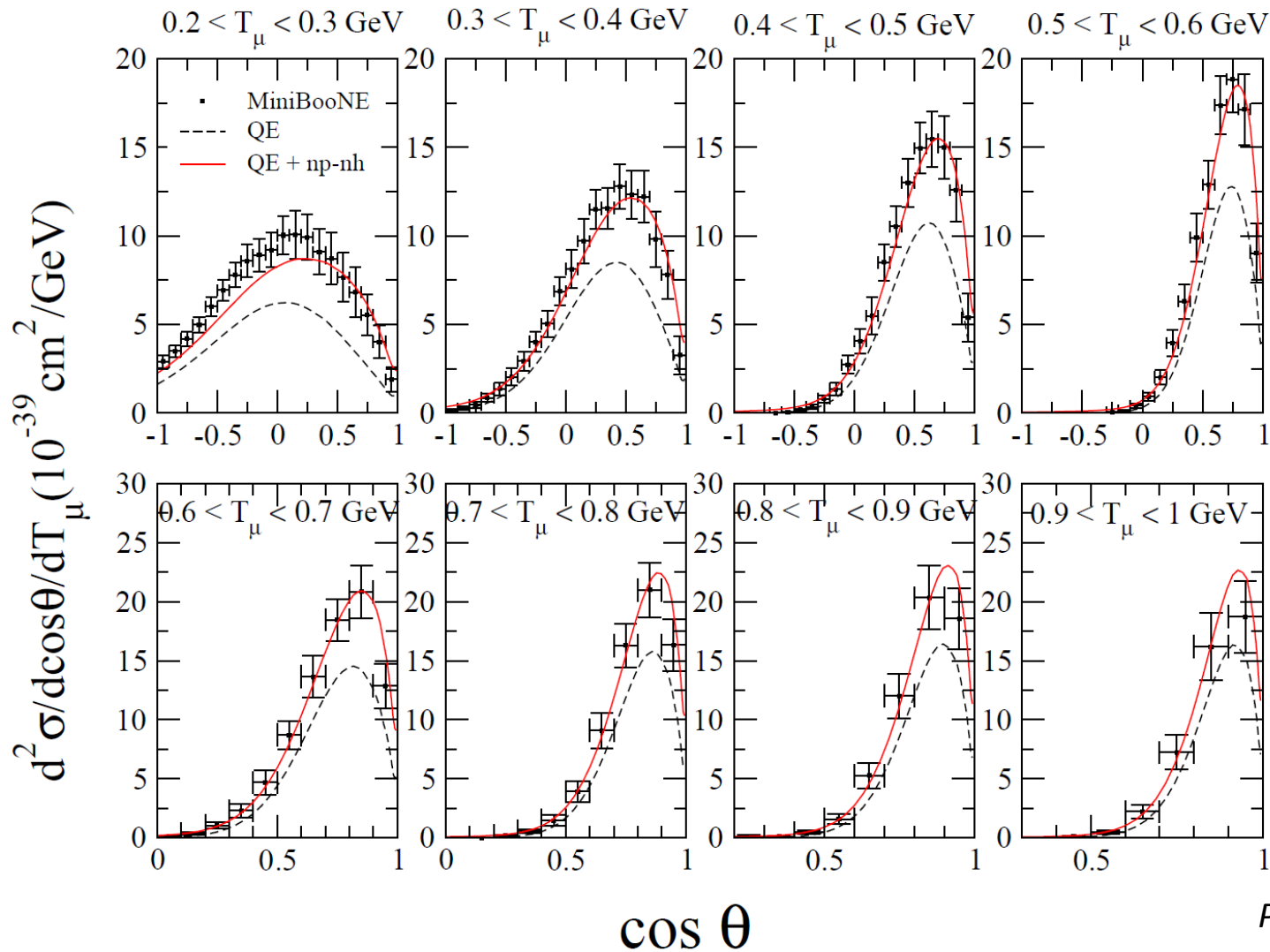
$$\frac{d^2\sigma}{dE_\mu d\cos\theta} = \int dE_\nu \left[\frac{d^2\sigma}{d\omega d\cos\theta} \right]_{\omega=E_\nu-E_\mu} \Phi(E_\nu)$$



Model independent measurement

MiniBooNE, Phys. Rev. D 81, 092005 (2010)

Flux-integrated double differential cross section



red: including np-nh

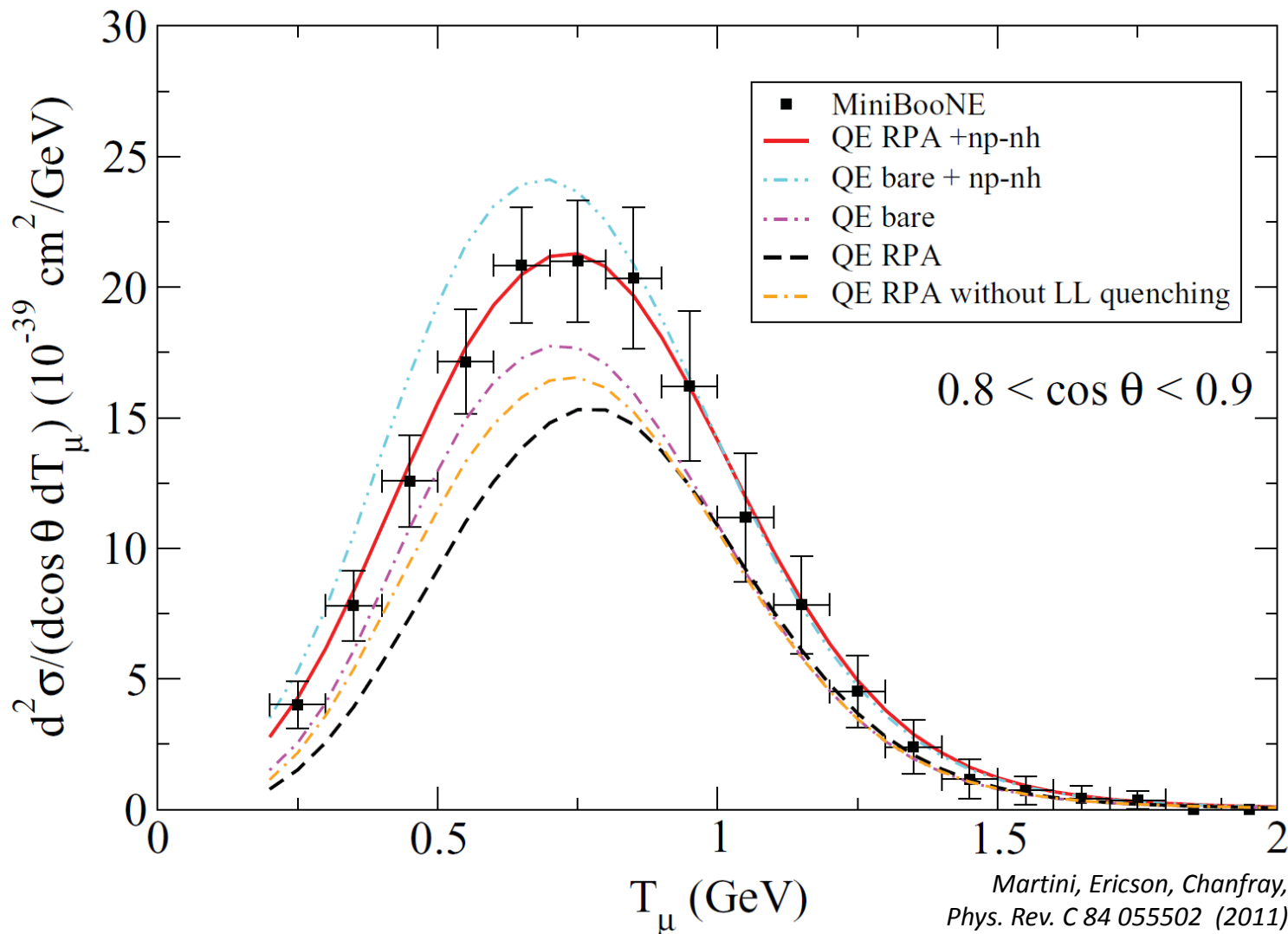
black: genuine QE

Martini, Ericson, Chanfray,
Phys. Rev. C 84 055502 (2011)

Agreement with MiniBooNE without increasing M_A once np-nh is included

Similar conclusions in Nieves *et al. PLB* 707, 72 (2012)

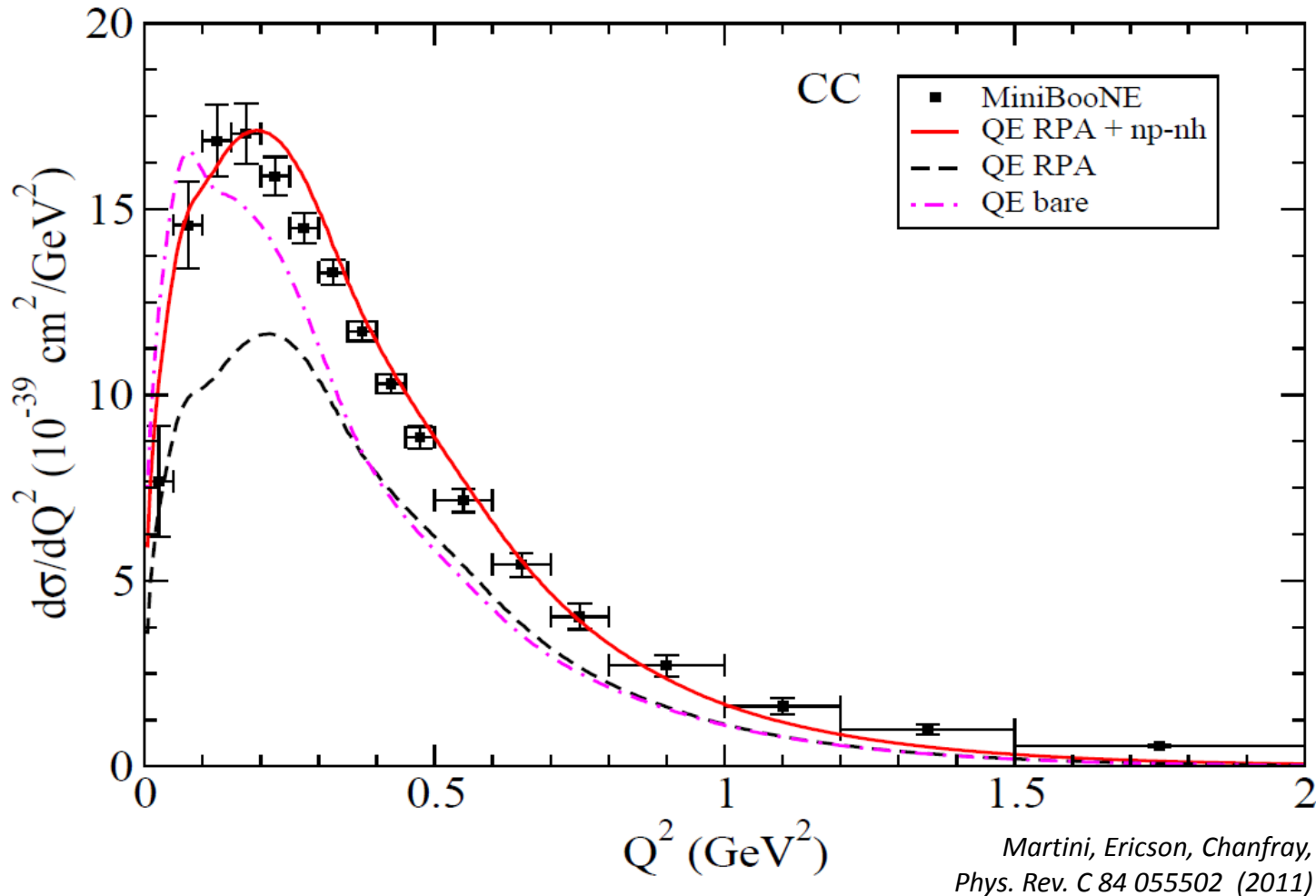
Flux-integrated ν CCQE double differential X section versus T_μ



Delicate balance between
RPA quenching and np-nh enhancement but...

Charged current Q^2 distribution

Historically of interest for the determination of the axial form factor



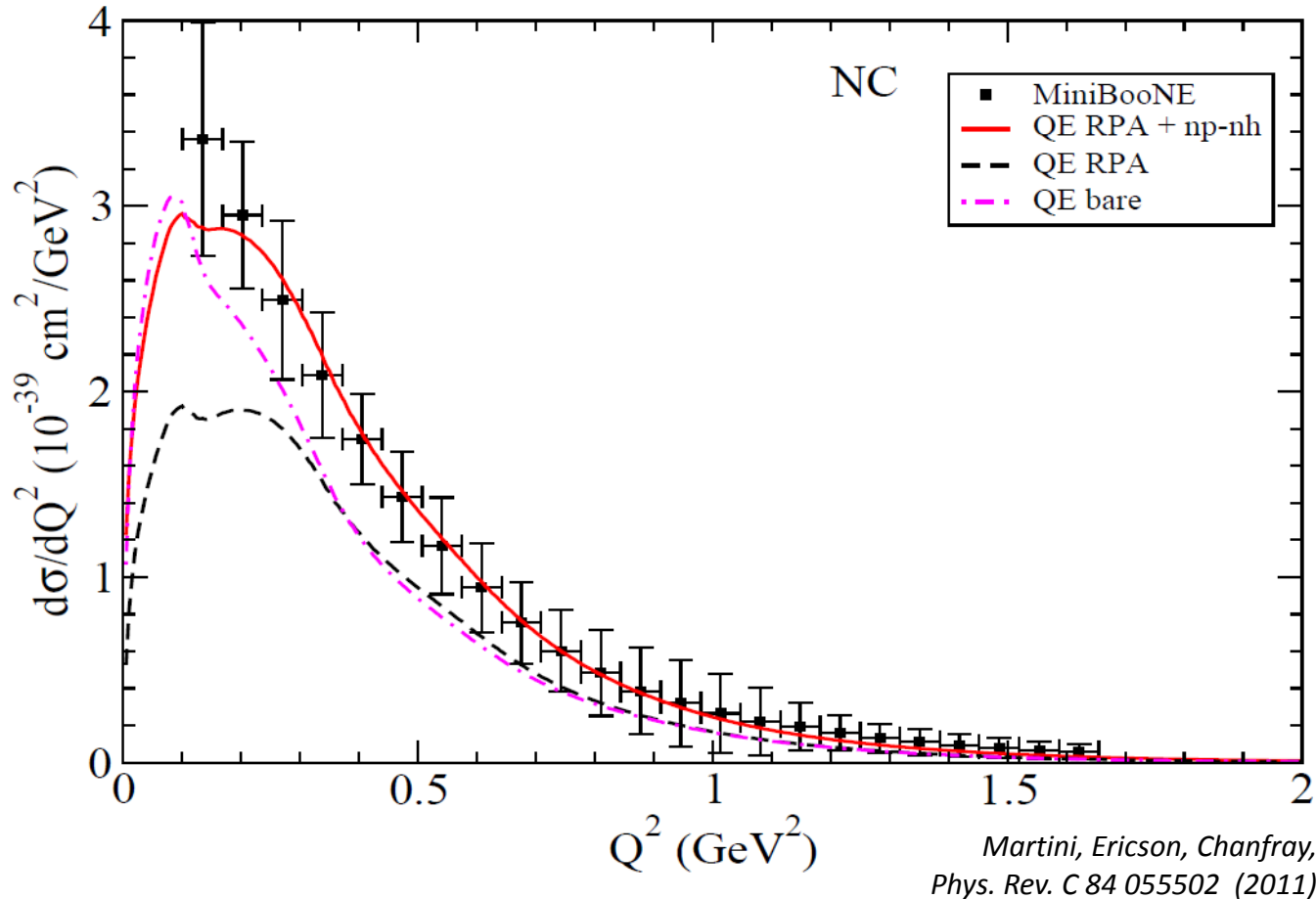
low Q^2 :
opposite actions of
RPA quenching and
np-nh enhancement

$Q^2 > 0.2$ GeV 2 :
np-nh contribution
singled out

Neutral current Q^2 distribution

Exp. Data: MiniBooNE, *Phys. Rev. D* 82, 092005 (2010)

obtained indirectly from the energy of ejected nucleons



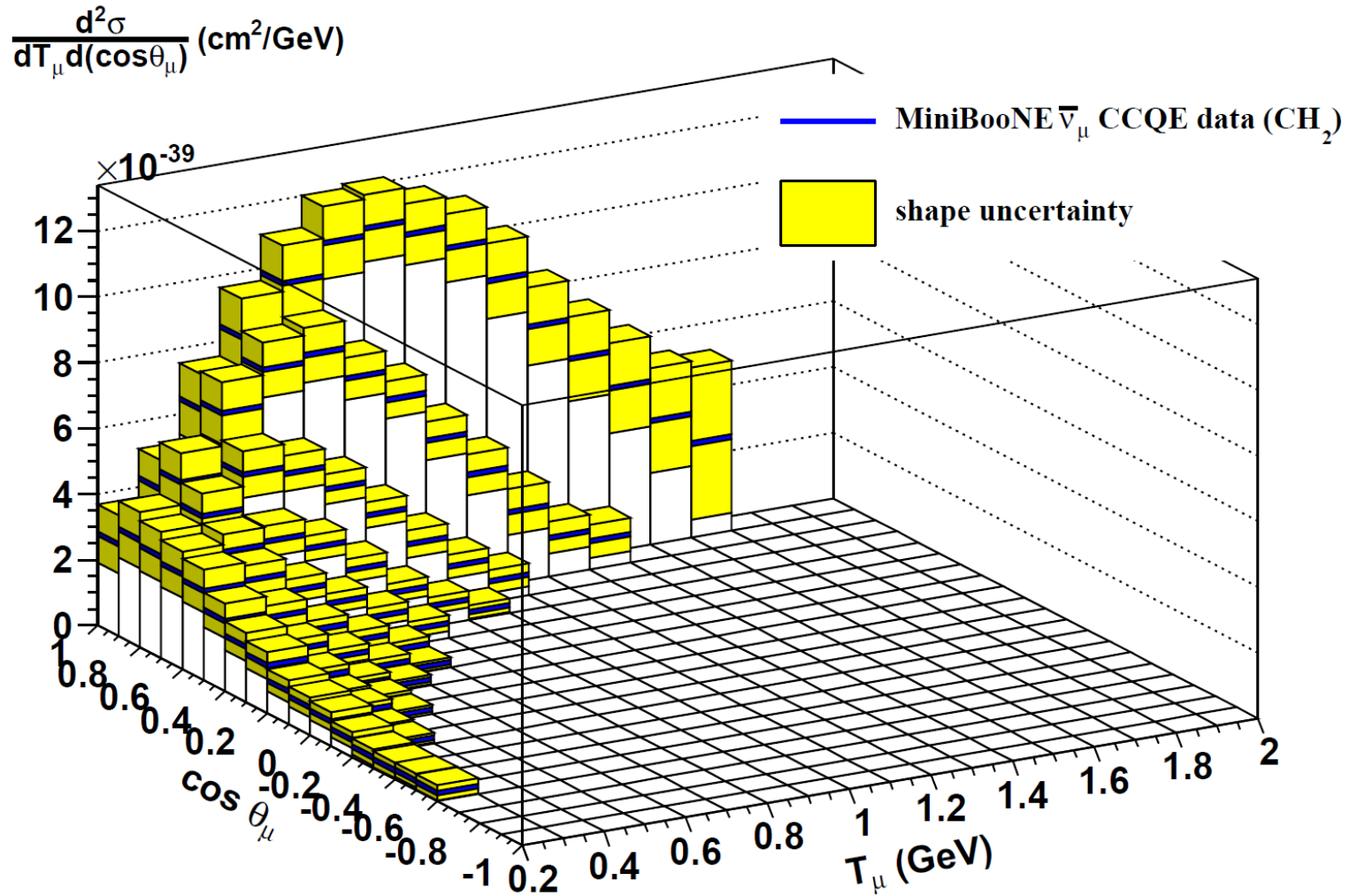
is not clear how multinucleon component shows up in the data

low Q^2 :
opposite actions of
RPA quenching and
np-nh enhancement

$Q^2 > 0.3 \text{ GeV}^2$:
np-nh contribution
singled out

Antineutrino MiniBooNE CCQE-like $d^2\sigma$

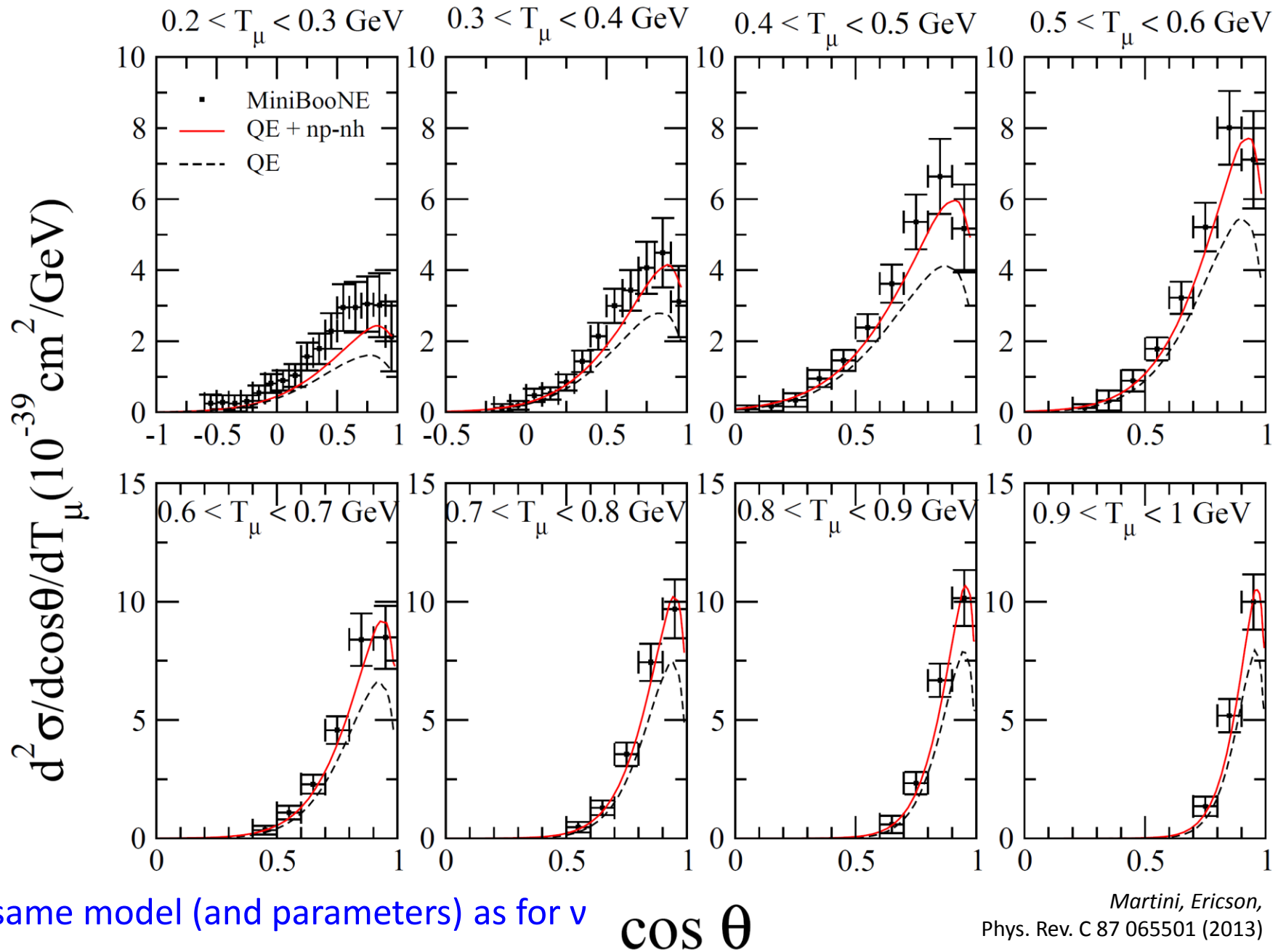
Recent Measurement



MiniBooNE, *Phys. Rev. D* 88 (2013) 032001

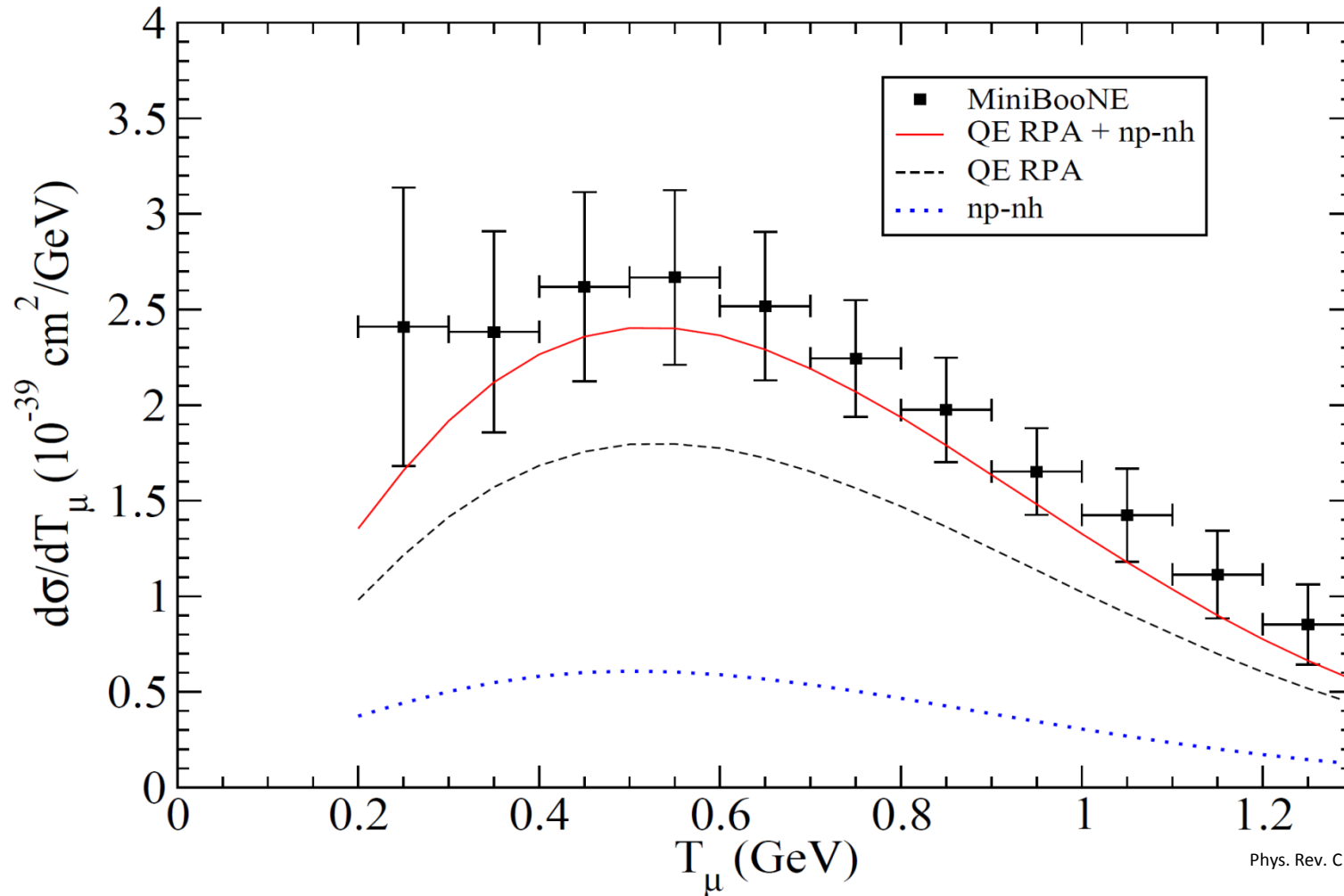
CH₂ and Carbon

Our results for $\bar{\nu}$



same model (and parameters) as for ν

Antineutrino $d\sigma/dT_\mu$



Martini, Ericson,
Phys. Rev. C 87 065501 (2013)

Our results are fully compatible with experimental data.

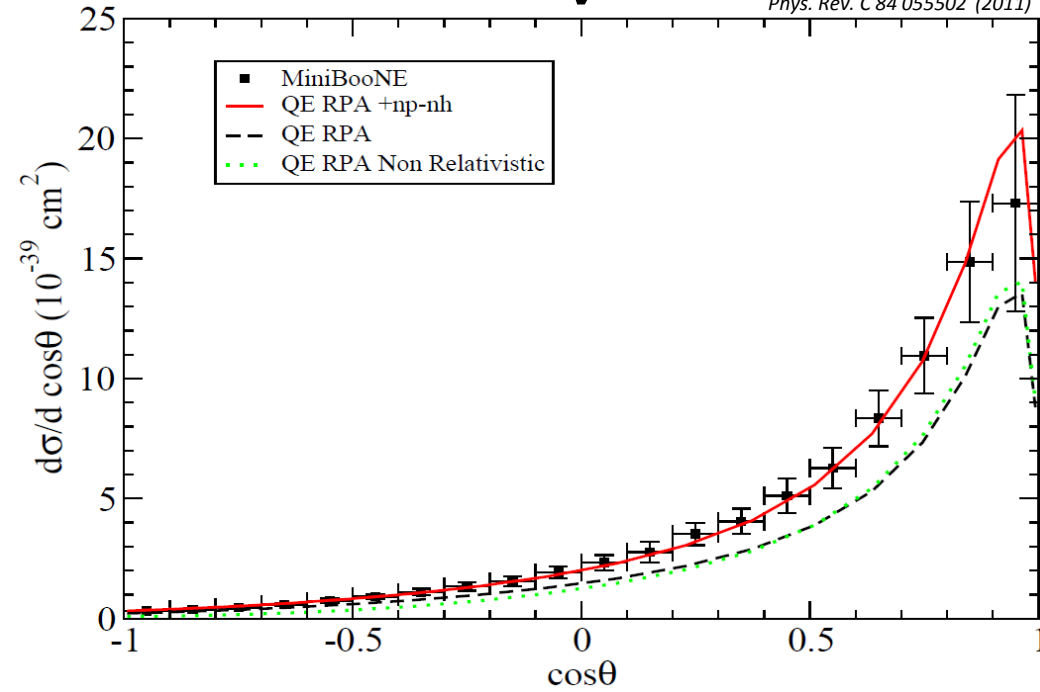
Nevertheless a small but systematic underestimation shows up.

We remind the additional normalization uncertainty of 17.2% in the MiniBooNE data not shown here.

$d\sigma/d\cos\theta$

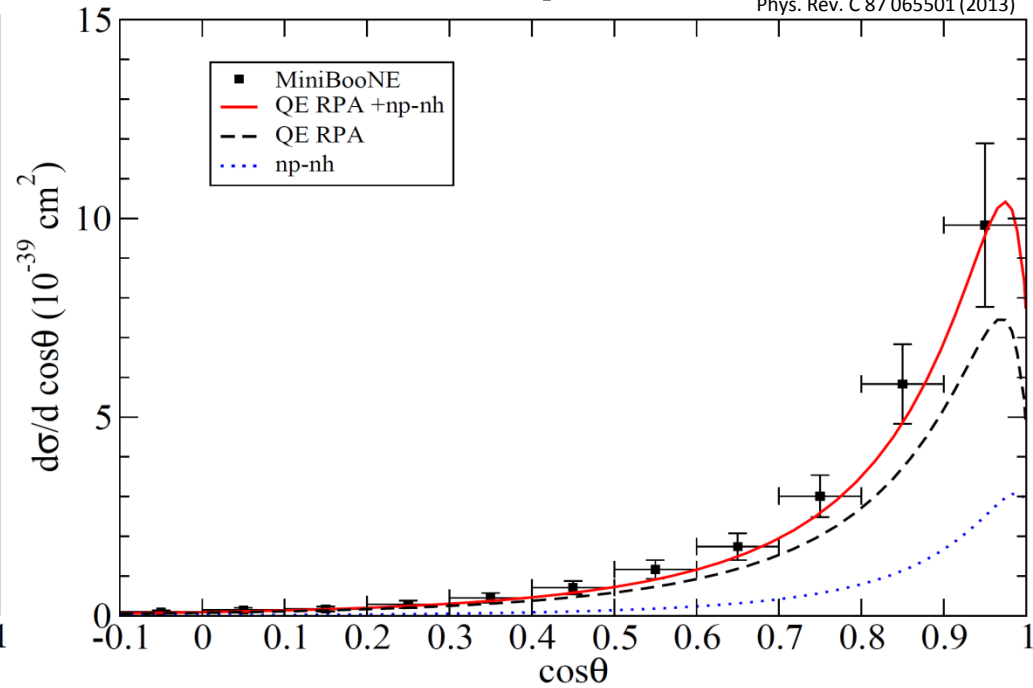
ν

Martini, Ericson, Chanfray,
Phys. Rev. C 84 055502 (2011)



$\bar{\nu}$

Martini, Ericson,
Phys. Rev. C 87 065501 (2013)

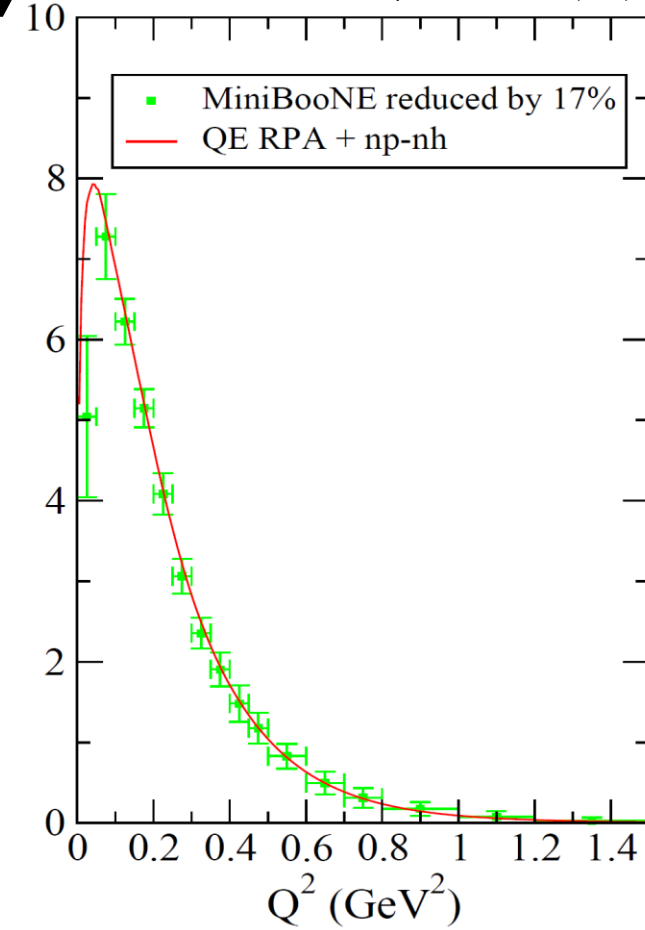
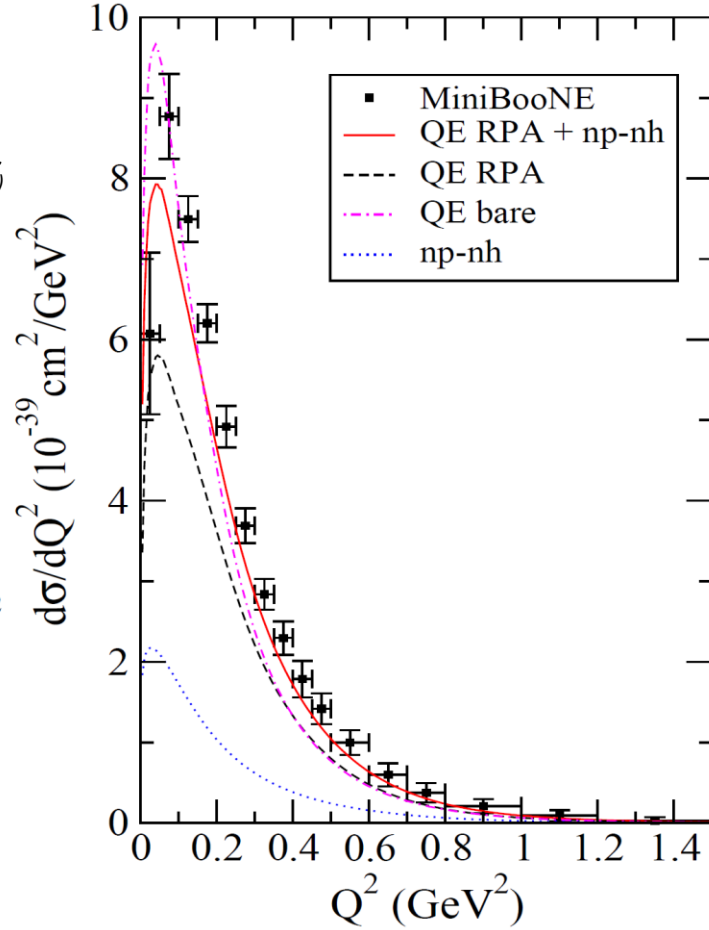
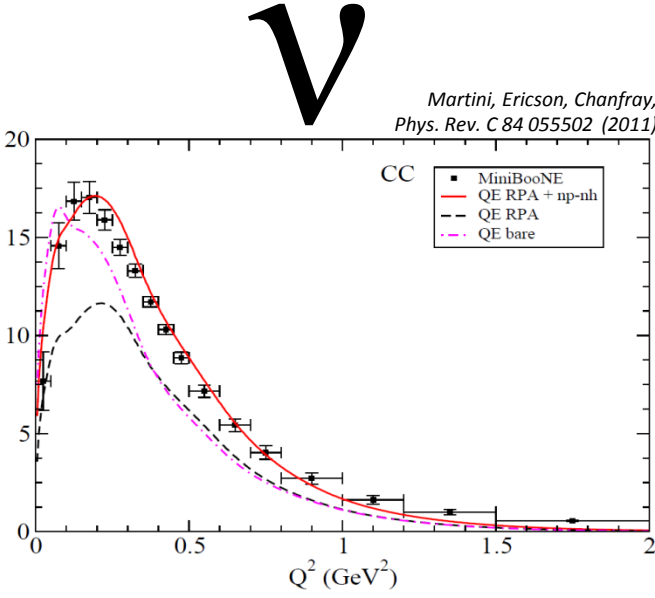


Antineutrino cross section falls more rapidly with angle than the neutrino one

CC Q^2 distribution

$\bar{\nu}$

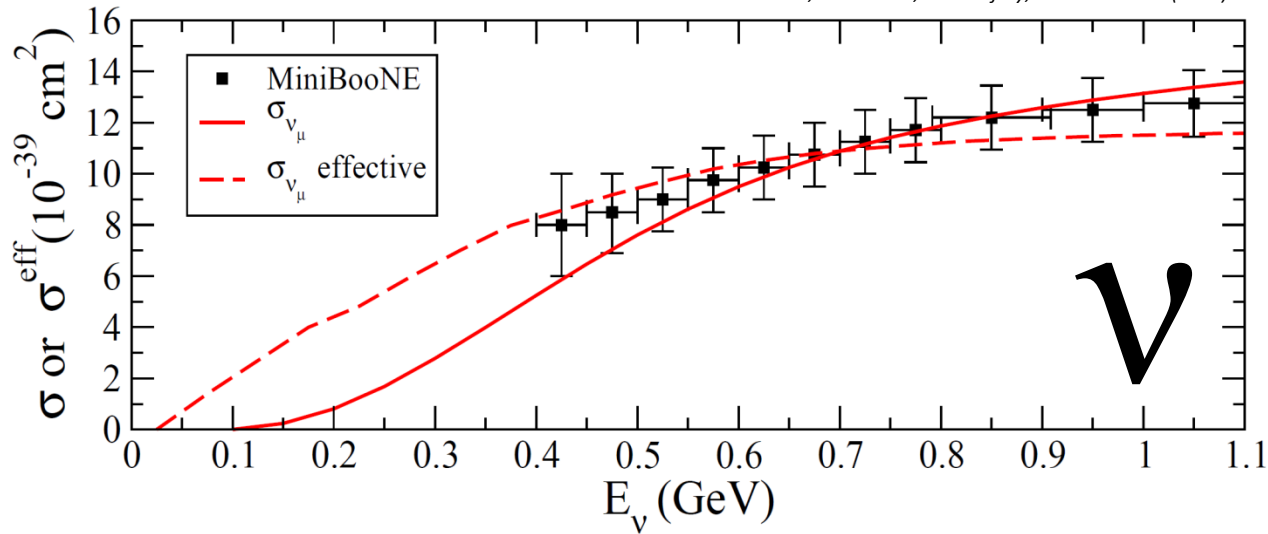
Martini, Ericson,
Phys. Rev. C 87 065501 (2013)



- Antineutrino Q^2 distribution peaks at smaller Q^2 values than the neutrino one
- RPA effects disappears beyond $Q^2 \geq 0.3 \text{ GeV}^2$ where the np-nh contribution is required

Real and effective cross sections versus $E_{\nu\mu}$

M. Martini, M. Ericson, G. Chanfray, PRD 87 013009 (2013)

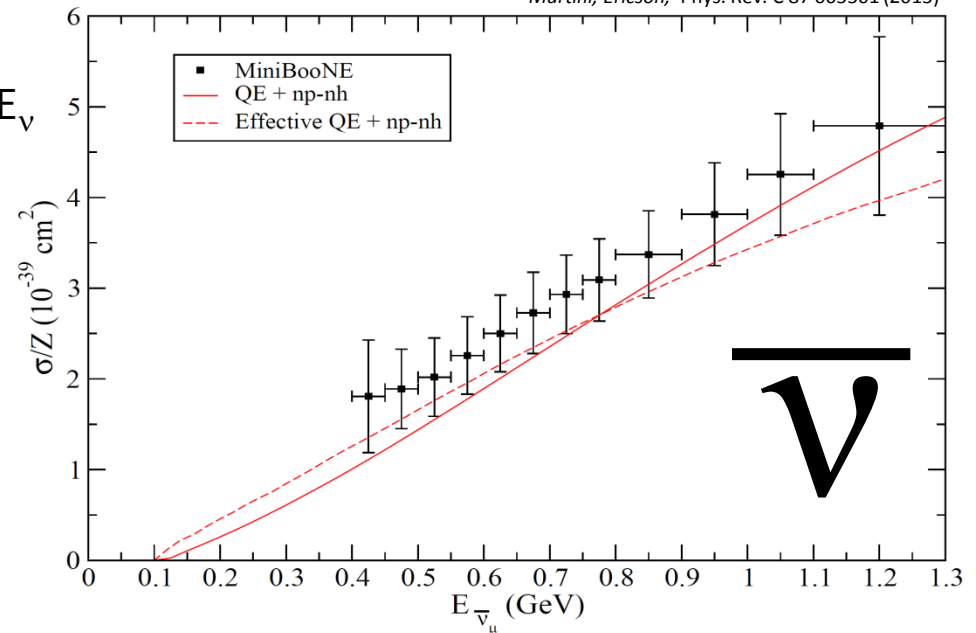


- Real: pure theoretical calculation (QE + np-nh) vs E_ν
- Effective: **taking into account the energy reconstruction corrections** as described in

M. Martini, M. Ericson, G. Chanfray,
PRD 85 093012 (2012); PRD 87 013009 (2013)

- crucial role of np-nh
- flux dependence

Martini, Ericson, Phys. Rev. C 87 065501 (2013)

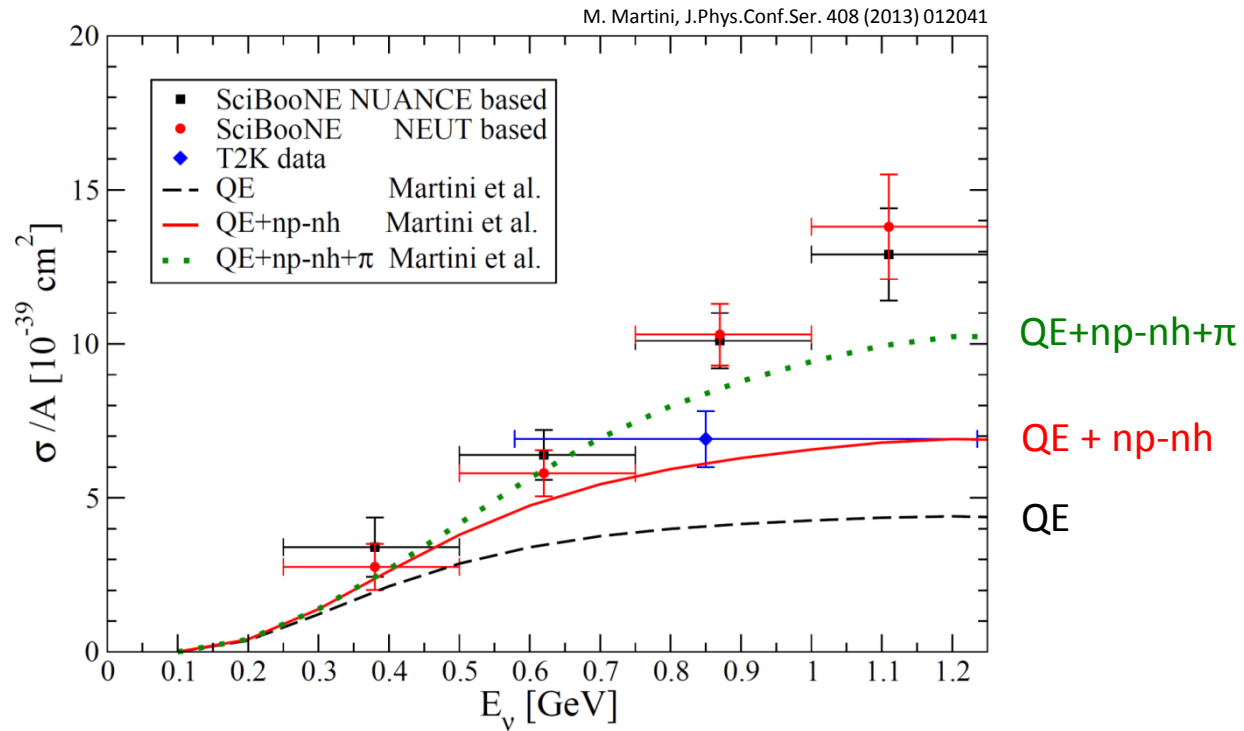


Inclusive CC cross section on Carbon

Less affected by background subtraction with respect to exclusive channels

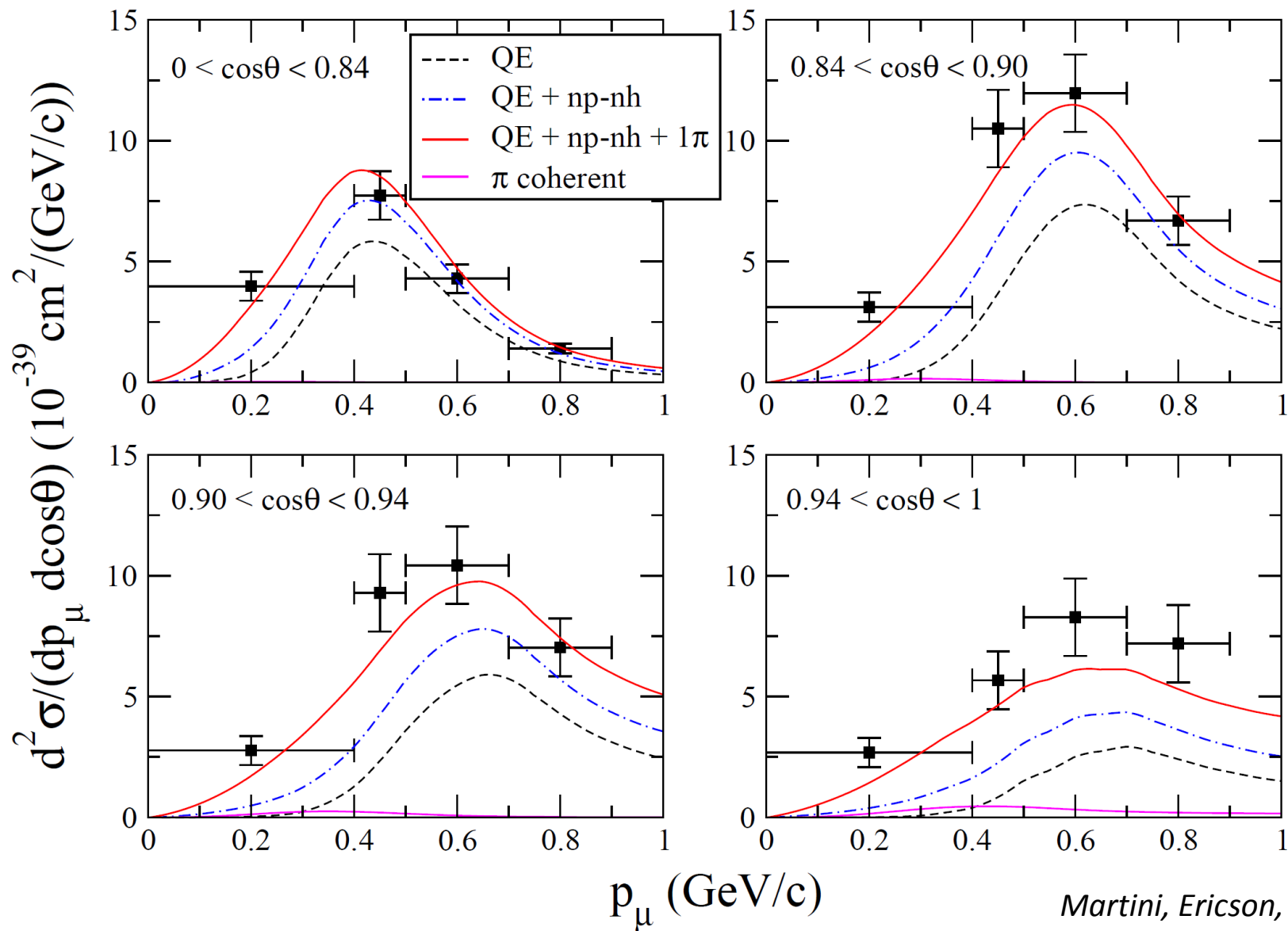
SciBooNE, *Phys. Rev. D* 83, 012005 (2011)

T2K, *Phys. Rev. D* 87, 092003 (2013)



M. Martini, M. Ericson, G. Chanfray, J. Marteau *Phys. Rev. C* 80 065501 (2009)

T2K Flux-integrated inclusive double differential cross section

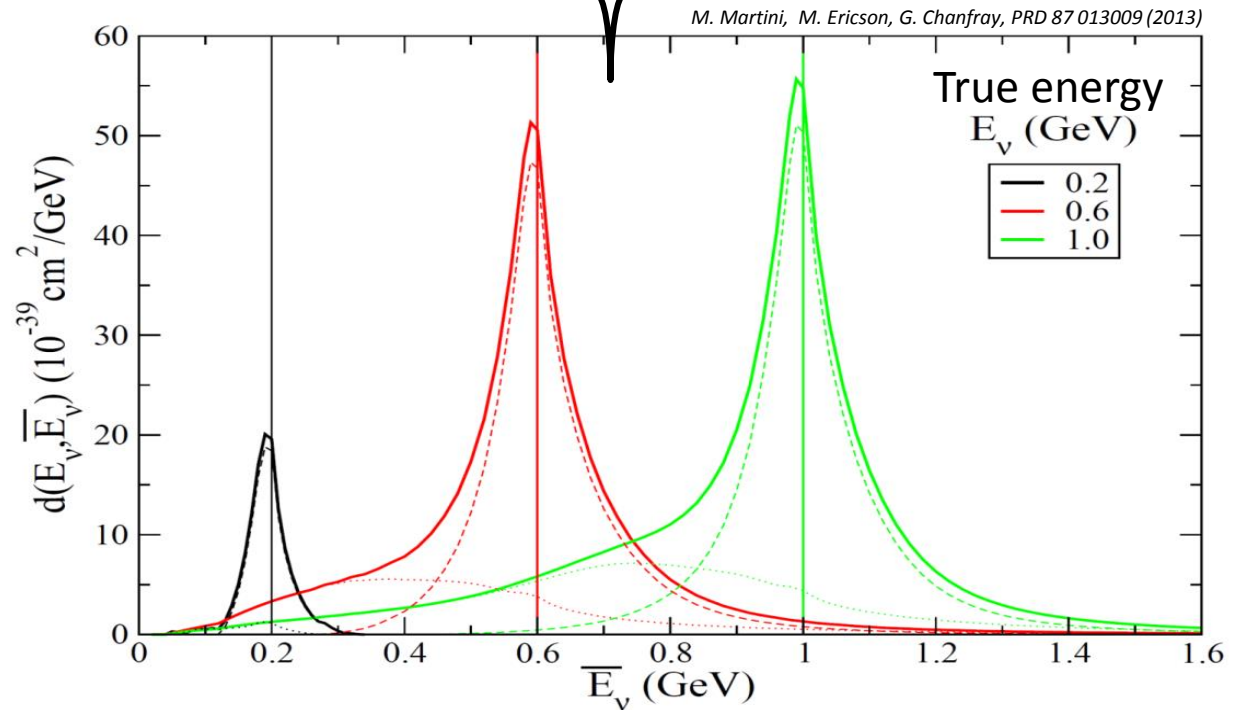


Martini, Ericson,
to be submitted

Neutrino energy reconstruction problems and neutrino oscillations

$$D_{rec}(\overline{E}_\nu) = \int dE_\nu \Phi(E_\nu) \left[\int_{E_l^{min}}^{E_l^{max}} dE_l \frac{M E_l - m_l^2/2}{\overline{E}_\nu^2 P_l} \left[\frac{d^2\sigma}{d\omega d\cos\theta} \right]_{\omega=E_\nu-E_l, \cos\theta=\cos\theta(E_l, \overline{E}_\nu)} \right]$$

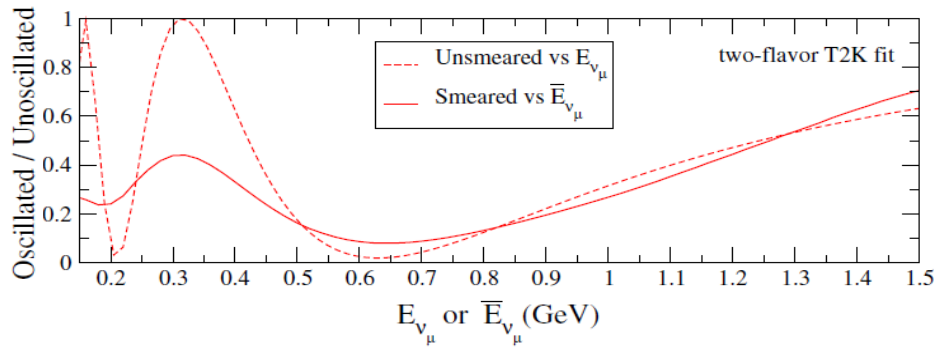
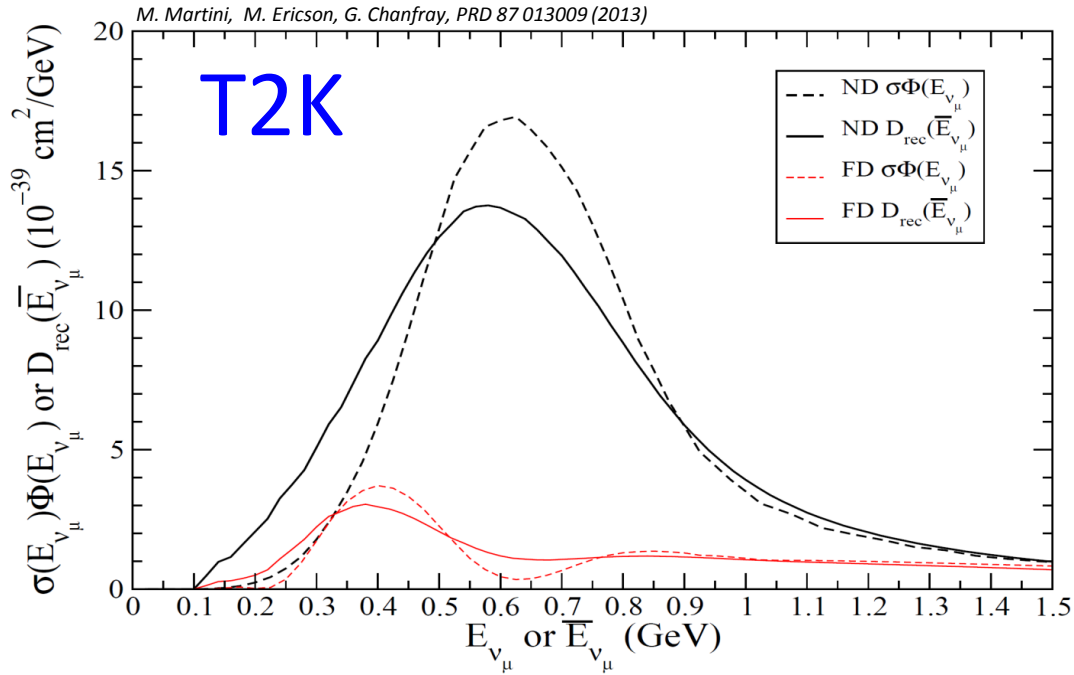
The quantity $D_{rec}(\overline{E}_\nu)$ corresponds to the product $\sigma(E_\nu)\Phi(E_\nu)$ but in terms of reconstructed ν energy \overline{E}_ν



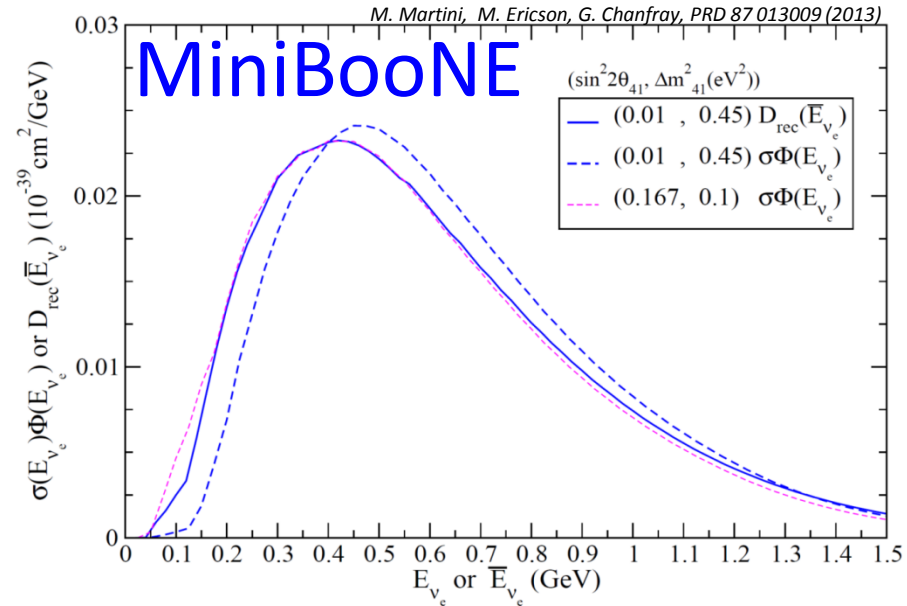
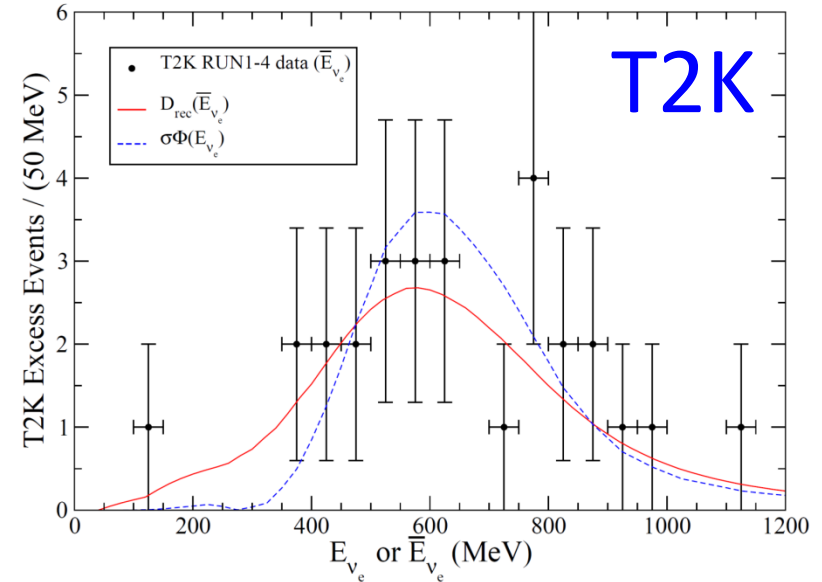
Crucial role of np-nh: low energy tail

M. Martini, M. Ericson, G. Chanfray
 - Phys. Rev. D 85 093012 (2012)
 - Phys. Rev. D 87 013009 (2013)

ν_μ disappearance



$\nu_\mu \rightarrow \nu_e$



Our theoretical model

Neutrino-nucleus interaction

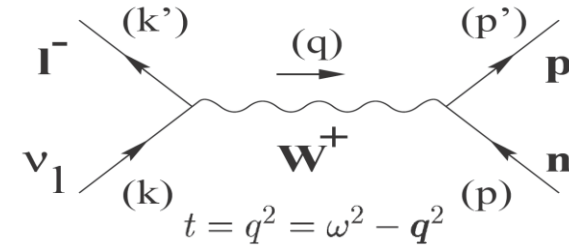
$$\mathcal{L}_W = \frac{G_F}{\sqrt{2}} \cos(\theta_C) l_\mu h^\mu$$

lepton

$$\langle k', s' | l_\mu | k, s \rangle = e^{-iqx} \bar{u}(k', s') [\gamma_\mu (1 - \gamma_5)] u(k, s)$$

hadron

$$\langle p', s' | h^\mu | p, s \rangle = e^{iqx} \bar{u}(p', s') \left[\underbrace{F_1(t) \gamma^\mu + F_2(t) \sigma^{\mu\nu} \frac{iq_\nu}{2M_N}}_{\text{Vector}} + \underbrace{G_A(t) \gamma^\mu \gamma_5 + G_P(t) \gamma_5 \frac{q^\mu}{2M_N}}_{\text{Axial}} \right] \tau_+ u(p, s)$$



Cross section:

$$\frac{\partial^2 \sigma}{\partial \Omega_{k'} \partial k'} = \frac{G_F^2 \cos^2 \theta_C \mathbf{k}'^2}{2\pi^2} \cos^2 \frac{\theta}{2} \left[\underbrace{G_E^2 (1 - \frac{\omega^2}{q^2})^2}_{\text{Vector}} \underbrace{R_C}_{\text{Charge}} + \underbrace{G_A^2 \frac{(M_\Delta - M_N)^2}{q^2}}_{\text{Axial}} \underbrace{R_L}_{\text{Isospin Spin-Longitudinal}} \right. \\ \left. + \underbrace{(G_M^2 \frac{\omega^2}{q^2} + G_A^2)}_{\text{Magnetic}} (1 - \frac{\omega^2}{q^2} + 2 \tan^2 \frac{\theta}{2}) \underbrace{R_T}_{\text{Isospin Spin Transverse}} \pm \underbrace{G_A G_M}_{\text{Magnetic}} 2 \frac{k + k'}{M_N} \tan^2 \frac{\theta}{2} \underbrace{R_T}_{\text{Isospin Spin Transverse}} \right]$$

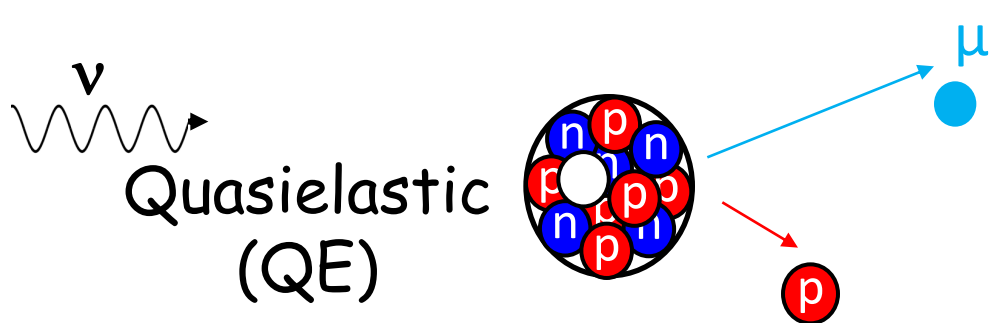
Nucleon properties → Form factors: Electric G_E , Magnetic G_M , Axial G_A

Nuclear dynamics → Nuclear Response Functions:

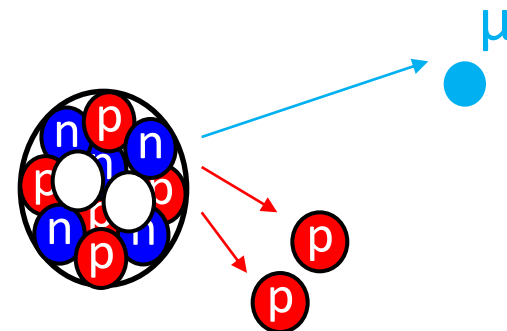
Charge $R_C(\tau)$, Isospin Spin-Longitudinal $R_L(\tau, \sigma \cdot q)$, Isospin Spin Transverse $R_T(\tau, \sigma \times q)$

Neutrino - nucleus interaction @ $E_\nu \sim 0$ (1 GeV)

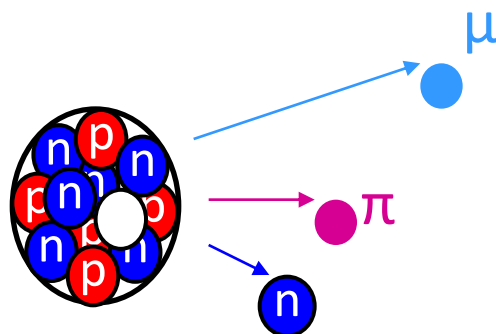
[MiniBooNE, T2K energies]



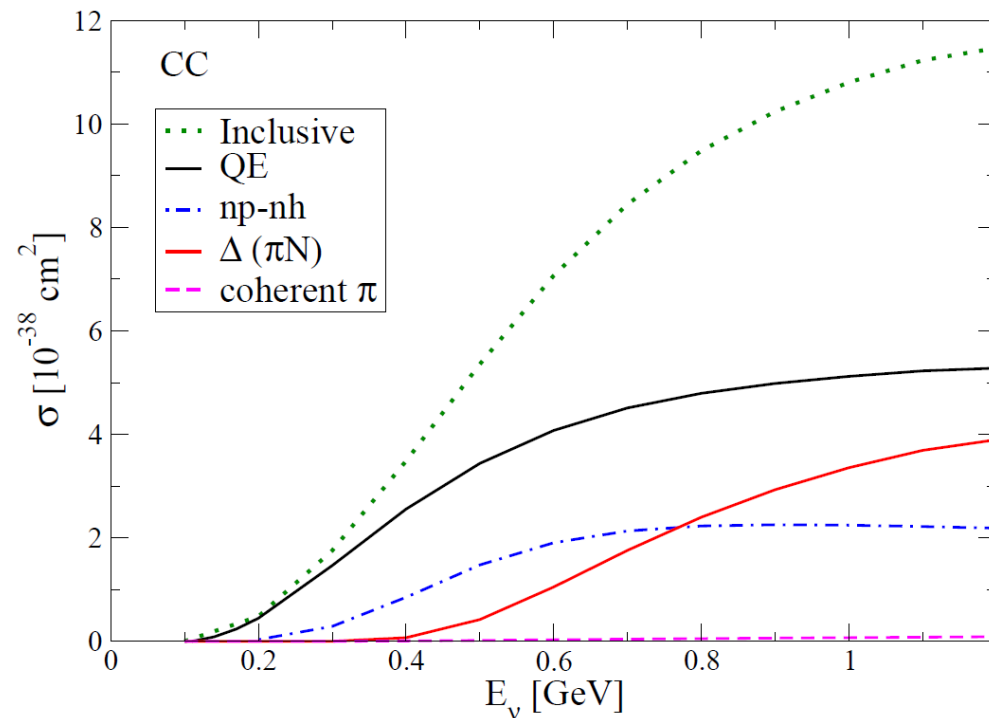
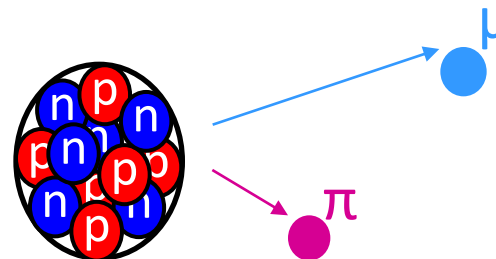
Two Nucleons knock-out (2p-2h)



Incoherent π production



Coherent π production



M. Martini, M. Ericson, G. Chanfray, J. Marteau, PRC 80 065501 (2009)

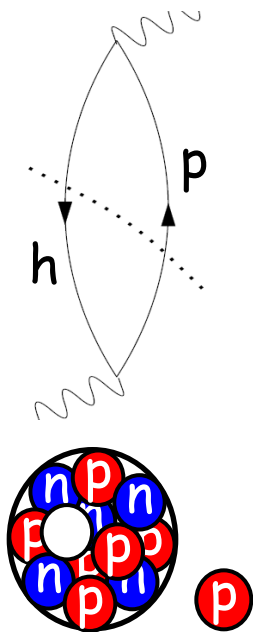
Response picture

Response functions

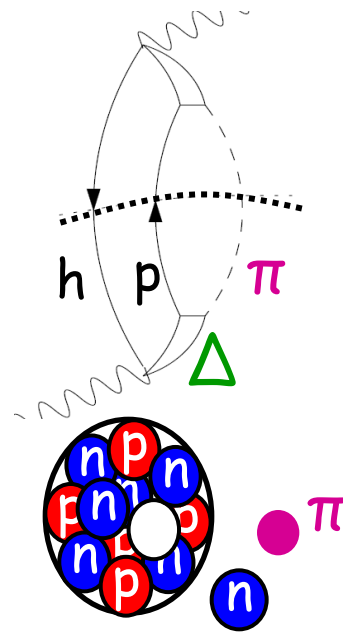
$$R(\omega, q) = -\frac{\mathcal{V}}{\pi} \text{Im}[\Pi(\omega, q, q)]$$

easy to separate the several channels

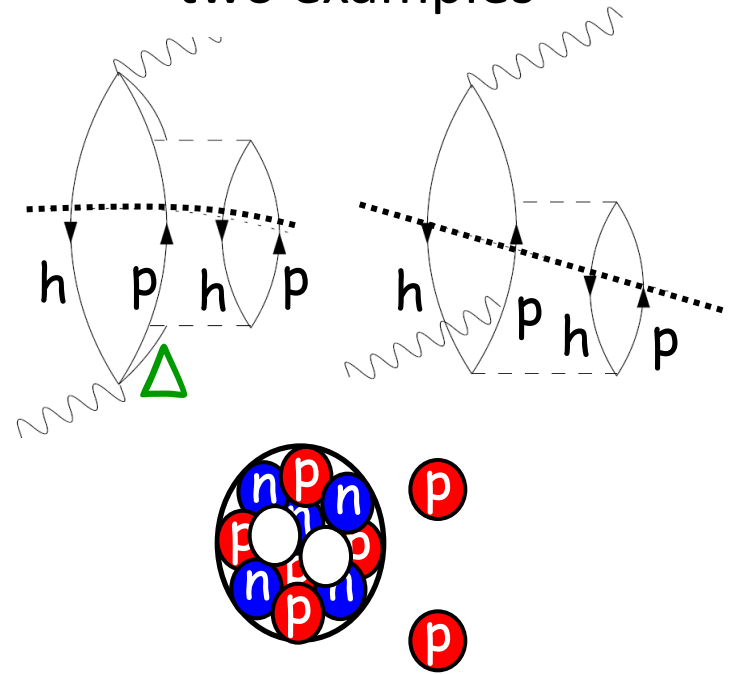
1p-1h
QE



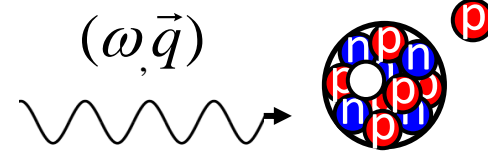
1p-1h
1 π production



2p-2h:
two examples



The nuclear response



Nucleon-Nucleon interaction switched off

Nucleons respond individually

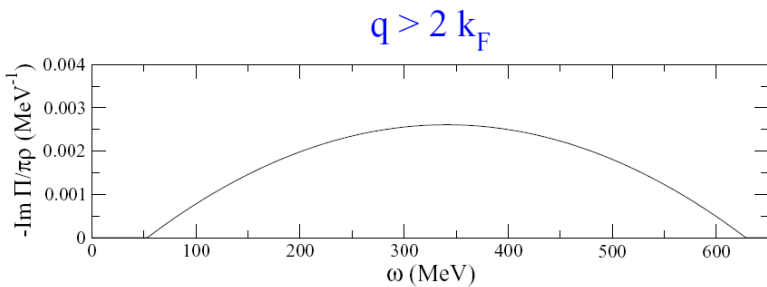
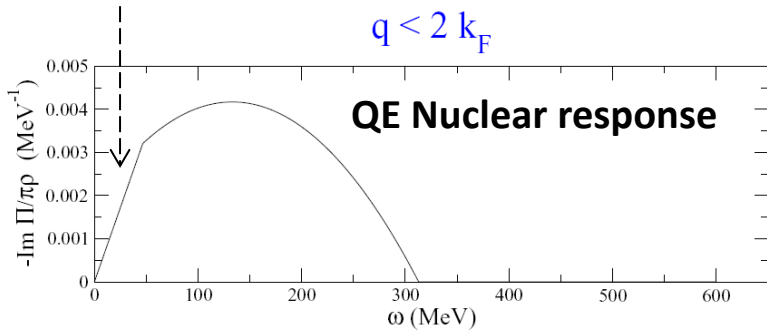
Nucleon at rest:

$$R \propto \delta\left(\omega - \left(\sqrt{q^2 + M^2} - M\right)\right)$$

Nucleon inside the nucleus:

Fermi motion spreads δ distribution (**Fermi Gas**)

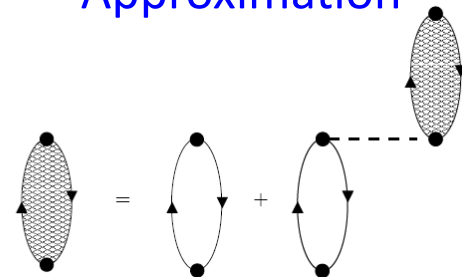
Pauli blocking cuts part of the low momentum Resp.



Nucleon-Nucleon interaction switched on

The nuclear response becomes collective

Random Phase Approximation

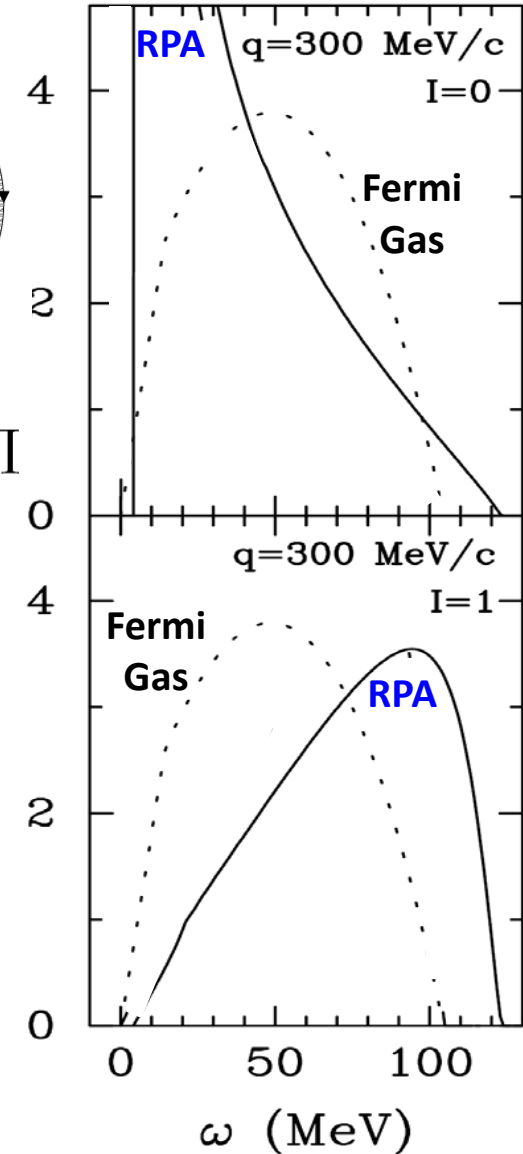


$$\Pi = \Pi^0 + \Pi^0 V \Pi$$

*Force acting on one nucleon is transmitted by the interaction

*Shift of the peak with respect to Fermi Gas, decrease, increase,...

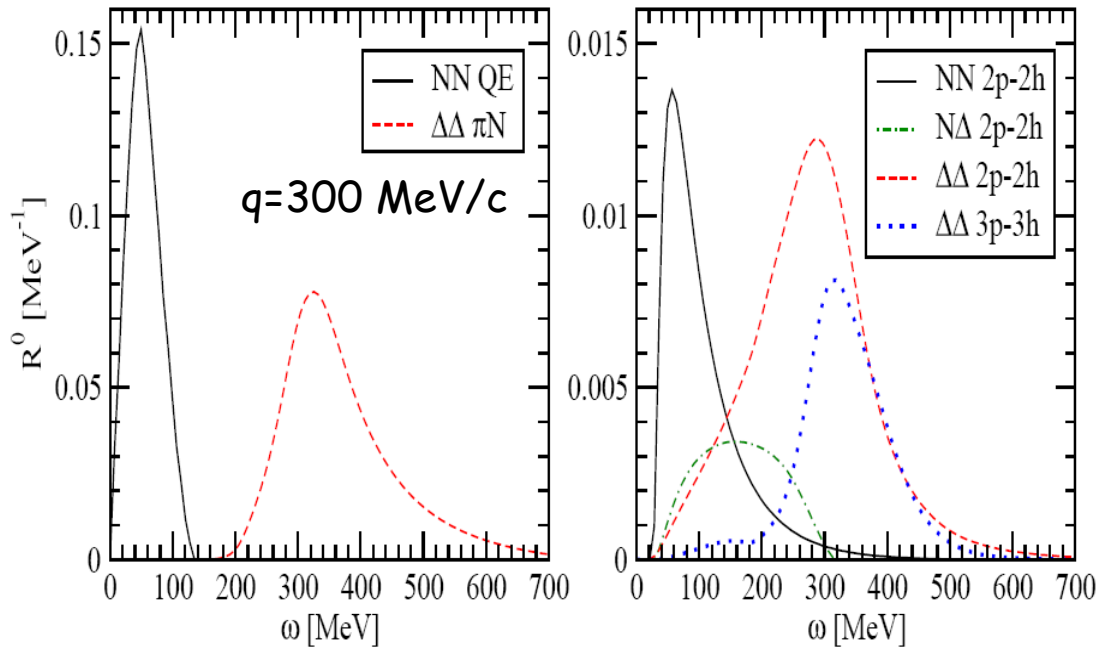
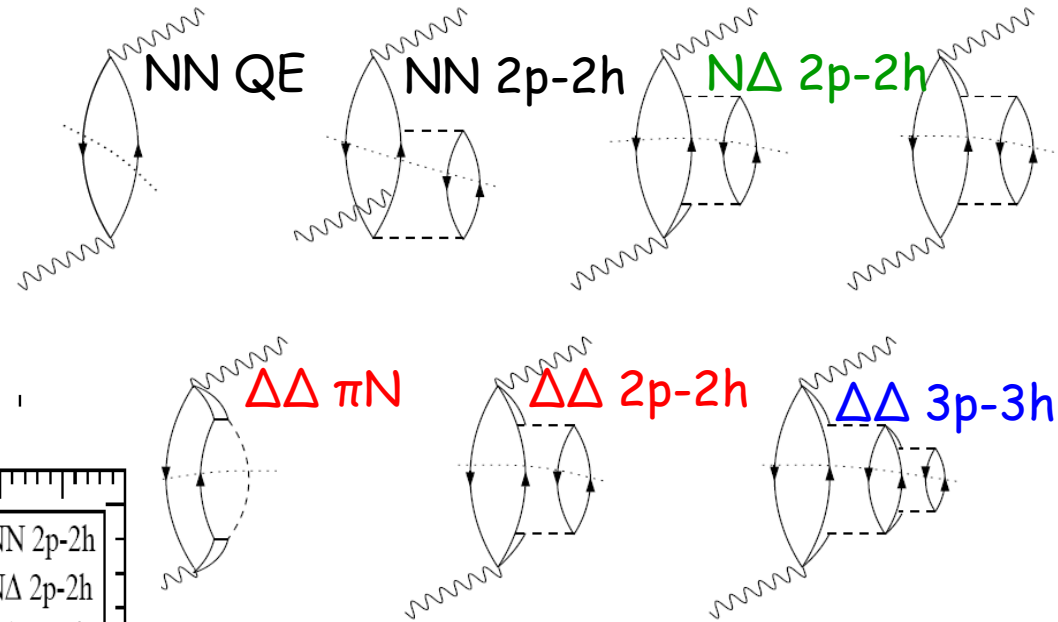
Alberico, Ericson, Molinari, Nucl. Phys. A 379, 429 (1982)



Bare nuclear responses

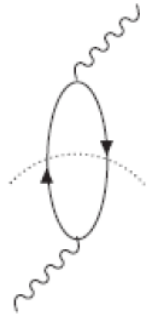
Several partial components
(final state channels)

- QE (1 nucleon knock-out)
- Pion production
- Multinucleon emission



Bare polarization propagators

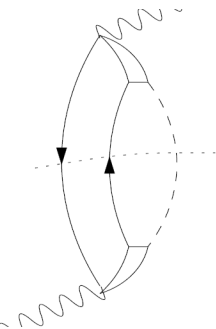
Quasielastic



$$\Pi^0(\vec{q}, \omega) = g \int \frac{d\vec{k}}{(2\pi)^3} \left[\frac{\theta(|\vec{k} + \vec{q}| - k_F)\theta(k_F - k)}{\omega - (\omega_{\vec{k}+\vec{q}} - \omega_{\vec{k}}) + i\eta} - \frac{\theta(k_F - |\vec{k} + \vec{q}|\theta(k - k_F))}{\omega + (\omega_{\vec{k}} - \omega_{\vec{k}+\vec{q}}) - i\eta} \right]$$

Nucleon-hole

Pion production



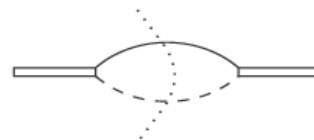
$$\Pi_{\Delta-h}(q) = \frac{32\tilde{M}_{\Delta}}{9} \int \frac{d^3k}{(2\pi)^3} \theta(k_F - k) \left[\frac{1}{s - \tilde{M}_{\Delta}^2 + i\tilde{M}_{\Delta}\tilde{\Gamma}_{\Delta}} - \frac{1}{u - \tilde{M}_{\Delta}^2} \right]$$

Delta-hole

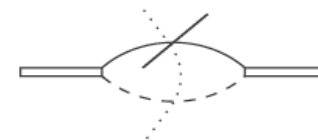
Delta in the medium

Mass

$$\tilde{M}_\Delta = M_\Delta + 40(\text{MeV}) \frac{\rho}{\rho_0}$$



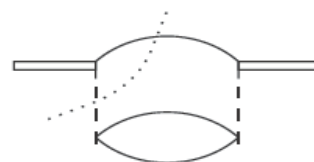
$\Delta \rightarrow \pi N$



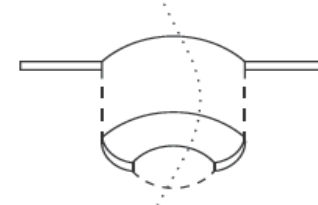
Pauli correction (F_P)

Width

$$\tilde{\Gamma}_\Delta = \Gamma_\Delta F_P - 2\text{Im}(\Sigma_\Delta)$$

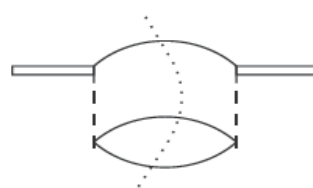


Pion distortion (C_Q)

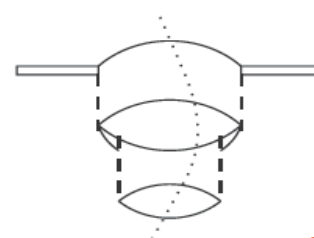


Self energy

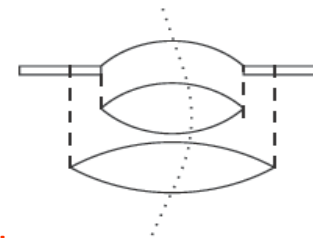
$$\text{Im}(\Sigma_\Delta(\omega)) = - \left[C_Q \left(\frac{\rho}{\rho_0} \right)^\alpha + C_{2p2h} \left(\frac{\rho}{\rho_0} \right)^\beta + C_{3p3h} \left(\frac{\rho}{\rho_0} \right)^\gamma \right]$$



2p-2h

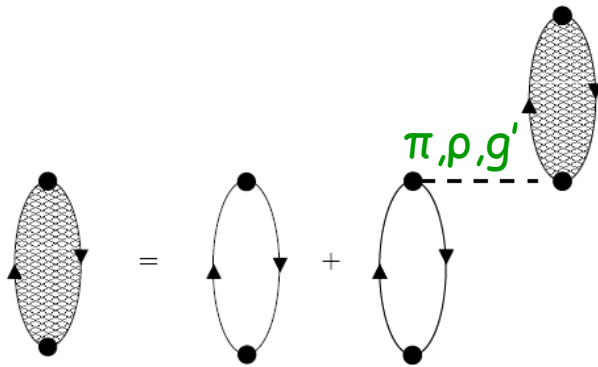


3p-3h



E. Oset and L. L. Salcedo, Nucl. Phys. A 468, 631 (1987)

Switching on the interaction: random phase approximation

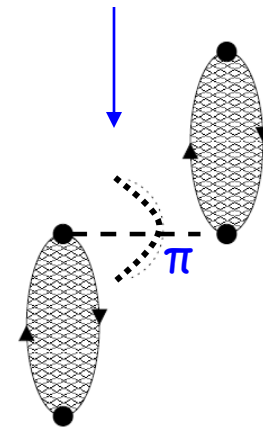
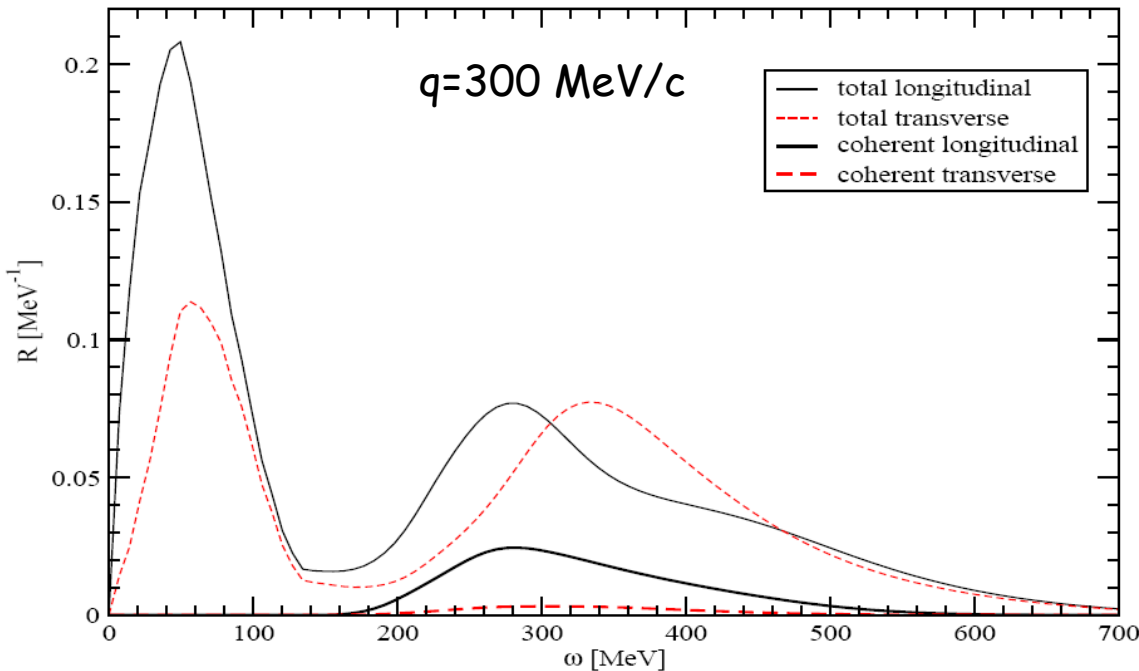


RPA

$$\Pi = \Pi^0 + \Pi^0 V \Pi$$

$$\text{Im}\Pi = |\Pi|^2 \text{Im}V + |1 + \Pi V|^2 \text{Im}\Pi^0$$

M. Martini, M. Ericson, G. Chanfray, J. Marteau, PRC 80 065501 (2009)



coherent π production

$$\Pi^0 = \sum_{k=1}^{N_k} \Pi_{(k)}^0$$

exclusive channels:
QE, 2p-2h, $\Delta \rightarrow \pi N$...

Several partial components
treated in self-consistent,
coupled and coherent way

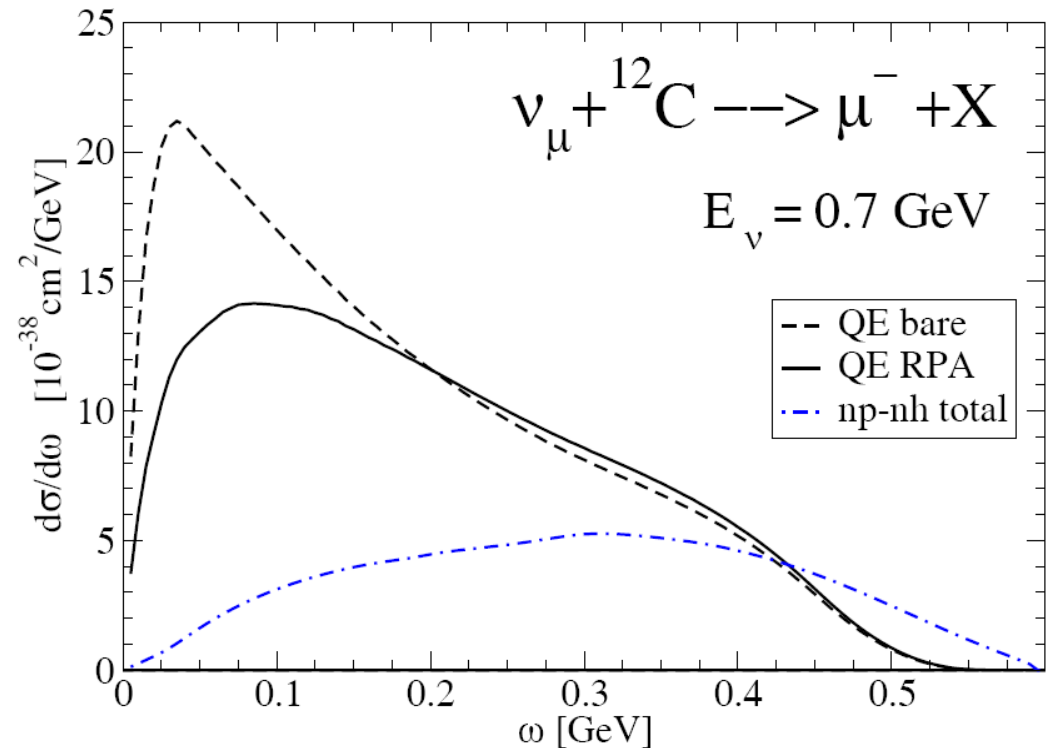
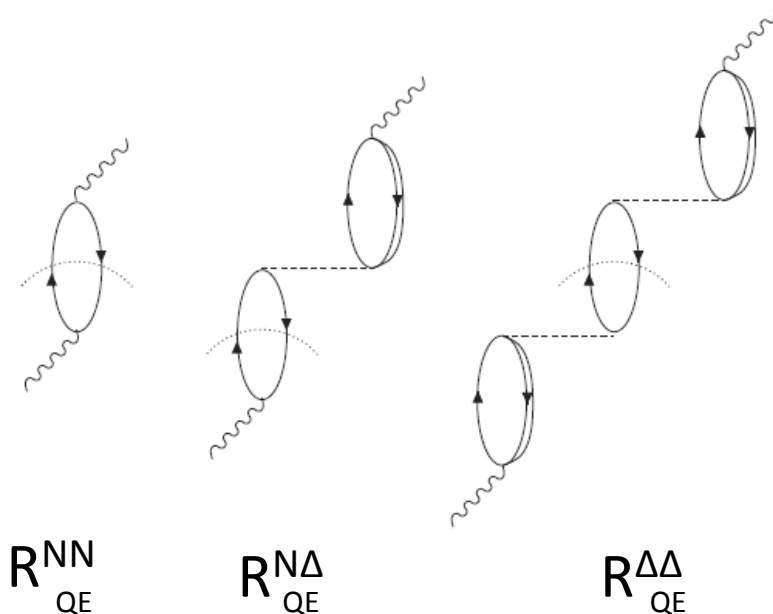
Effects of the RPA in the ν genuine quasielastic scattering

QE totally dominated by isospin spin-transverse response $R_{\sigma\tau(T)}$

RPA reduction

- expected from the repulsive character of p-h interaction in T channel
- mostly due to interference term $R^{N\Delta} < 0$
(Lorentz-Lorenz or Ericson-Ericson effect)

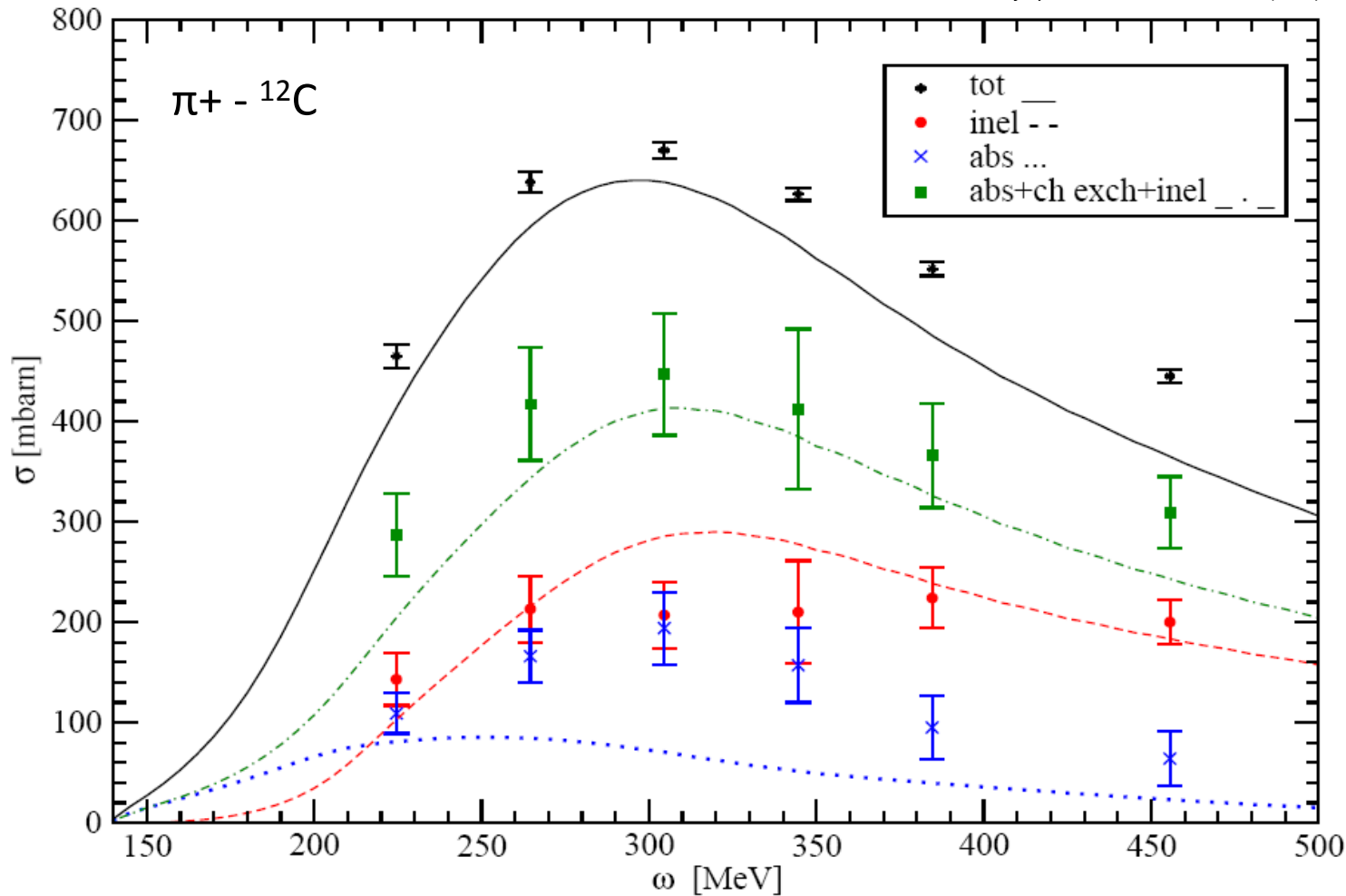
Lowest order contribution to QE



Testing our model: pion-nucleus cross-section

$$\sigma^{tot}(\omega) = \left(\frac{g_r}{2M_N} \right)^2 \pi q_\pi R_L(\omega, q_\pi)$$

M. Martini, M. Ericson, G. Chanfray, J. Marteau, PRC 80 065501 (2009)

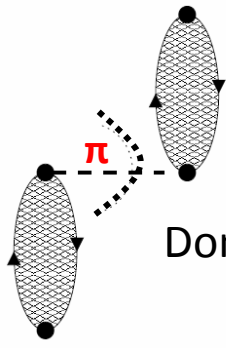


Overestimation of inelastic ch. in the delta peak region

Underestimation of absorption

Absence of π FSI

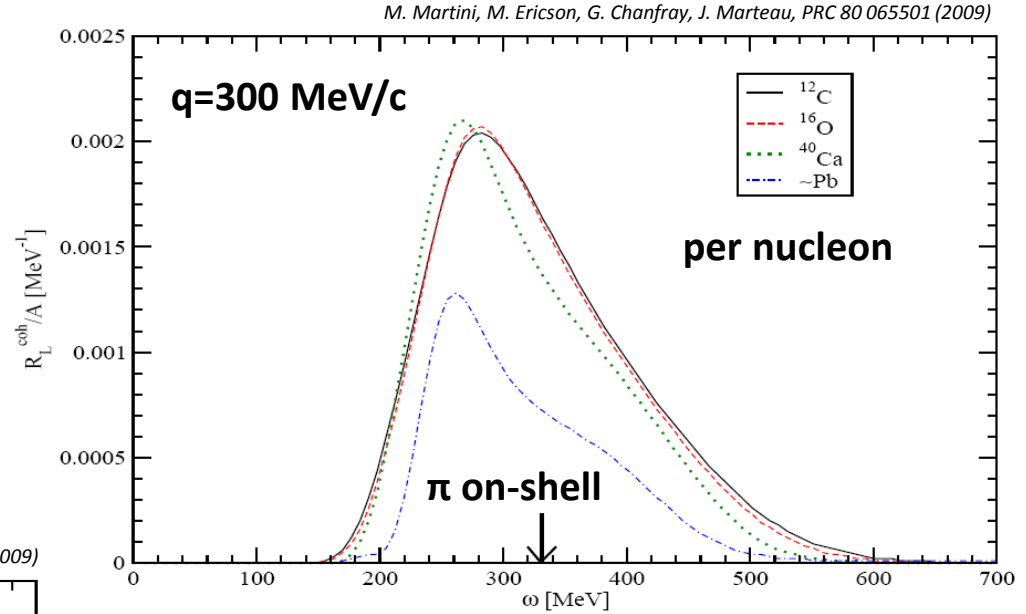
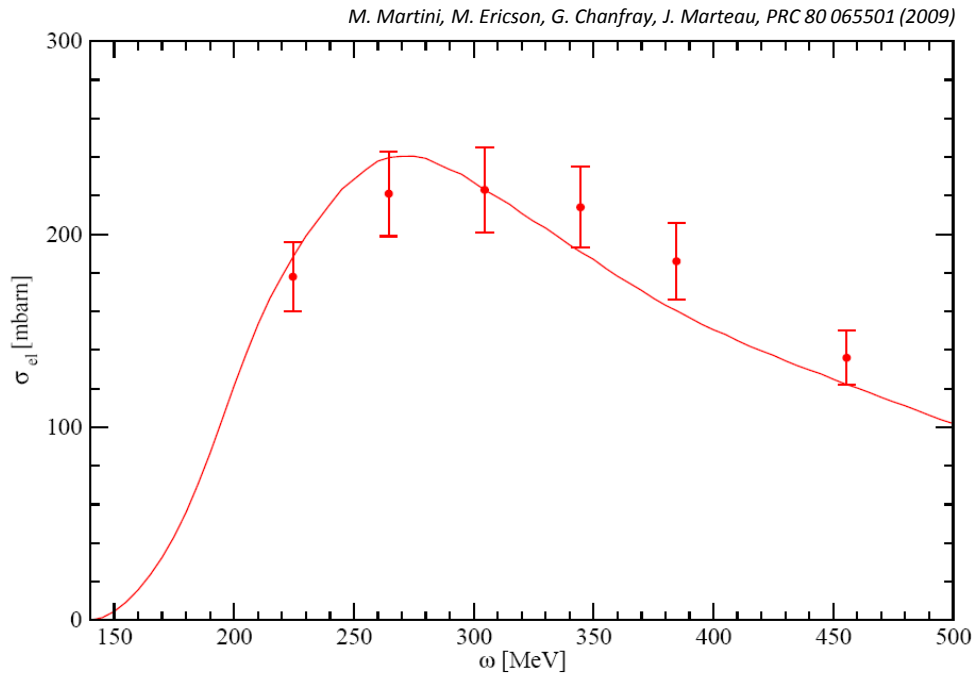
Coherent channel



Dominated by $R_{\sigma\pi}$ longitudinal

Reshaped by collective effects

Softening of the responses



Test: π - ^{12}C elastic cross-section

$$\sigma^{\text{elas}}(\omega) = \left(\frac{g_r}{2M_N} \right)^2 \pi q_\pi R_L^{\text{coh}}(\omega, q_\pi)$$

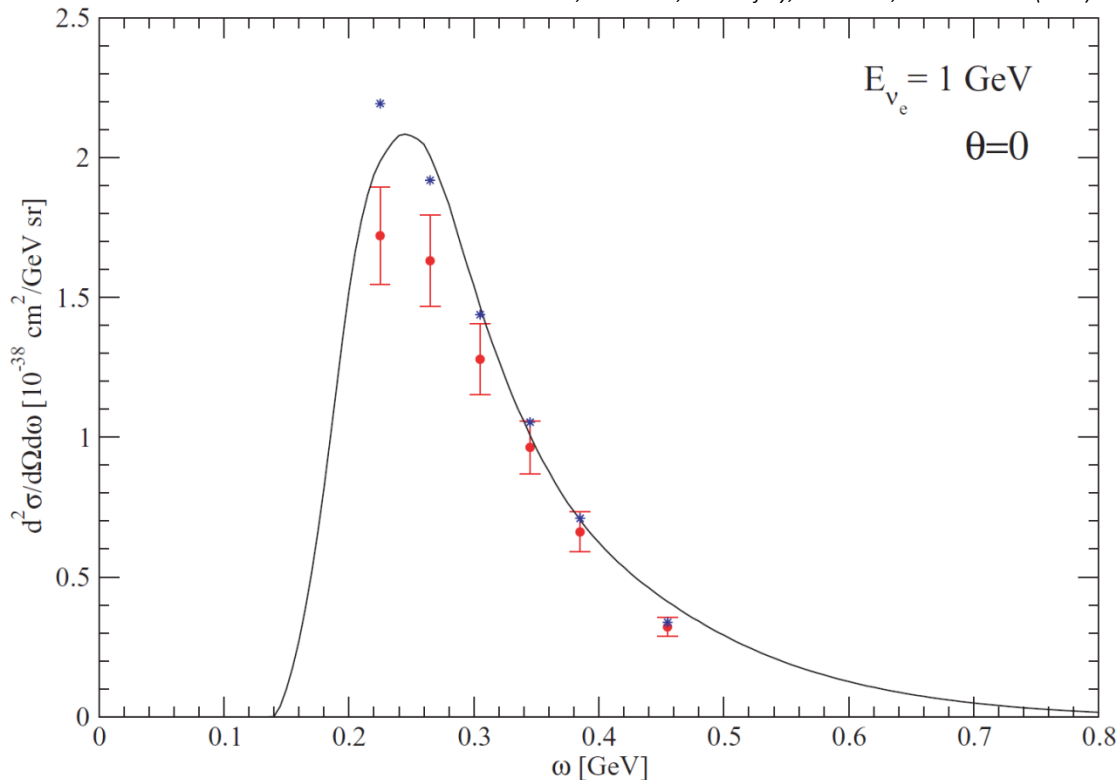
$$q_\pi^2 = \omega^2 - m_\pi^2$$

Coherent pion production and Adler's theorem

In the forward direction where $q=\omega$ only the spin longitudinal response contribution survives.
It is possible to relate the forward neutrino cross section to the pion cross section (Adler's theorem)

$$\left(\frac{\partial^2 \sigma}{\partial \Omega \partial \omega} \right)_{\theta=0}^{\text{coh}} = \frac{G_F^2 \cos^2 \theta_c}{\pi^3} f_\pi^2 \frac{(E_\nu - \omega)^2}{\omega} \sigma^{\text{elas}}(\omega)$$

M. Martini, M. Ericson, G. Chanfray, J. Marteau, PRC 80 065501 (2009)



Difference in kinematics:

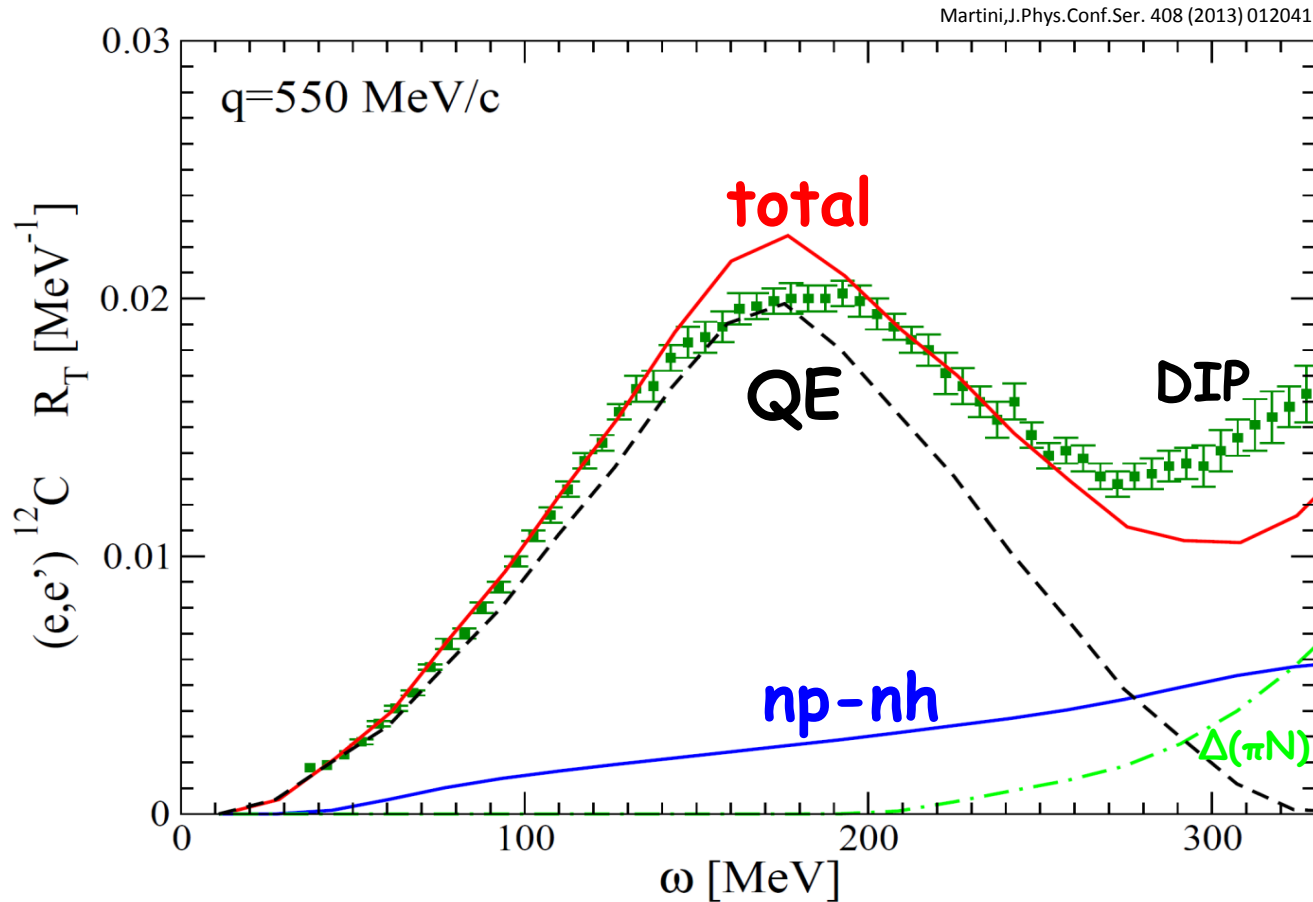
$$q = \omega \text{ for } \nu$$

$$q = q_\pi = \sqrt{\omega^2 - m_\pi^2} \text{ for } \pi$$

By multiplying the result obtained from the experimental $\sigma^{\text{elas}}(\omega)$ (red points) by the factor $\frac{\omega}{q_\pi}$ (blue points)

the agreement with our theoretical prediction (black line) improves

Testing our model: transverse response in electron scattering



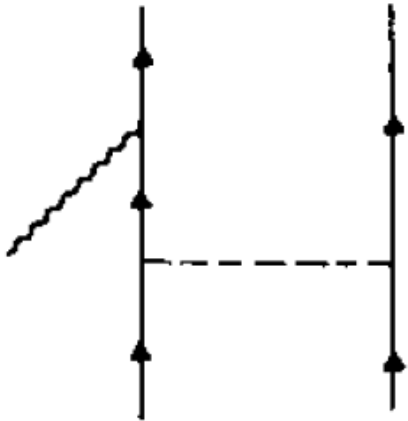
np-nh creates a high energy tail in the nuclear response above the QE peak

Alberico, Ericson, Molinari, Ann. Phys. 154, 356 (1984)

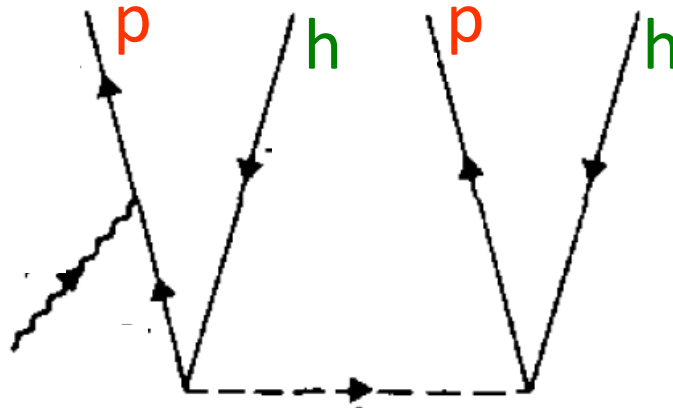
Two particle-two hole sector (2p-2h)

Three equivalent representations of the same process

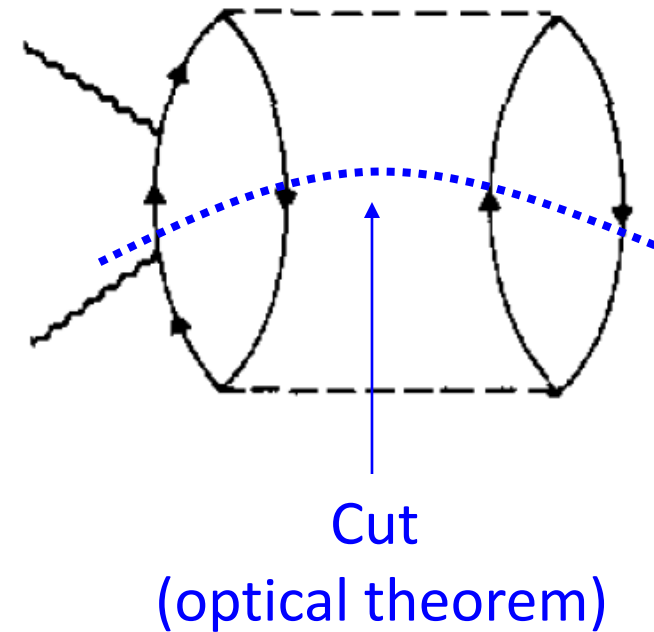
2 body current



2p-2h matrix element



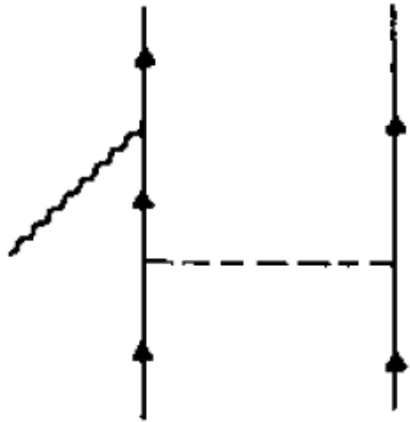
2p-2h response



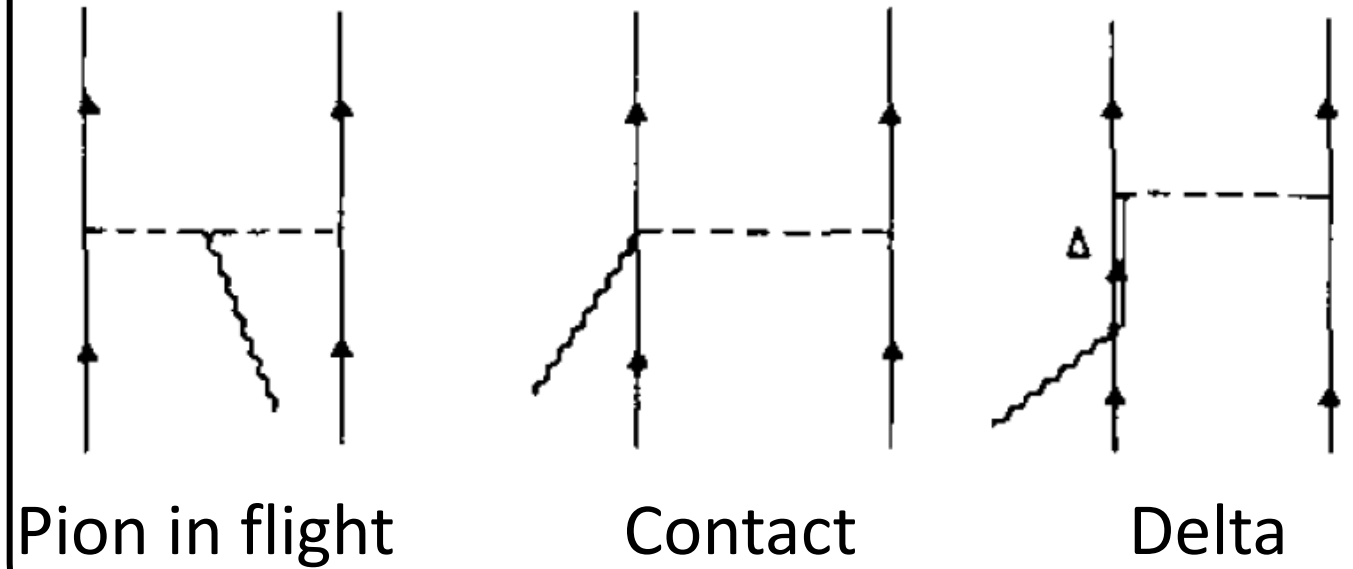
Final state: two particles-two holes

Some diagrams for 2 body currents

Nucleon-Nucleon correlations

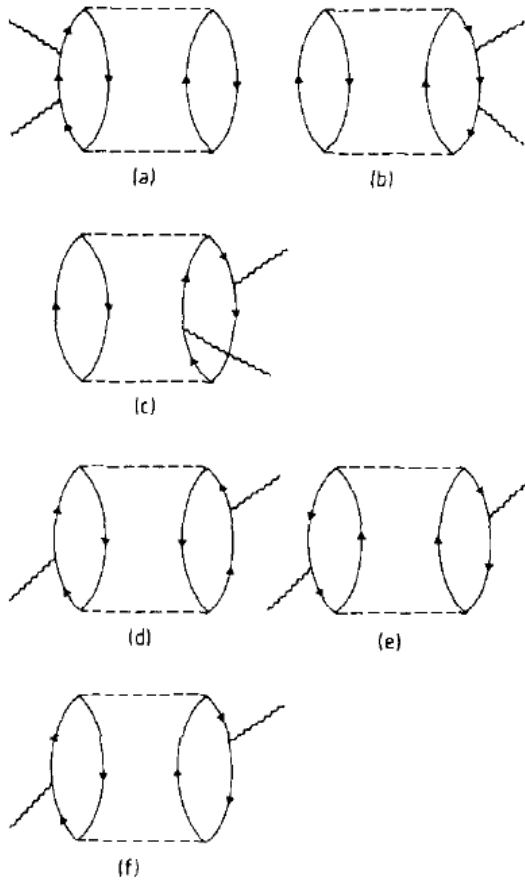


Meson Exchange Currents (MEC)



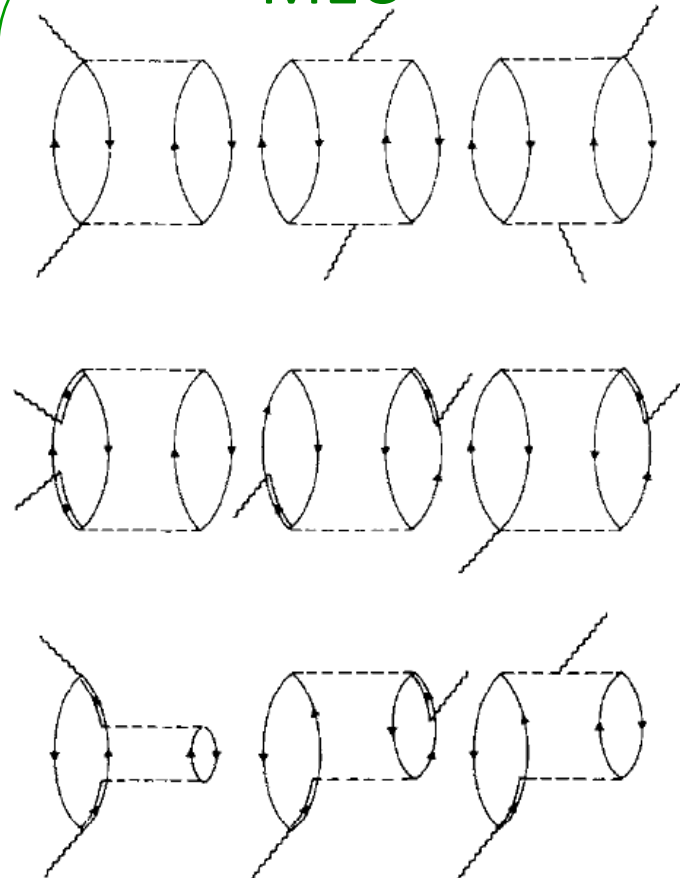
Some diagrams for 2p-2h responses

NN correlations

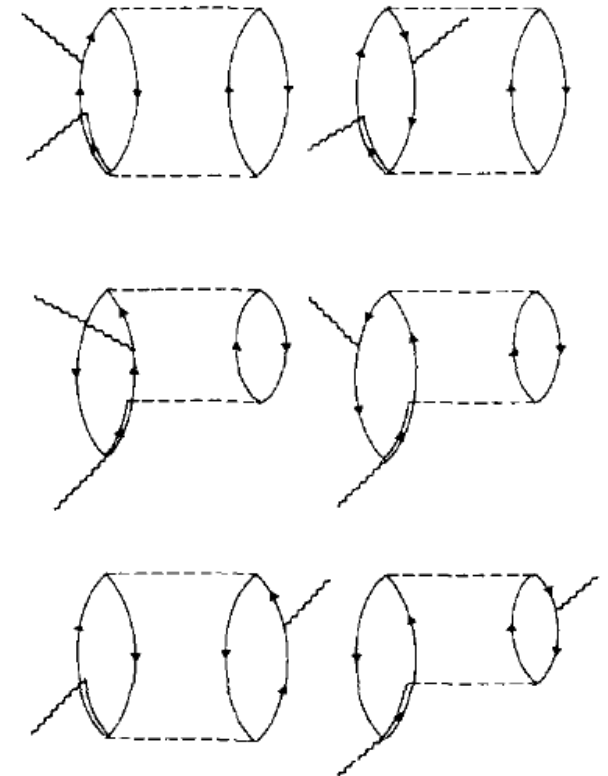


16 diagrams

MEC



NN correlation-MEC interference



Where 2p-2h enter in V -nucleus cross-section?

$$\begin{aligned}
 \frac{\partial^2 \sigma}{\partial \Omega \partial k'} &= \frac{G_F^2 \cos^2 \theta_c (\mathbf{k}')^2}{2 \pi^2} \cos^2 \frac{\theta}{2} \left[G_E^2 \left(\frac{q_\mu^2}{\mathbf{q}^2} \right)^2 R_\tau^{NN} \text{ isovector nuclear response} \right. \\
 &+ G_A^2 \frac{(M_\Delta - M_N)^2}{2 \mathbf{q}^2} R_{\sigma\tau(L)} \text{ isospin spin-longitudinal} \\
 &+ \left(G_M^2 \frac{\omega^2}{\mathbf{q}^2} + G_A^2 \right) \left(-\frac{q_\mu^2}{\mathbf{q}^2} + 2 \tan^2 \frac{\theta}{2} \right) R_{\sigma\tau(T)} \text{ isospin spin-transverse} \\
 &\left. \pm 2 G_A G_M \frac{k + k'}{M_N} \tan^2 \frac{\theta}{2} R_{\sigma\tau(T)} \right] \text{ interference V-A}
 \end{aligned}$$

The 2p-2h term affects the magnetic and axial responses
 (terms in G_M, G_A)
 (spin-isospin, σ excitation operator)

Other processes, with the same excitation operator ($\sigma\tau$),
where 2p-2h are relevant

• Pion absorption

Two-nucleon mechanism: $\pi NN \rightarrow NN$ ($\pi N \rightarrow N$ strongly suppressed)

Dominated by p-n initial pairs

V scattering 

Ejected pairs will be predominantly:

p-p for ν CC

n-n for anti- ν CC

p-n for NC

First results of MINER νA seem to confirm this prediction (PRL 111 022501; 022502 2013)

• Photon absorption

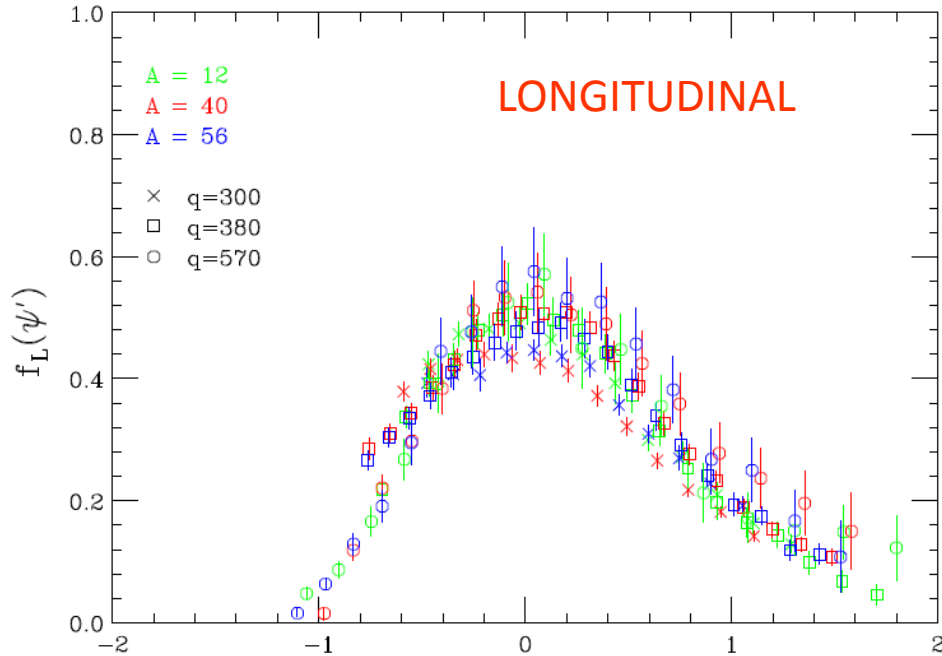
$$\sigma_{\gamma}^{\text{tot}} = 2\pi^2 \frac{\alpha}{\omega} R_T(q, q)$$

• Transverse response in electron scattering

$$\frac{d^2\sigma}{d\theta d\omega} = \sigma_M \left\{ \frac{(\omega^2 - q^2)^2}{q^4} R_L(\omega, q) + \left[\tan^2 \left(\frac{\theta}{2} \right) - \frac{\omega^2 - q^2}{2q^2} \right] R_T(\omega, q) \right\}$$

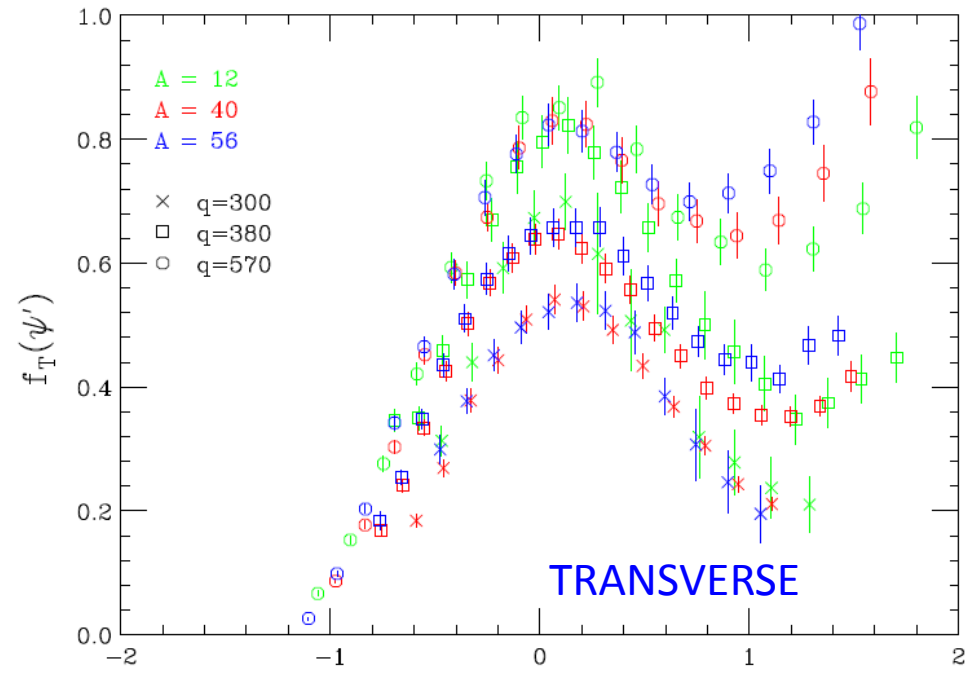
Electron scattering

$$\frac{d^2\sigma}{d\theta d\omega} = \sigma_M \left\{ \frac{(\omega^2 - q^2)^2}{q^4} R_L(\omega, q) + \left[\tan^2\left(\frac{\theta}{2}\right) - \frac{\omega^2 - q^2}{2q^2} \right] R_T(\omega, q) \right\}$$



$$f_L(\Psi) = k_F \frac{q^2 - \omega^2}{q m} \frac{R_L(\omega, q)}{Z(G_E^p)^2 + N(G_E^n)^2}$$

Donnelly et al. PRC 60 '99, ...

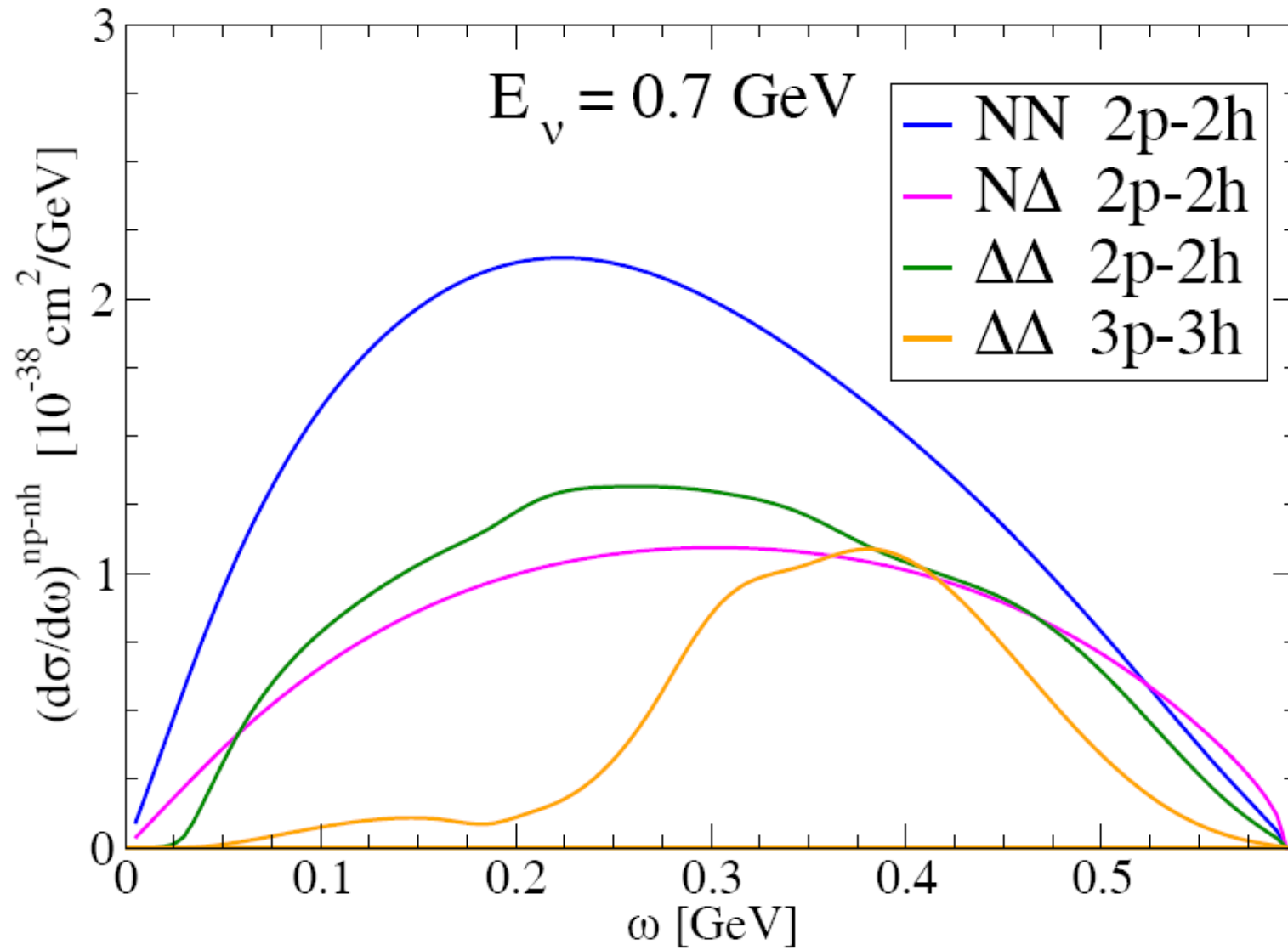


$$f_T(\Psi) = 2 k_F \frac{q m}{q^2 - \omega^2} \frac{R_T(\omega, q)}{Z(G_M^p)^2 + N(G_M^n)^2}$$

Excess in the transverse channel likely due to 2-body currents (MEC and correlations)

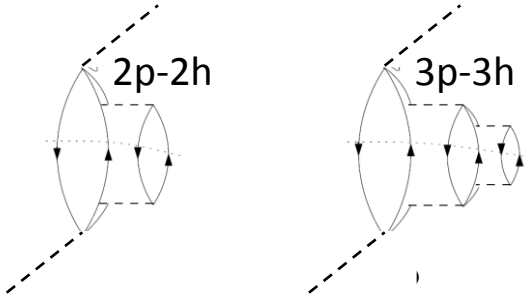
A. Bodek et al. Eur.Phys.J. C71 (2011) : parametrization of the enhancement in T channel in terms of correction to $G_M(Q^2)$

Further details of our model in the np-nh sector



$\Delta\Delta$ contributions to np-nh in our model

- Reducible to a modification of the Delta width in the medium



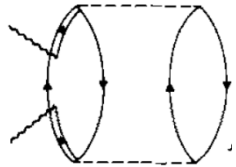
E. Oset and L. L. Salcedo, Nucl. Phys. A 468, 631 (1987):

$$\widetilde{\Gamma}_\Delta = \Gamma_\Delta F_P - 2\text{Im}(\Sigma_\Delta)$$

$$\text{Im}(\Sigma_\Delta(\omega)) = - \left[C_Q \left(\frac{\rho}{\rho_0}\right)^\alpha + C_{2p2h} \left(\frac{\rho}{\rho_0}\right)^\beta + C_{3p3h} \left(\frac{\rho}{\rho_0}\right)^\gamma \right]$$

Nieves et al. in PRC 83 (2011) and in PLB 707 (2012) use the same model for these contributions

- Not reducible to a modification of the Delta width



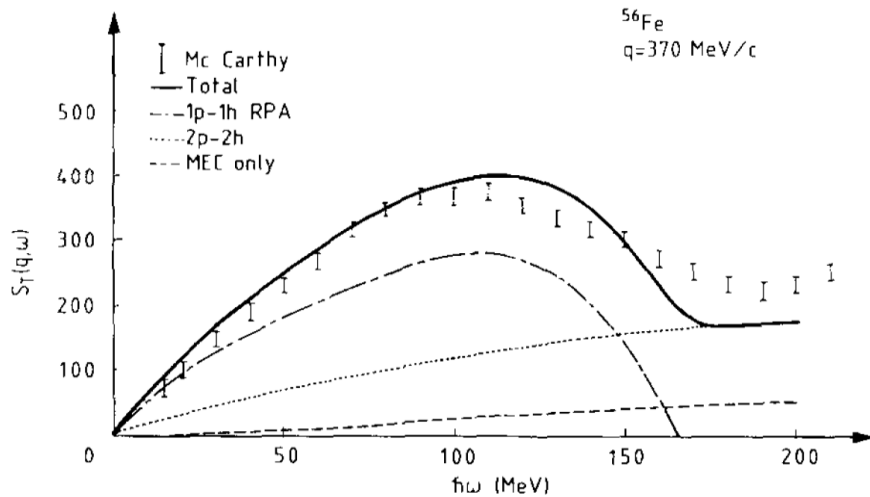
Microscopic calculation of π absorption at threshold: $\omega = m_\pi$

Shimizu, Faessler, Nucl. Phys. A 333,495 (1980)

Extrapolation to other energies

$$\text{Im}(\Pi_{\Delta\Delta}^0) = -4\pi\rho^2 \frac{(2M_N + m_\pi)^2}{(2M_N + \omega)^2} C_3 \Phi_3(\omega) \left[\frac{1}{(\omega + M_\Delta - M_N)^2} \right]$$

NN correlations and $N\Delta$ interference contributions to 2p-2h



Starting point: a microscopic evaluation of R_T
Alberico, Ericson, Molinari, Ann. Phys. 154, 356 (1984)

Transverse magnetic response of (e, e')
 for some values of q and ω , but:

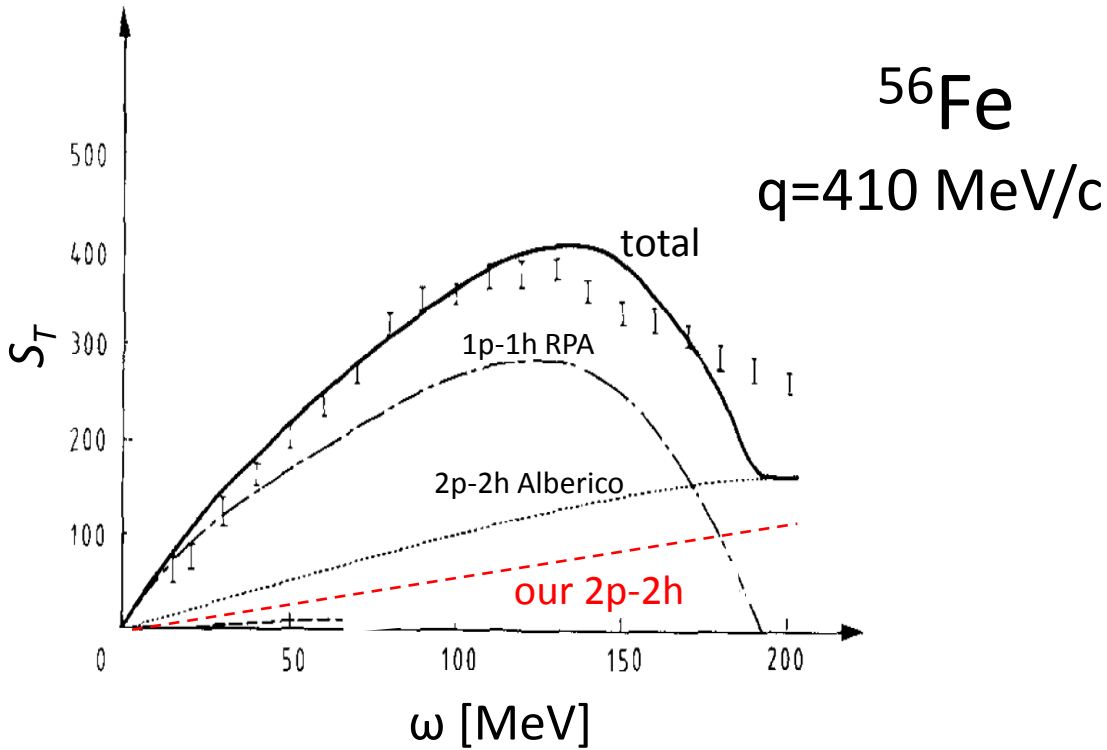
^{56}Fe , instead of ^{12}C and responses available
 only for few q and ω values

Our work

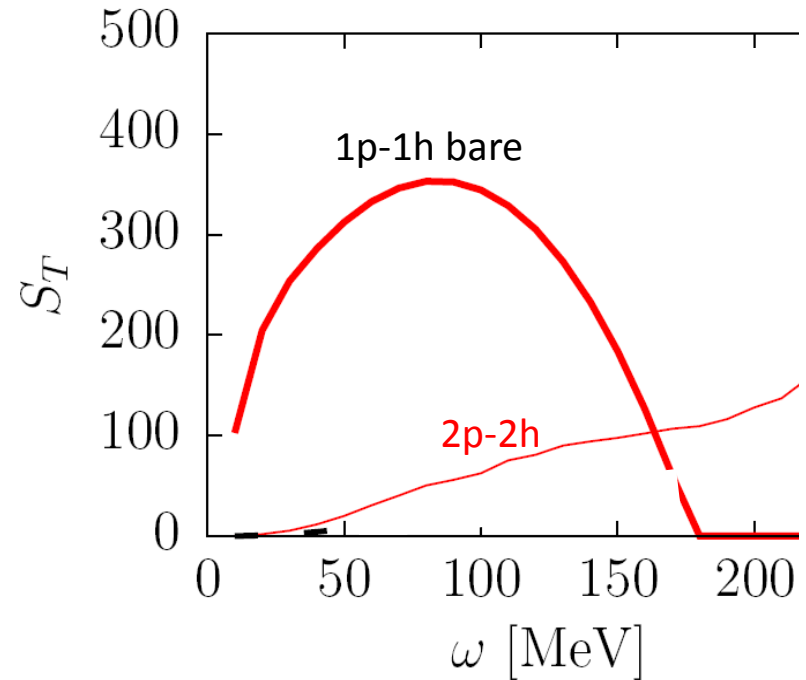
- Parameterization of these contributions in terms of $\mathcal{X} = \frac{q^2 - \omega^2}{2M_N\omega} \longrightarrow$ Extrapolation to cover neutrino region
- Global reduction ≈ 0.5 applied to reproduce the absorptive p-wave π -A optical potential

A comparison between our parameterization of 2p-2h (PRC 2009) and the one of the PRC (2010) paper of Amaro et al. on electron scattering

Alberico et al. Ann. Phys. 154, 356 (1984)



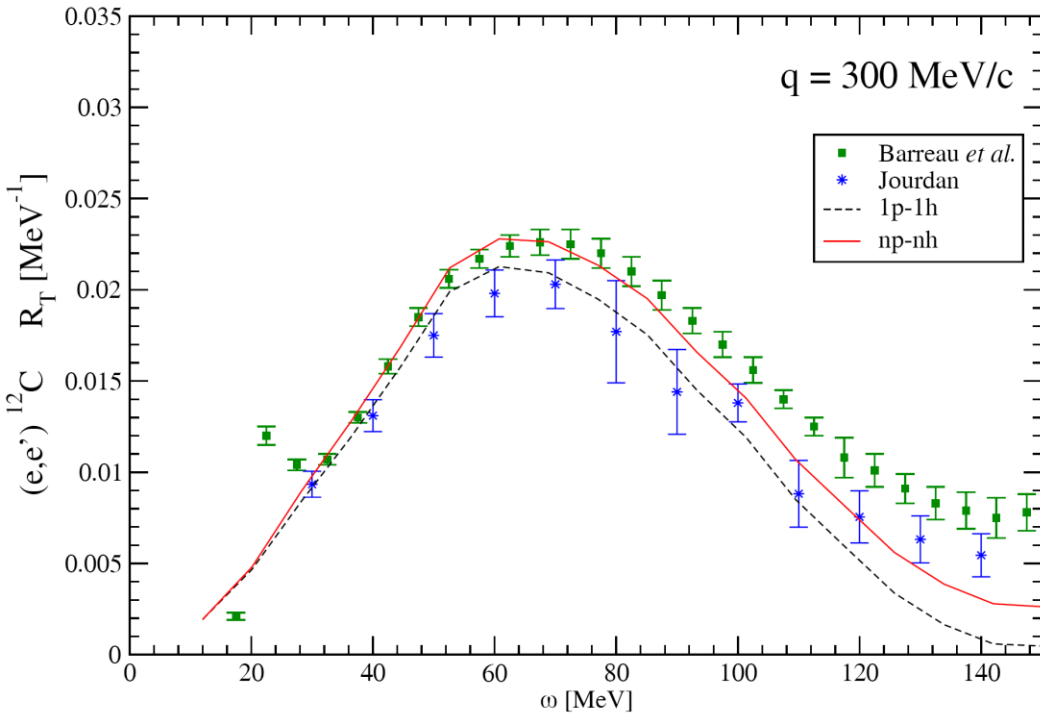
Amaro et al. PRC 82 044601 (2010)
(not yet inserted in neutrino calculations)



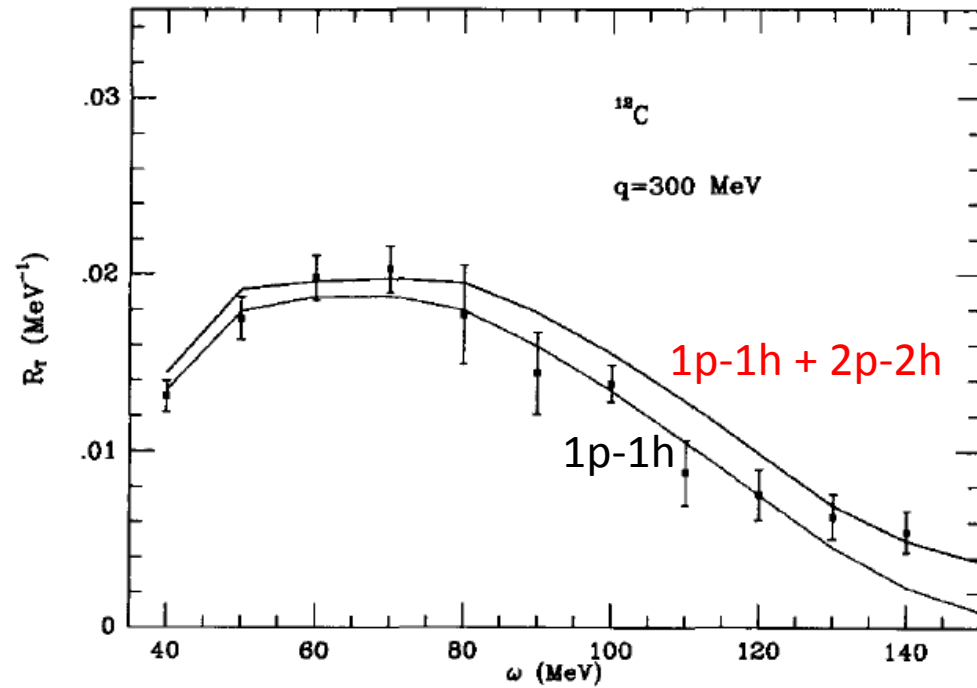
Our parameterization is quite close to the results of Amaro et al.

R_T of ^{12}C : comparison with data and with calculations of Gil, Nieves, Oset

Our calculation J.Phys.Conf.Ser. 408 (2013) 012041



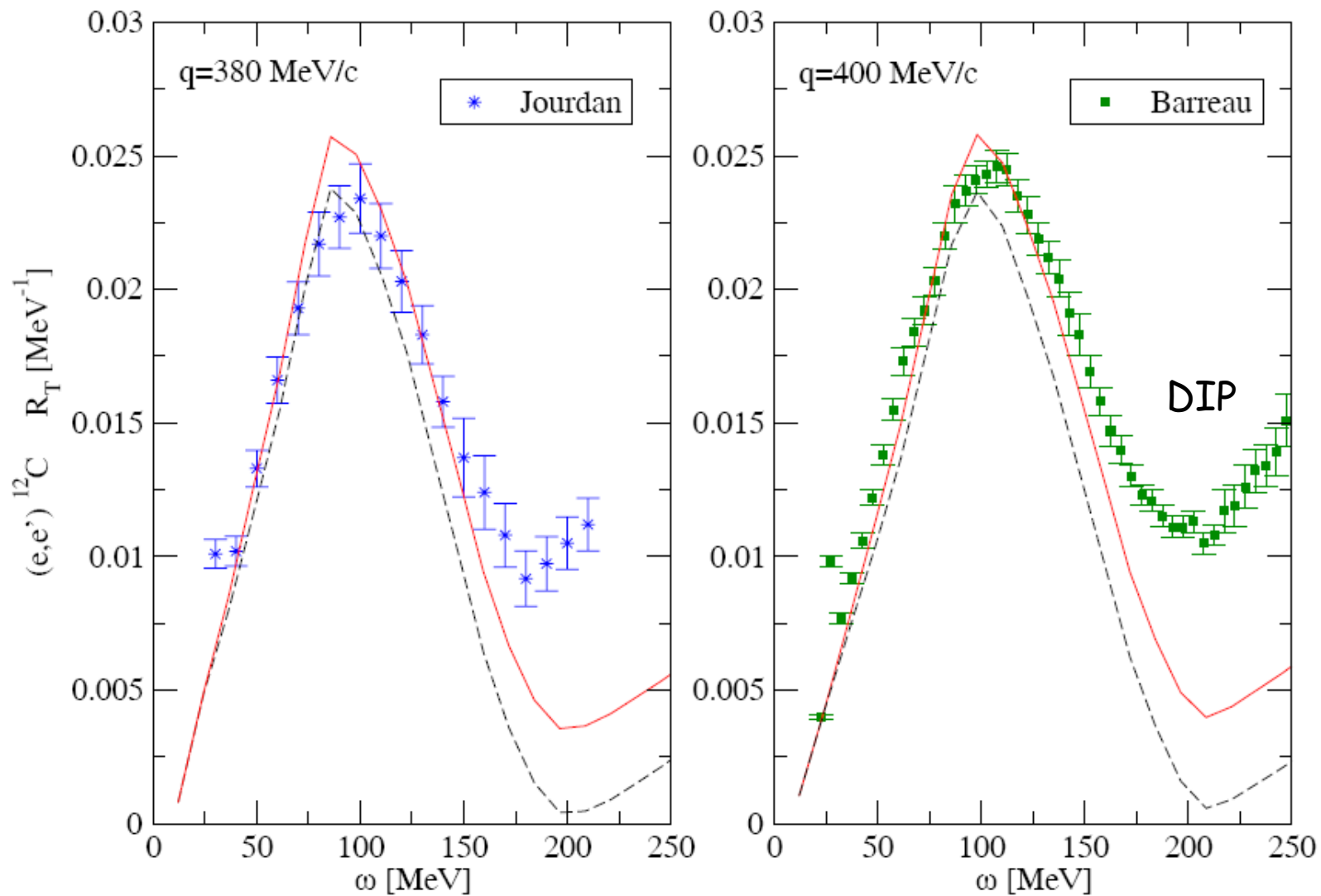
Gil, Nieves, Oset NPA 627, 543 (1997)



- Two evaluations of 2p-2h: same order of magnitude
- Agreement with data

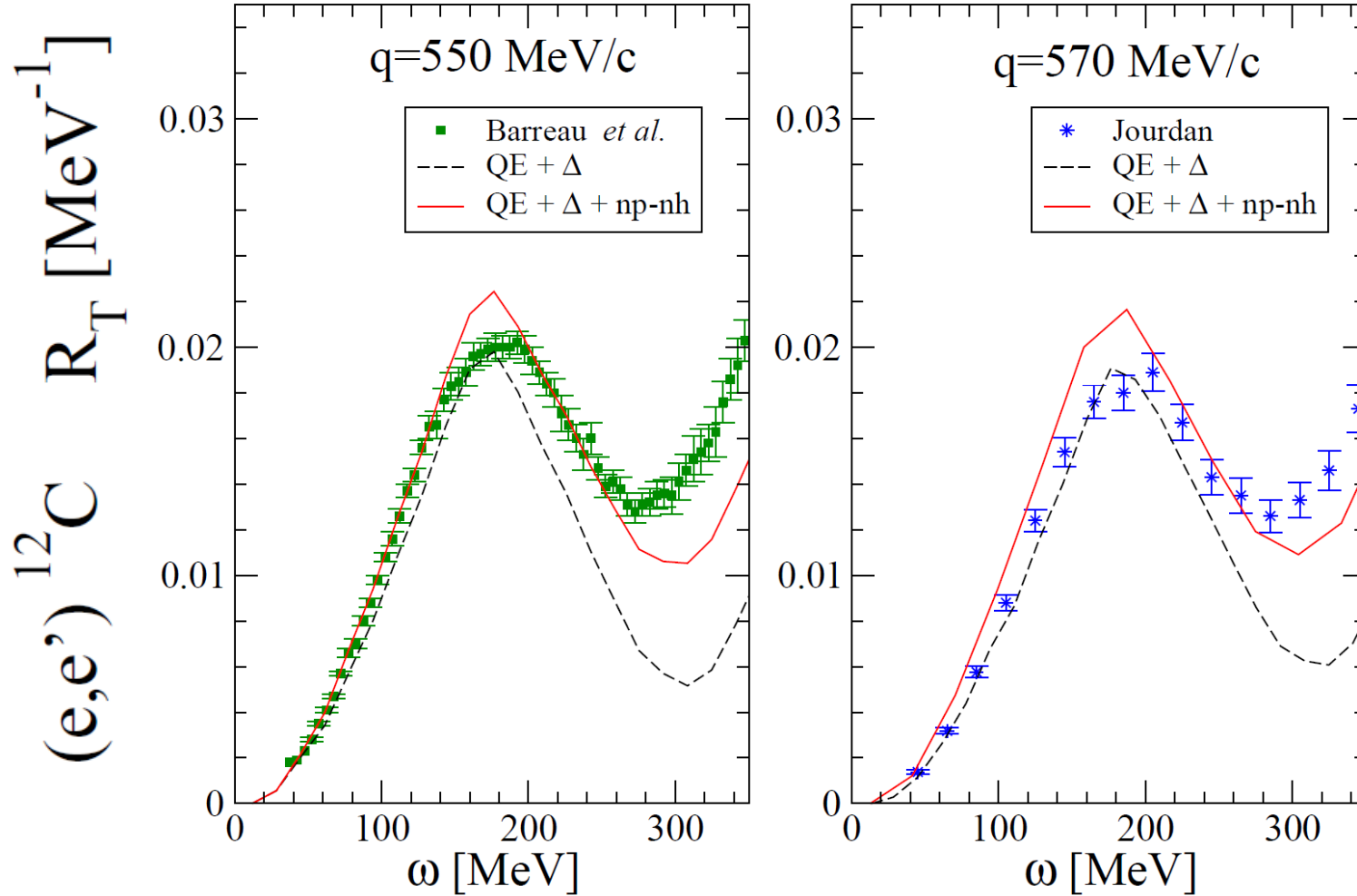
N.B. Some discrepancies in the two experimental data sets

Our results vs experiment for other q values



Our results vs experiment for other q values

M. Martini, J.Phys.Conf.Ser. 408 (2013) 012041



Conclusions: various evaluations of 2p-2h contributions to R_T are compatible among them and the data in the DIP region are never overestimated.

This test is important for ν cross section which is dominated by the transverse response.

In order to avoid confusion further comments on our model for the np-nh sector

In the paper

Martini, Ericson, Chanfray, Marteau, PRC 80 (2009)
we considered 2 different parameterizations of np-nh

“old” red:

Extrapolation from **2p-2h π absorption** (Delorme, Guichon) used for **total** neutrino cross section in:
Marteau, Delorme, Ericson, NIM A451 (2000) 76-80
Sometimes called “The Marteau model”

Oversimplified: no q dependence in NN and $N\Delta$ contributions only ω dependence

“new” blue:

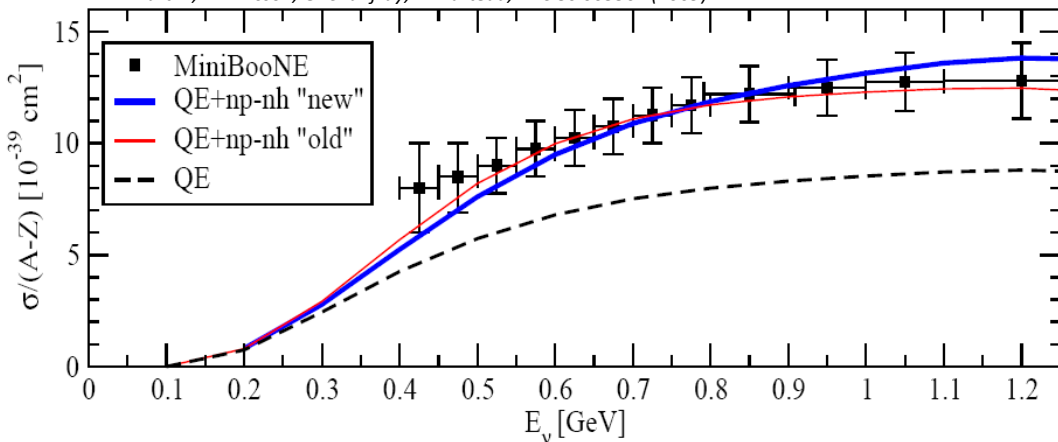
The more realistic parameterization described previously deduced from Alberico et al.

These two approaches give results

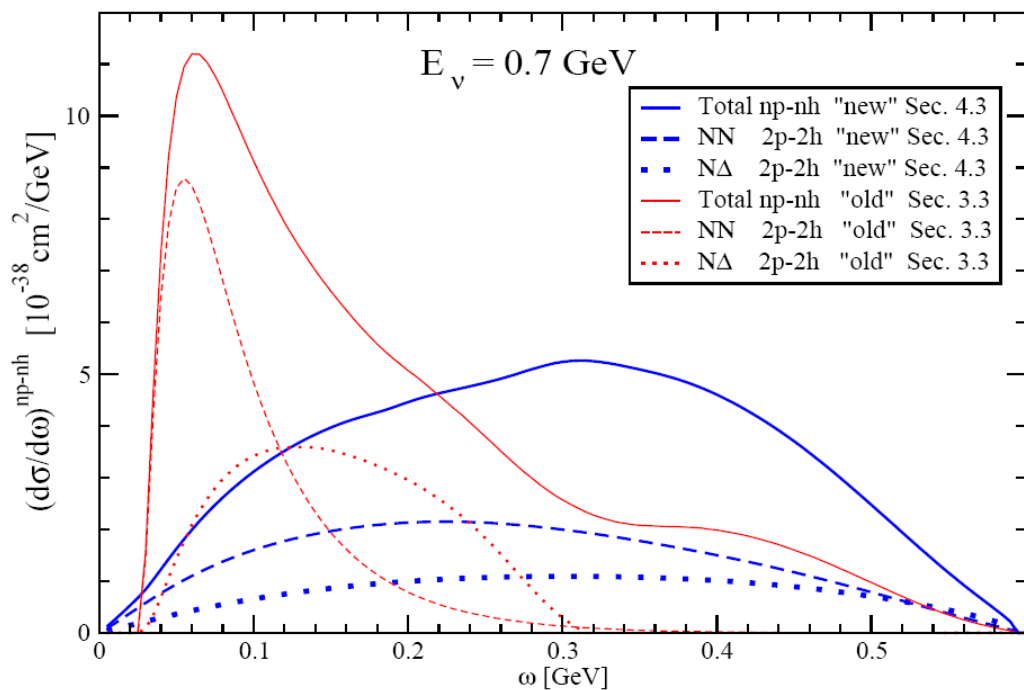
- very similar for the total cross section
- very different for the differential cross sections

The “New” parameterization is the only one we have considered in all our later papers

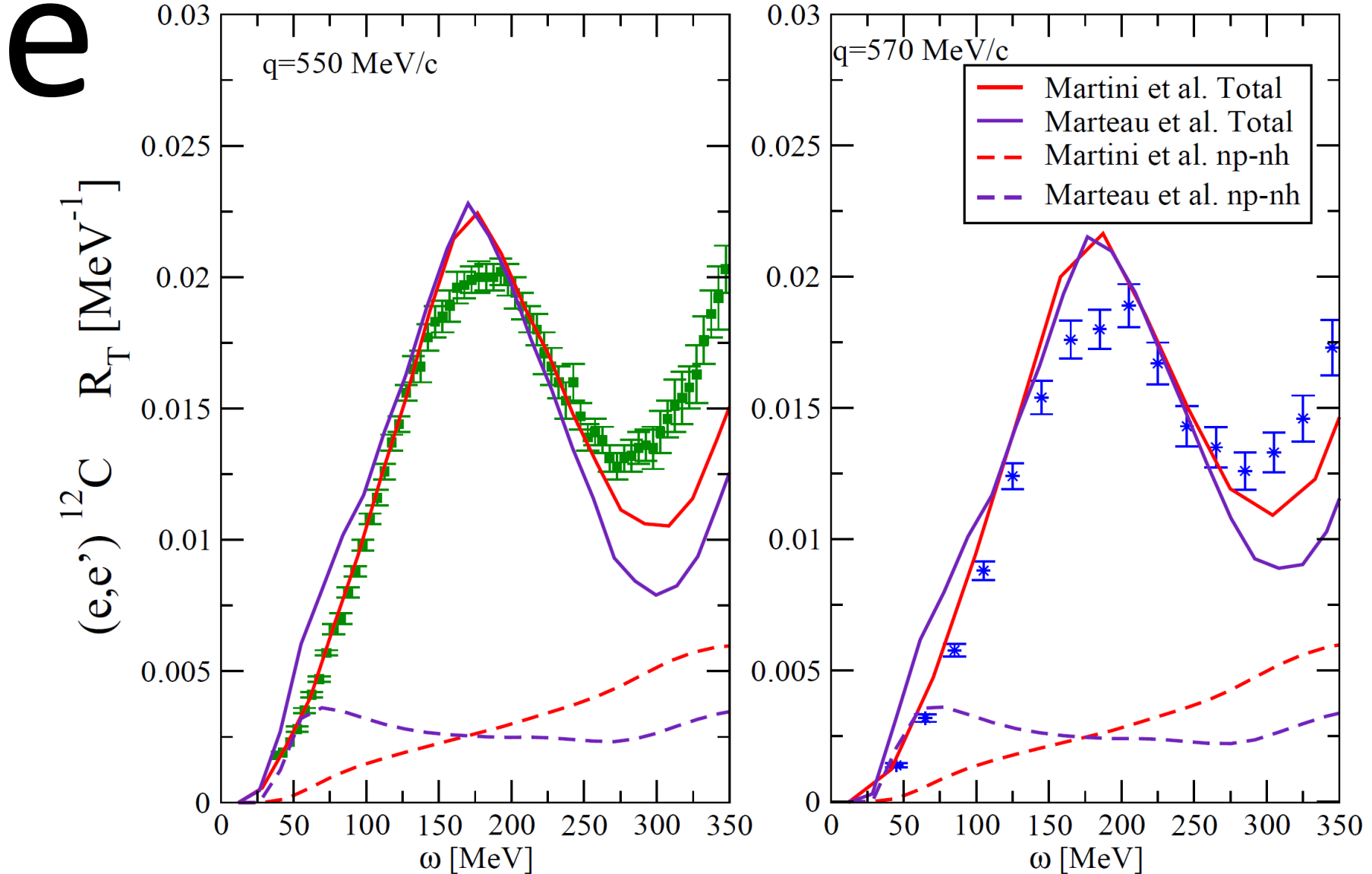
M. Martini, M. Ericson, G. Chanfray, J. Marteau, PRC 80 065501 (2009)



M. Martini, M. Ericson, G. Chanfray, J. Marteau, PRC 80 065501 (2009)



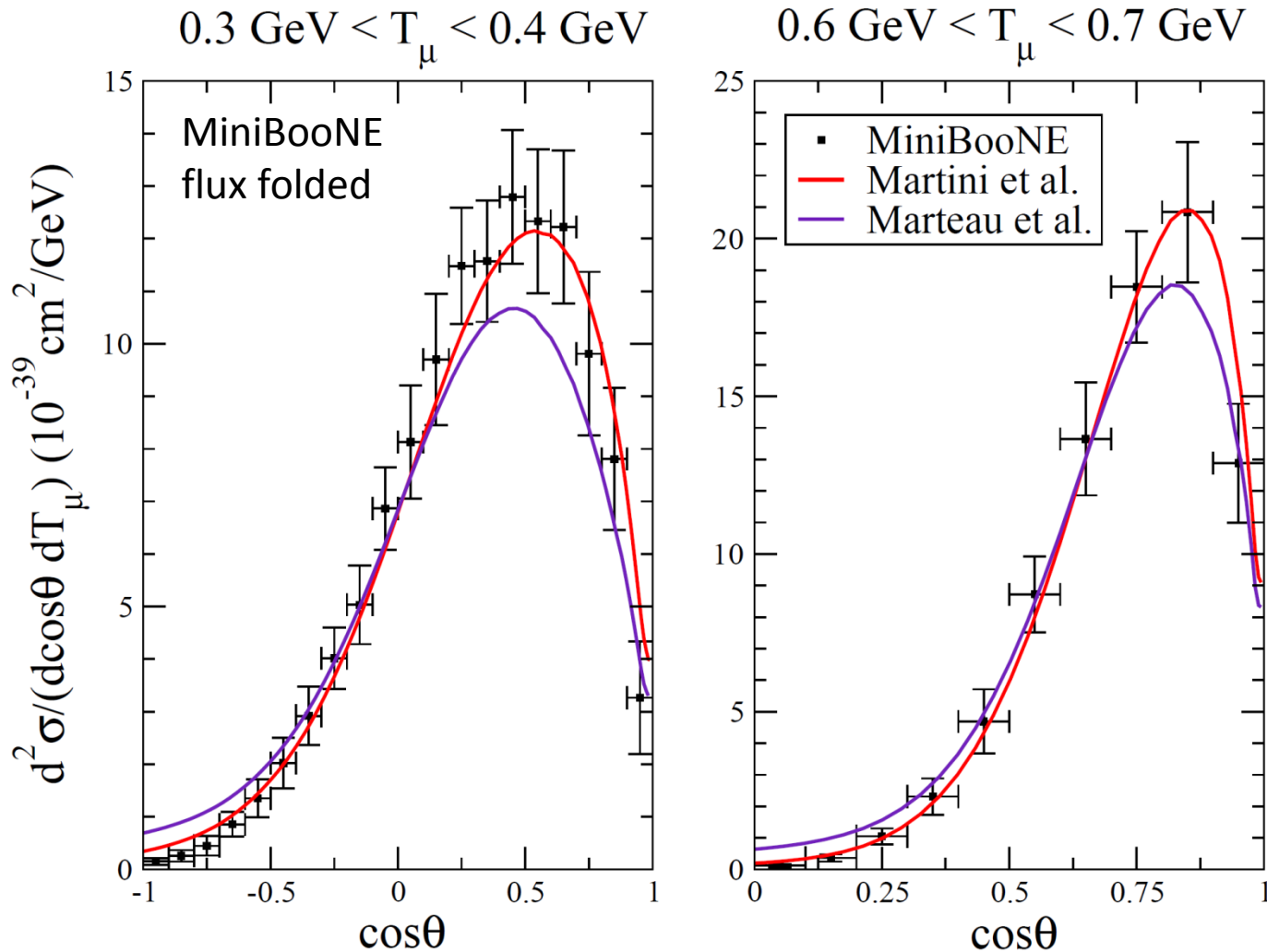
Comparison between the two approaches in electron scattering



In the “old” model too much accumulation of strength at low ω reflecting the absence of q dependence

Comparison between the two approaches in neutrino scattering

ν



The “old” (Marteau) model does not hold for the neutrino double differential cross sections. It is obsolete. Don’t use it to study neutrino energy reconstruction problems.

Some comparison among models treating the np-nh excitations

Theoretical studies on np-nh excitations in CCQE-like

M. Martini, M. Ericson, G. Chanfray, J. Marteau (Lyon, IPNL)

Phys. Rev. C 80 065501 (2009) ν σ_{total}

Phys. Rev. C 81 045502 (2010) ν vs antiv (σ_{total})

Phys. Rev. C 84 055502 (2011) ν $d^2\sigma$, $d\sigma/dQ^2$

Phys. Rev. D 85 093012 (2012) impact of np-nh on ν energy reconstruction

Phys. Rev. D 87 013009 (2013) impact of np-nh on ν energy reconstruction and ν oscillation

Phys. Rev. C 87 065501 (2013) antiv $d^2\sigma$, $d\sigma/dQ^2$

J. Nieves, I. Ruiz Simo, M.J. Vicente Vacas, F. Sanchez, R. Gran (Valencia, IFIC)

Phys. Rev. C 83 045501 (2011) ν , antiv σ_{total}

Phys. Lett. B 707 72-75 (2012) ν $d^2\sigma$

Phys. Rev. D 85 113008 (2012) impact of np-nh on ν energy reconstruction

Phys. Lett. B 721 90-93 (2013) antiv $d^2\sigma$

arXiv 1307.8105 (2013) extension of np-nh up to 10 GeV

J.E. Amaro, M.B. Barbaro, J.A. Caballero, T.W. Donnelly, J.M. Udias, C. F. Williamson

Phys. Lett. B 696 151-155 (2011) ν $d^2\sigma$

Phys. Rev. D 84 033004 (2011) ν $d^2\sigma$, σ_{total}

Phys. Rev. Lett. 108 152501 (2012) antiv $d^2\sigma$, σ_{total}

(Superscaling)

Sources and References of 2p-2h

M. Martini, M. Ericson, G. Chanfray, J. Marteau

Alberico, Ericson, Molinari, Ann. Phys. 154, 356 (1984) (e,e') γ π

**Oset and Salcedo, Nucl. Phys. A 468, 631 (1987)* π γ

Shimizu, Faessler, Nucl. Phys. A 333, 495 (1980) π

Delorme, Ericson, Phys.Lett. B156 263 (1985)

Marteau, Eur.Phys.J. A5 183-190 (1999); PhD thesis

Marteau, Delorme, Ericson, NIM A 451 76 (2000)

} ν pioneer works

J. Nieves, I. Ruiz Simo, M.J. Vicente Vacas et al.

Gil, Nieves, Oset, Nucl. Phys. A 627, 543 (1997) (e,e') γ

**Oset and Salcedo, Nucl. Phys. A 468, 631 (1987)* π γ

J.E. Amaro, M.B. Barbaro, J.A. Caballero, T.W. Donnelly et al.

De Pace, Nardi, Alberico, Donnelly, Molinari, Nucl. Phys. A741, 249 (2004) (e,e') γ

Analogies and differences of 2p-2h

M. Martini, M. Ericson, G. Chanfray, J. Marteau

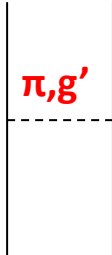
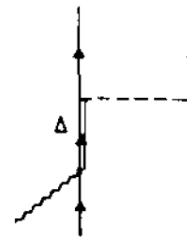
[Genuine CCQE (1p-1h): LRF+RPA]

Axial and Vector

NN corr.

Δ -MEC

$N\Delta$ interf.



J. Nieves, I. Ruiz Simo, M.J. Vicente Vacas et al.

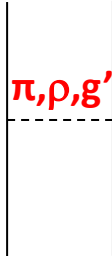
[Genuine CCQE (1p-1h): LRF+SF+RPA]

Axial and Vector

NN corr.

MEC

N -MEC interf.

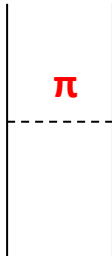


J.E. Amaro, M.B. Barbaro, J.A. Caballero, T.W. Donnelly et al.

[Genuine CCQE (1p-1h): Superscaling]

Only Vector

MEC



2p-2h contributions in the different approaches

$$\begin{aligned}
 \frac{\partial^2 \sigma}{\partial \Omega \partial k'} &= \frac{G_F^2 \cos^2 \theta_c (\mathbf{k}')^2}{2 \pi^2} \cos^2 \frac{\theta}{2} \left[G_E^2 \left(\frac{q_\mu^2}{q^2} \right)^2 R_\tau^{NN} \right. \\
 &+ G_A^2 \frac{(M_\Delta - M_N)^2}{2 q^2} R_{\sigma\tau(L)} \\
 &+ \left(\underline{G_M^2} \frac{\omega^2}{q^2} + G_A^2 \right) \left(-\frac{q_\mu^2}{q^2} + 2 \tan^2 \frac{\theta}{2} \right) R_{\sigma\tau(T)} \\
 &\left. \pm 2 G_A G_M \frac{k + k'}{M_N} \tan^2 \frac{\theta}{2} R_{\sigma\tau(T)} \right]
 \end{aligned}$$

M. Martini, M. Ericson, G. Chanfray, J. Marteau

Contribution to all terms in G_M and G_A

J. Nieves, I. Ruiz Simo, M.J. Vicente Vacas et al.

to all the terms

J.E. Amaro, M.B. Barbaro, J.A. Caballero, T.W. Donnelly et al.

only to the G_M^2 term

Relative role of 2p-2h for neutrinos and antineutrinos is different

Neutrino vs Antineutrino-nucleus cross-section

The asymmetry between neutrinos and antineutrinos interactions is important for the investigation of CP violation effects.

Nuclear effects generate an additional asymmetry due to **interference term**

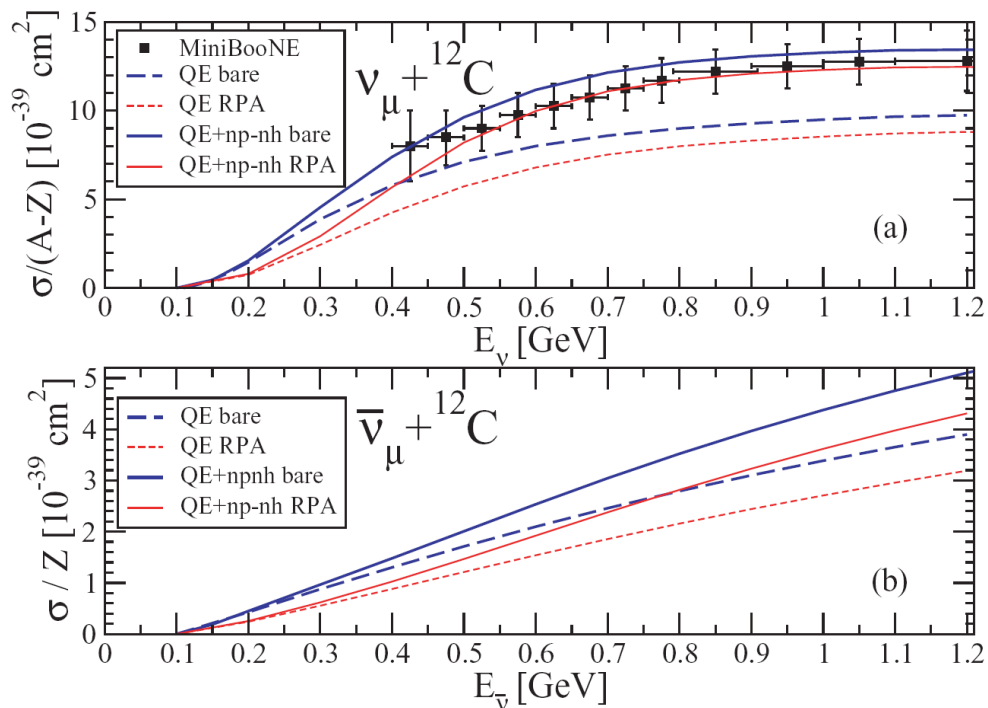
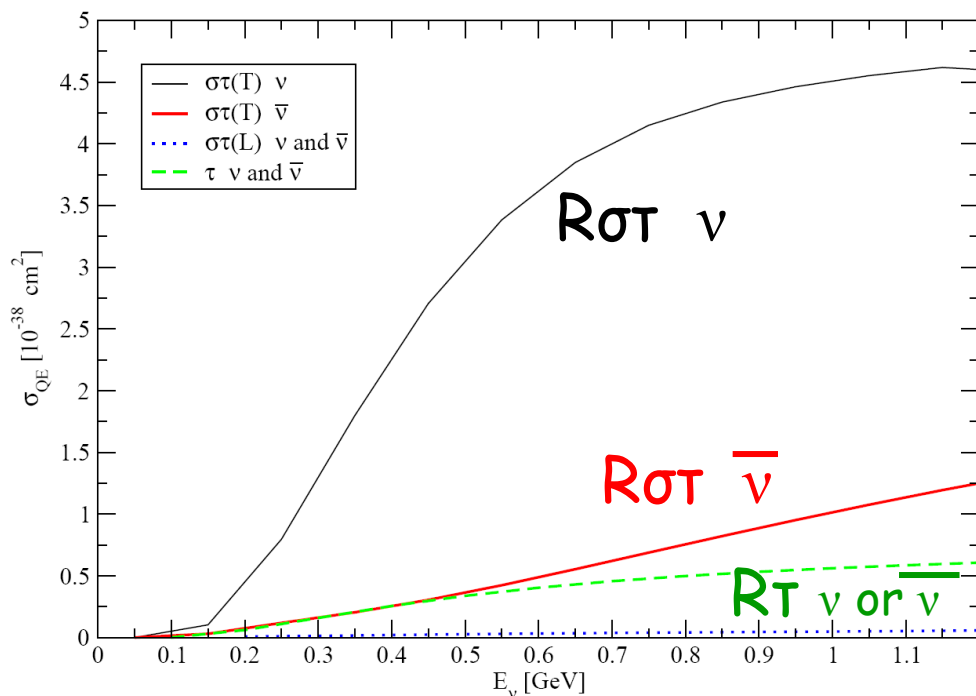
$$\begin{aligned}
 \frac{\partial^2 \sigma}{\partial \Omega \partial k'} &= \frac{G_F^2 \cos^2 \theta_c (\mathbf{k}')^2}{2 \pi^2} \cos^2 \frac{\theta}{2} \left[G_E^2 \left(\frac{q_\mu^2}{\mathbf{q}^2} \right)^2 \boxed{R_\tau^{NN}} \text{ isovector nuclear response} \right. \\
 &+ G_A^2 \frac{(M_\Delta - M_N)^2}{2 \mathbf{q}^2} \boxed{R_{\sigma\tau(L)}} \text{ isospin spin-longitudinal} \\
 &+ \left(G_M^2 \frac{\omega^2}{\mathbf{q}^2} + G_A^2 \right) \left(-\frac{q_\mu^2}{\mathbf{q}^2} + 2 \tan^2 \frac{\theta}{2} \right) \boxed{R_{\sigma\tau(T)}} \text{ isospin spin-transverse} \\
 &\left. \left\{ \begin{array}{l} + \quad (\nu) \\ - \quad (\bar{\nu}) \end{array} \right. \boxed{\pm} 2 G_A G_M \frac{k + k'}{M_N} \tan^2 \frac{\theta}{2} \boxed{R_{\sigma\tau(T)}} \right] \text{ interference V-A}
 \end{aligned}$$

In our model :

- The 2p-2h term affects the magnetic and axial responses (terms in $G_A^2, G_M^2, G_A G_M$)
- The isovector response R_τ (term in G_E^2) is not affected
(remember: Superscaling analysis of (e, e') data displays no tail above the QE peak in R_τ)

Various response contributions to the ν and $\bar{\nu}$ CCQE

The role of interference term (in $G_A G_M$) is crucial: it enhances the contribution of $R\sigma_T(T)$ for neutrinos. For antineutrinos instead the destructive interference partially suppresses this contribution leaving a larger role for isovector $R\tau$ which is insensitive to 2p-2h.

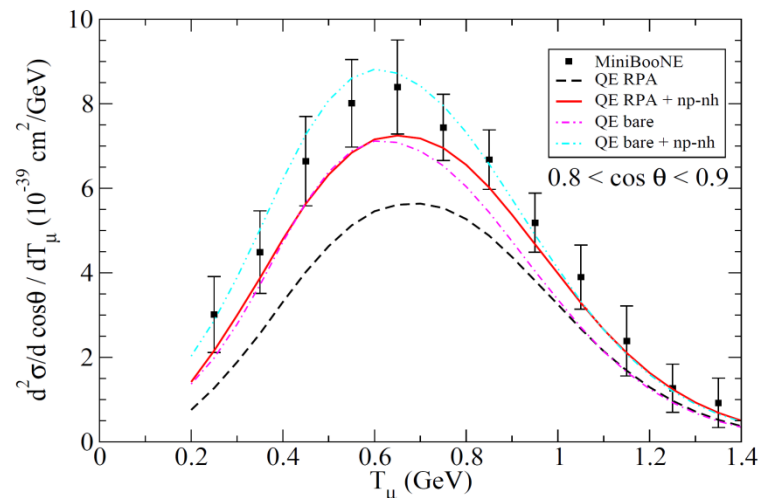
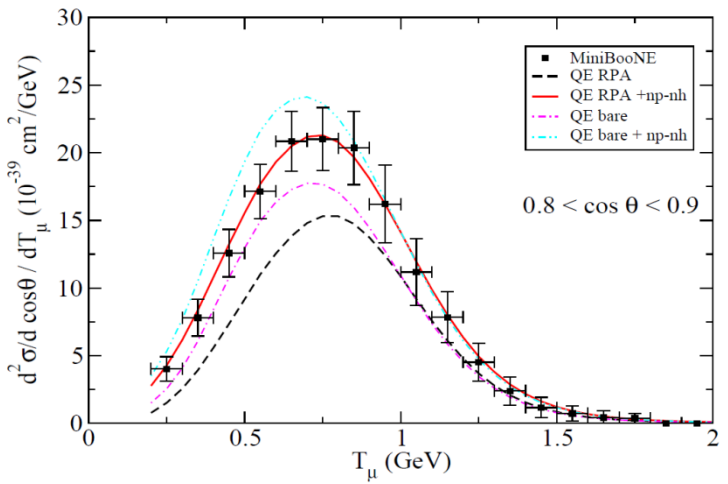


Relative role of 2p-2h smaller for antineutrinos

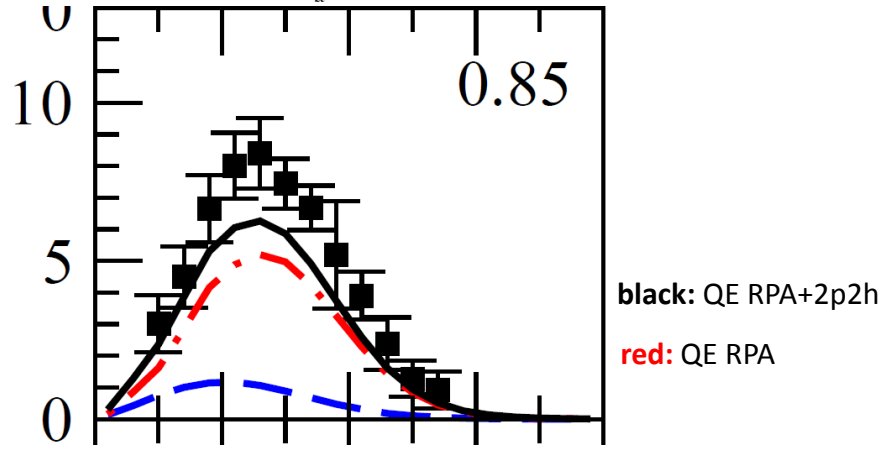
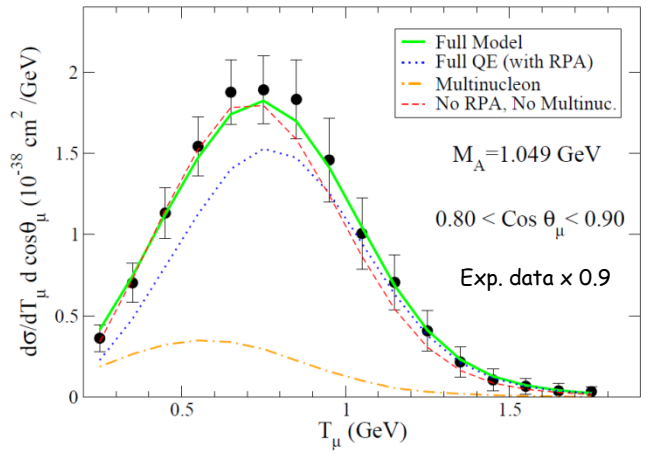
M. Martini, M. Ericson, G. Chanfray, J. Marteau, Phys. Rev. C 81 045502 (2010)

ν

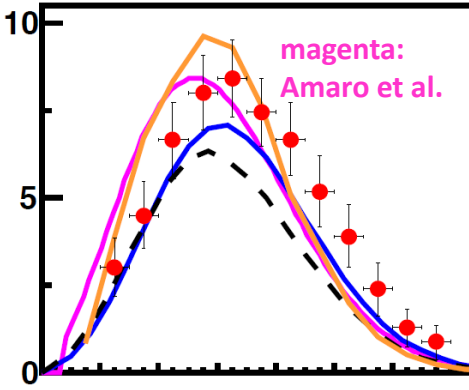
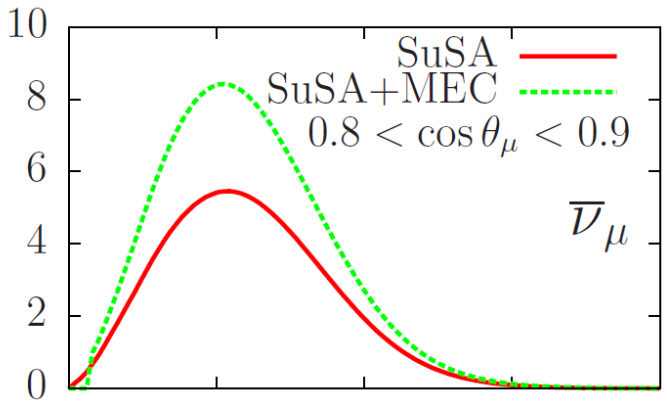
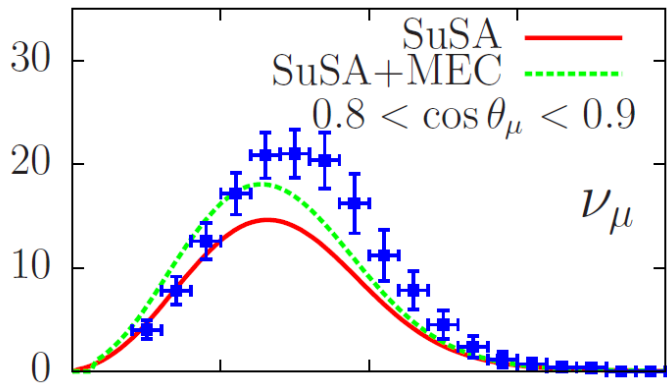
Martini et al.



Nieves et al.



Amaro et al.



$\bar{\nu}$

Summary

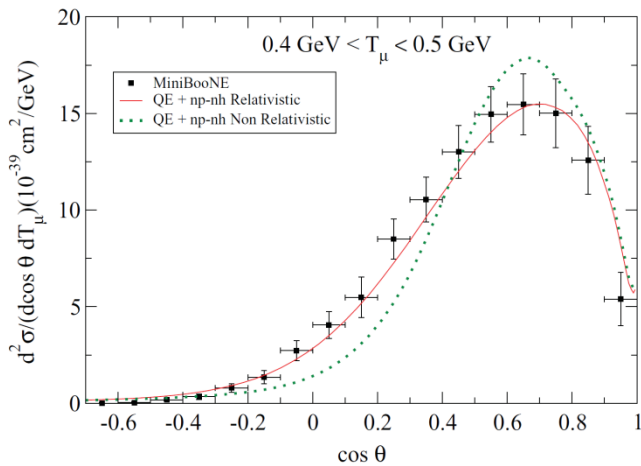
- Nuclear responses treated in RPA
 - Quasielastic
 - Pion production
 - Multinucleon emission** (np-nh excitations)
- There are some differences in the various theoretical approaches in the way to include the np-nh
- Our model (PRC 2009) with the inclusion of np-nh is in good agreement with all available neutrino experimental data
 - ν CCQE σ , $d^2\sigma/(dT_\mu d\cos\theta)$, $d\sigma/dQ^2$ (MiniBooNE PRD 2010)
 - ν NCQE $d\sigma/dQ^2$ (MiniBooNE PRD 2010)
 - $\bar{\nu}$ CCQE σ , $d^2\sigma/(dT_\mu d\cos\theta)$, $d\sigma/dQ^2$ (MiniBooNE PRD 2013)
 - ν CC Inclusive σ (SciBooNE PRD 2011)
 - ν CC Inclusive $d^2\sigma/(dp_\mu d\cos\theta)$ (T2K PRD 2013)
- Next step: extension at higher energies

Spares

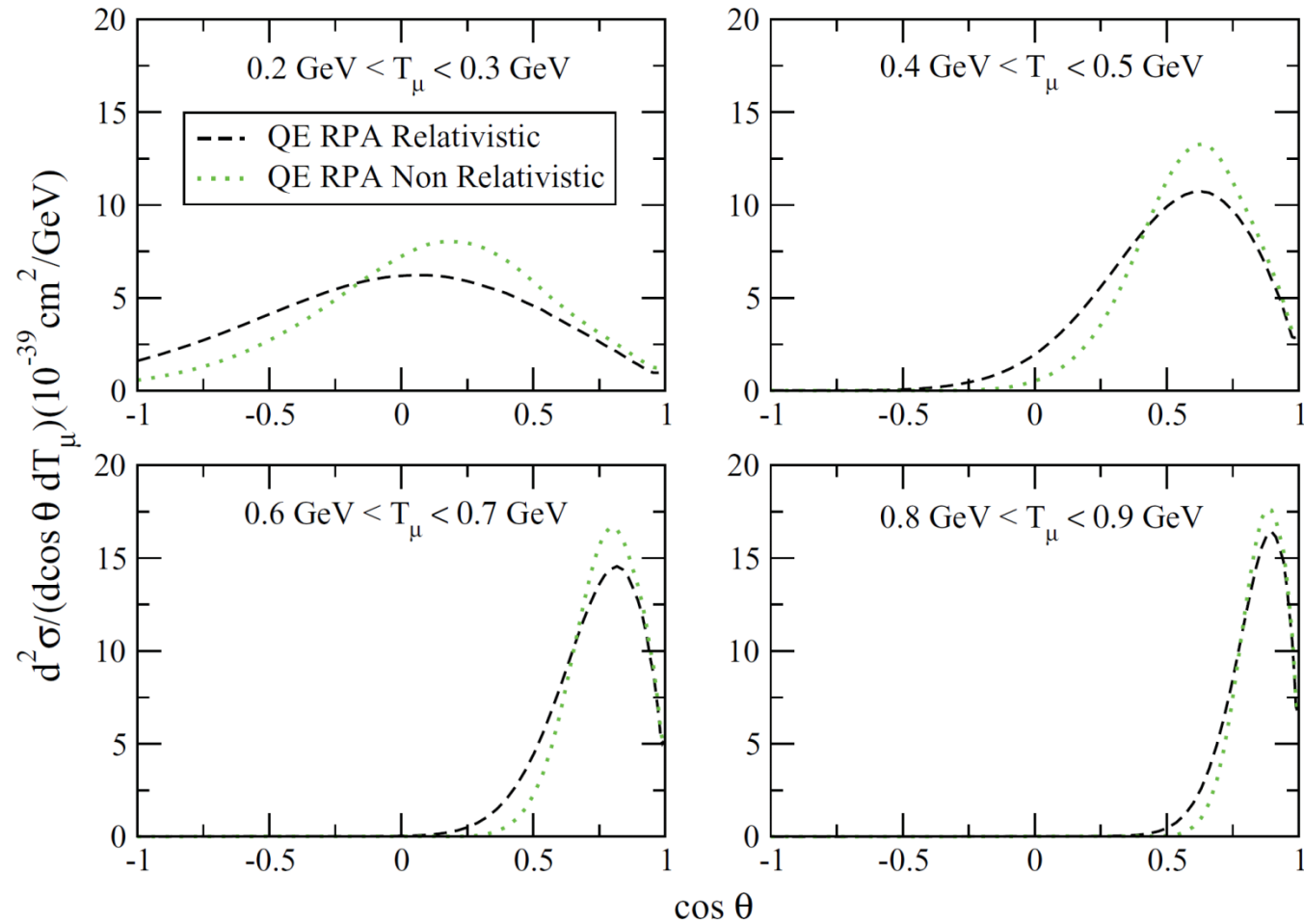
Relativistic corrections

$$\omega \rightarrow \omega \left(1 + \frac{\omega}{2M_N} \right)$$

$$\Pi \rightarrow \left(1 + \frac{\omega}{M_N} \right) \Pi$$

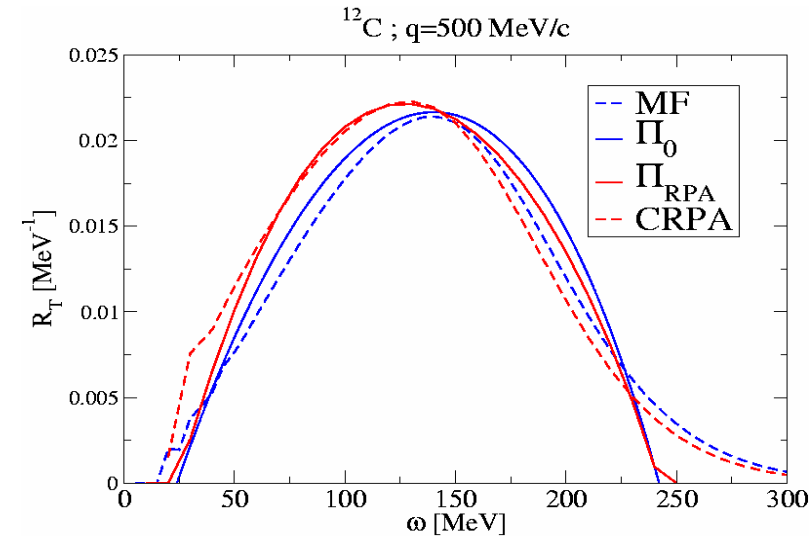


Martini, Ericson, Chanfray, Phys. Rev. C 84 055502 (2011)



From nuclear matter to finite nuclei

A comparison between a finite nucleus (Woods-Saxon + continuum RPA) and pure nuclear matter calculation shows that nuclear matter at these energies is a good approximation of the nucleus.



The agreement further improves introducing the Local density Approximation (this is the case of our model)

Local Density Approximation

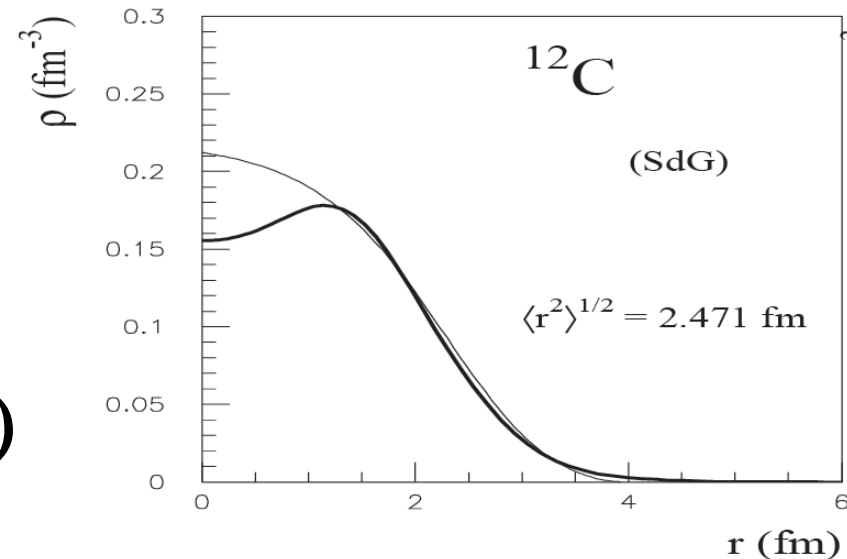
$$\rho = \frac{2k_F^3}{3\pi^2}$$

constant
in nuclear matter

$$k_F \rightarrow k_F(r)$$

$$k_F(r) = [3/2 \pi^2 \rho(r)]^{1/3}$$

$$\Pi_{k_F}(q, \omega) \rightarrow \Pi_{k_F(r)}(q, \omega)$$



From nuclear matter to finite nuclei

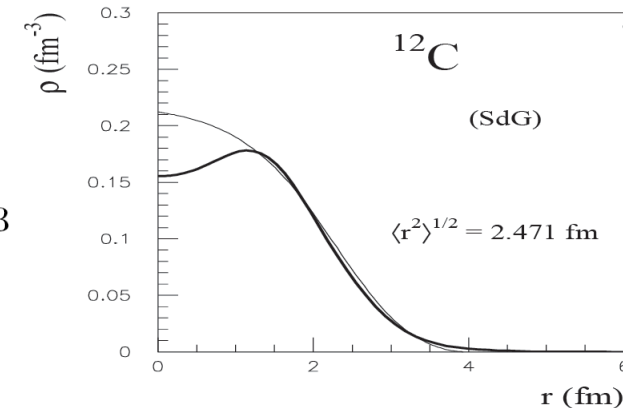
Semi-classical approximation

$$\Pi^0(\omega, \mathbf{q}, \mathbf{q}') = \int d\mathbf{r} e^{-i(\mathbf{q}-\mathbf{q}')\cdot\mathbf{r}} \Pi^0\left(\omega, \frac{1}{2}(\mathbf{q} + \mathbf{q}'), \mathbf{r}\right)$$

Local density approximation

$$k_F(r) = (3/2 \pi^2 \rho(r))^{1/3}$$

$$\Pi^0\left(\omega, \frac{\mathbf{q} + \mathbf{q}'}{2}, \mathbf{r}\right) = \Pi_{k_F(r)}^0\left(\omega, \frac{\mathbf{q} + \mathbf{q}'}{2}\right)$$



e.g. Lindhard funct. for QE

$$\Pi_{k_F(R)}^{0(L)}(\omega, q, q') = 2\pi \int du P_L(u) \Pi_{k_F(R)}^0\left(\omega, \frac{\mathbf{q} + \mathbf{q}'}{2}\right)$$

$$\begin{aligned} \Pi^{0(L)}(\omega, q, q') &= 4\pi \sum_{l_1, l_2} (2l_1 + 1)(2l_2 + 1) \begin{pmatrix} l_1 & l_2 & L \\ 0 & 0 & 0 \end{pmatrix}^2 \\ &\times \int dR R^2 j_{l_1}(qR) j_{l_2}(q'R) \Pi_{k_F(R)}^{0(l_2)}(\omega, q, q') \end{aligned}$$

$$R_{(k)xy}^{0PP'}(\omega, q) = -\frac{\mathcal{V}}{\pi} \sum_J \frac{2J+1}{4\pi} \text{Im}[\Pi_{(k)xyPP'}^{0(J)}(\omega, q, q)]$$

QE, 2p-2h, ... Longit., Transv., Charge

Details: p-h effective interaction

$$V_{NN} = (f' + V_\pi + V_\rho + V_{g'}) \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$$

$$V_{N\Delta} = (V_\pi + V_\rho + V_{g'}) \boldsymbol{\tau}_1 \cdot \mathbf{T}_2^\dagger$$

$$V_{\Delta N} = (V_\pi + V_\rho + V_{g'}) \mathbf{T}_1 \cdot \boldsymbol{\tau}_2$$

$$V_{\Delta\Delta} = (V_\pi + V_\rho + V_{g'}) \mathbf{T}_1 \cdot \mathbf{T}_2^\dagger$$

$$f' = 0.6 \quad g'_{NN} = 0.7 \quad g'_{N\Delta} = g'_{\Delta\Delta} = 0.5$$

$$G_M^*/G_M = G_A^*/G_A = f^*/f = 2.2$$

$$V_\pi = \left(\frac{g_r}{2M_N} \right)^2 F_\pi^2 \frac{q^2}{\omega^2 - q^2 - m_\pi^2} \boldsymbol{\sigma}_1 \cdot \hat{q} \boldsymbol{\sigma}_2 \cdot \hat{q}$$

$$V_\rho = \left(\frac{g_r}{2M_N} \right)^2 C_\rho F_\rho^2 \frac{q^2}{\omega^2 - q^2 - m_\rho^2} \boldsymbol{\sigma}_1 \times \hat{q} \boldsymbol{\sigma}_2 \times \hat{q}$$

$$V_{g'} = \left(\frac{g_r}{2M_N} \right)^2 F_\pi^2 g' \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$$

$$C_\rho = 1.5 \quad F_\pi(q) = (\Lambda_\pi^2 - m_\pi^2)/(\Lambda_\pi^2 - q^2)$$

$$\Lambda_\pi = 1 \text{ GeV} \quad \Lambda_\rho = 1.5 \text{ GeV}$$

RPA

$$\Pi = \Pi^0 + \Pi^0 V \Pi$$

$$(1 + \Pi V)^* \Pi = (1 + \Pi V)^* \Pi^0 + (1 + \Pi V)^* \Pi^0 V \Pi$$

$$\Pi + \Pi^* V^* \Pi = (1 + \Pi V)^* \Pi^0 (1 + V \Pi)$$

$$\text{Im}(\Pi) = |\Pi|^2 \text{Im}(V) + |1 + V \Pi|^2 \text{Im}(\Pi^0)$$

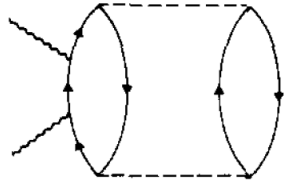
coherent

exclusive channels:

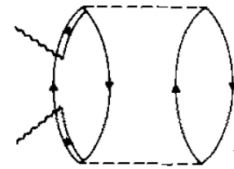
QE, 2p-2h, $\Delta \rightarrow \pi N$...

Main difficulties in the 2p-2h sector

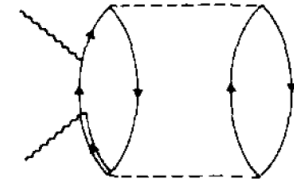
- Huge number of diagrams and terms



16 from NN correlations



49 from MEC



56 from interference

Alberico, Ericson, Molinari, Ann. Phys. 154, 356 (1984)

fully relativistic calculation (just of MEC !):

3000 direct terms

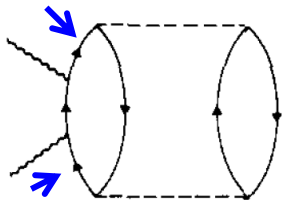
More than 100 000 exchange terms

De Pace, Nardi, Alberico, Donnelly, Molinari, Nucl. Phys. A741, 249 (2004)

- Divergences in NN correlations

prescriptions:

- nucleon propagator only off the mass shell (*Alberico et al. Ann. Phys. 1984*)
- kinematical constraints + nucleon self energy in the medium (*Nieves et al PRC 83*)
- regularization parameter taking into account the finite size of the nucleus to be fitted to data (*Amaro et al. PRC 82 044601 2010*)



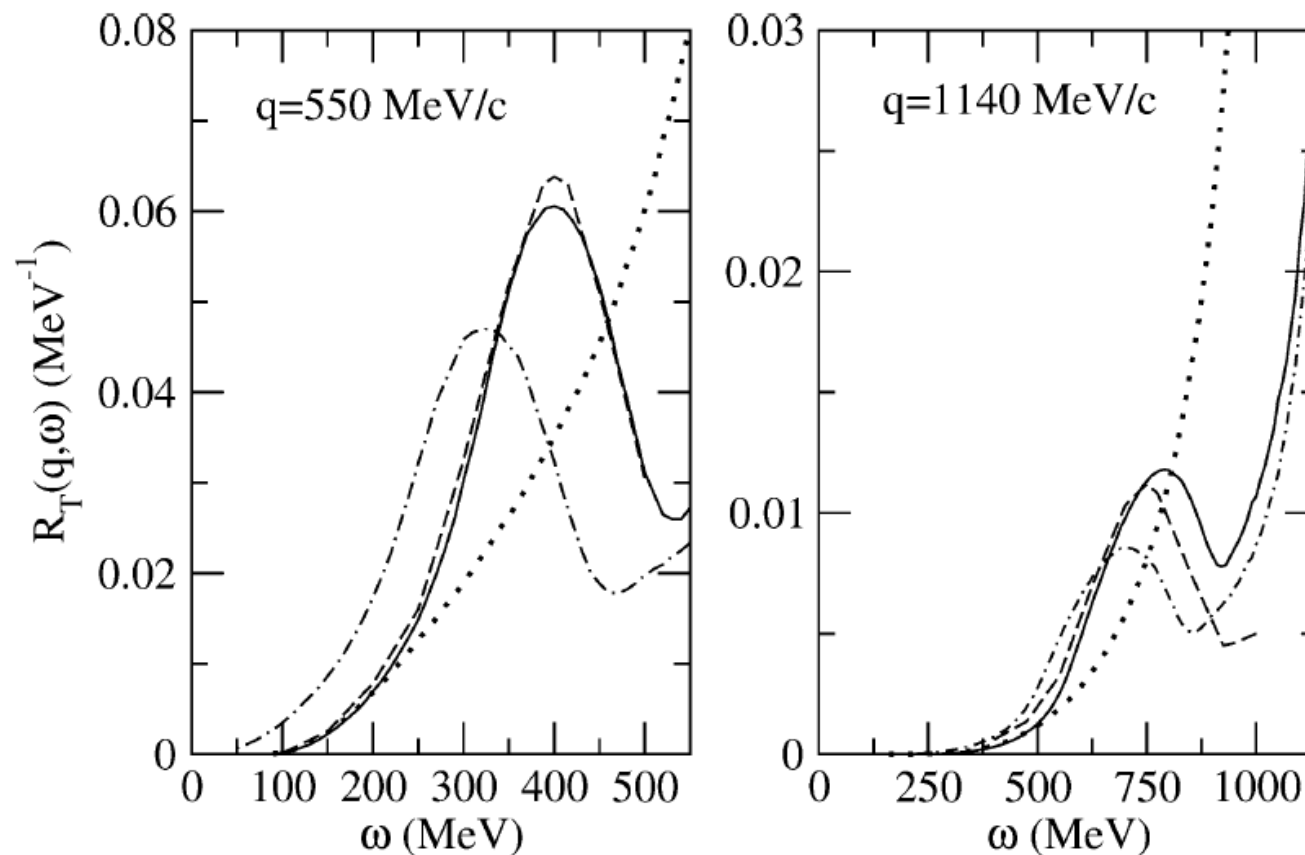


Fig. 8. The relativistic transverse response function $R_T(q, \omega)$ at $q = 550 \text{ MeV}/c$ and $q = 1140 \text{ MeV}/c$ calculated with $\bar{\epsilon}_2 = 70 \text{ MeV}$ (solid) and with $\bar{\epsilon}_2 = 0$ (dot-dashed). Only the direct contribution is shown. The non-relativistic results are also displayed in order to shed light on the role of relativity in the response (dotted). For the sake of comparison the relativistic results obtained in DBT are displayed (dashed). In all instances $k_F = 1.3 \text{ fm}^{-1}$.

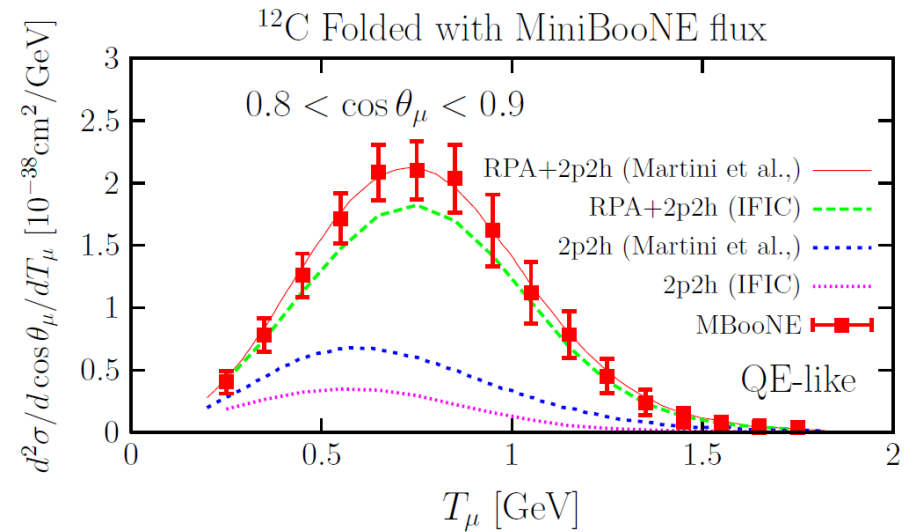
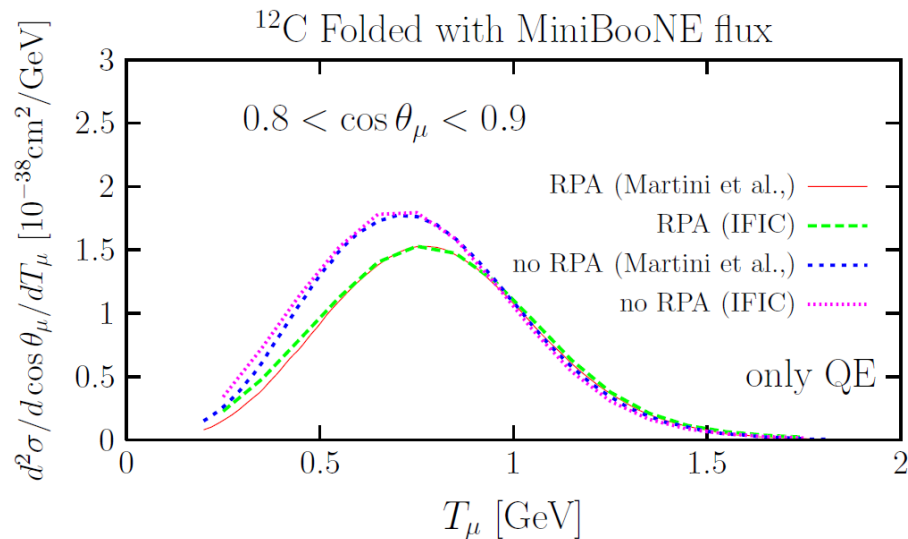
Comparison between our approach and the one of Nieves et al.

Morfin, Nieves, Sobczyk Adv.High Energy Phys. 2012 (2012) 934597

Genuine QE

V

QE + np-nh



- Genuine QE bare and RPA very similar in Martini et al. and Nieves et al.

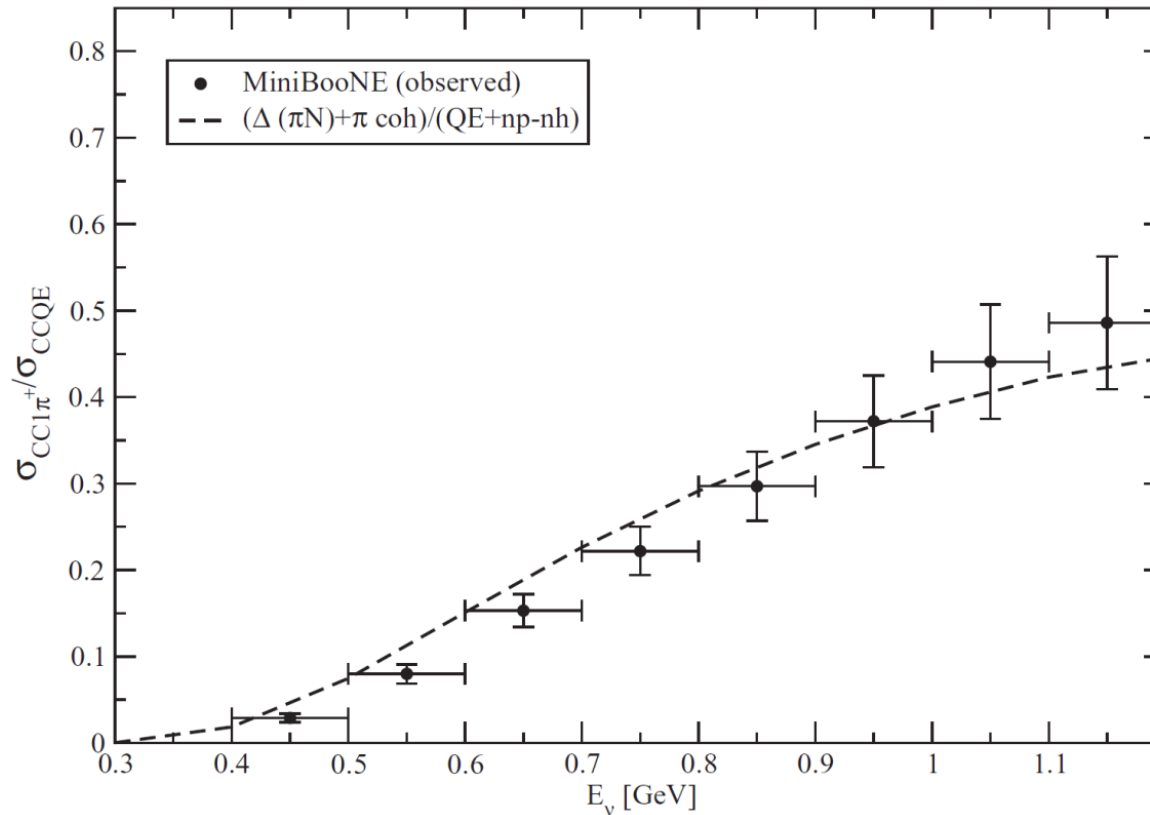
- Factor ~ 2 for the np-nh contribution

Both models compatible with MiniBooNE
(additional normalization uncertainty of 10% in the MB data not shown here)

Comparison with ν data pion production

Charged current total $1\pi^+$ production over QE ratio

MiniBooNE, Phys. Rev. Lett. 103, 081801 (2009)



In our model π FSI are not included;
a reduction of $\sim 15\%$ is expected

NC π^0 production cross sections

Total cross section	MiniBooNE PRD 81, 013005 2010 σ [10^{-40} cm ² /nucleon]	Our model σ [10^{-40} cm ² /nucleon]
ν @ 808 MeV	4.76 ± 0.05 st ± 0.76 sy	5.42
$\bar{\nu}$ @ 664 MeV	1.48 ± 0.05 st ± 0.23 sy	1.37

	Incoherent exclusive NC $1\pi^0$ corrected for FSI effects	Our model
ν @ 808 MeV	5.71 ± 0.08 st ± 1.45 sy	5.14
$\bar{\nu}$ @ 664 MeV	1.28 ± 0.07 st ± 0.35 sy	1.17

π^0 total NC/ σ total CC	SciBooNE PRD 81 033004 '10	Our model
ν @ 1.1 GeV	$(7.7 \pm 0.5$ st ± 0.5 sy) 10^{-2}	$7.9 \cdot 10^{-2}$ (without np-nh: 9.8)

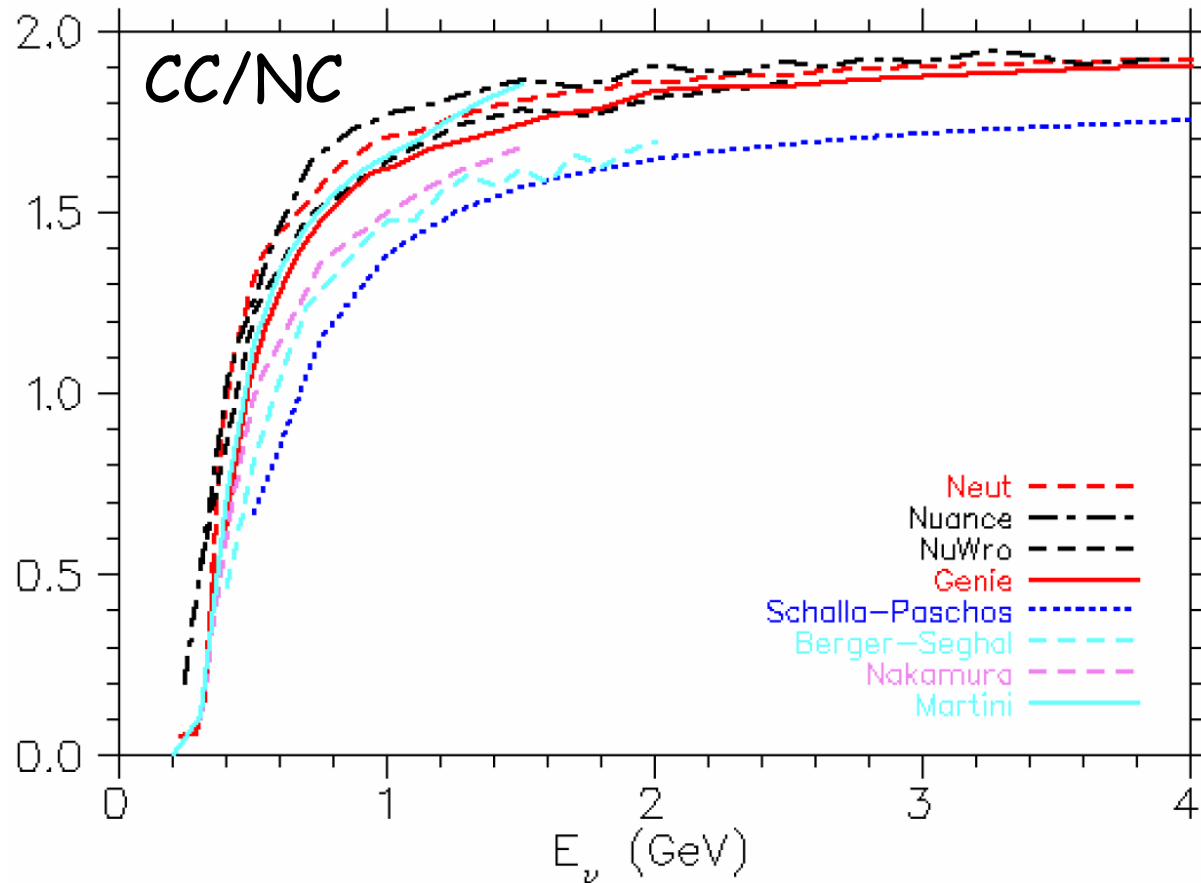
Coherent pion production

Ratio of σ	Experiment	Our model
π^+ coherent CC/ σ total CC	SciBooNE u.l.@ E_ν 1.1 GeV $0.67 \cdot 10^{-2}$ PRD 78 112004 '08	$0.71 \cdot 10^{-2}$ (without np-nh: 0.89)
π^0 coherent NC/ π^0 total NC	MiniBooNE $(19.5 \pm 1.1 \pm 2.5) \cdot 10^{-2}$ Phys. Lett. B 664,41 '08	$6 \cdot 10^{-2}$
π^0 coherent NC/ σ total CC	SciBooNE $(0.7 \pm 0.4) \cdot 10^{-2}$ PRD 81 033004 '10	$0.4 \cdot 10^{-2}$ (without np-nh: 0.5)
π^+ coherent CC/ π^0 coherent NC	SciBooNE $0.14^{+0.30}_{-0.28}$ PRD 81 111102 '10	1.5

coherent puzzle

Coherent puzzle

Boyd S. et al. AIP Conf. Proc. 1189 60 (2009)



SciBooNE:

$$\frac{\pi^+ \text{ coh. CC}}{\pi^0 \text{ coh. NC}} = 0.14^{+0.30}_{-0.28}$$

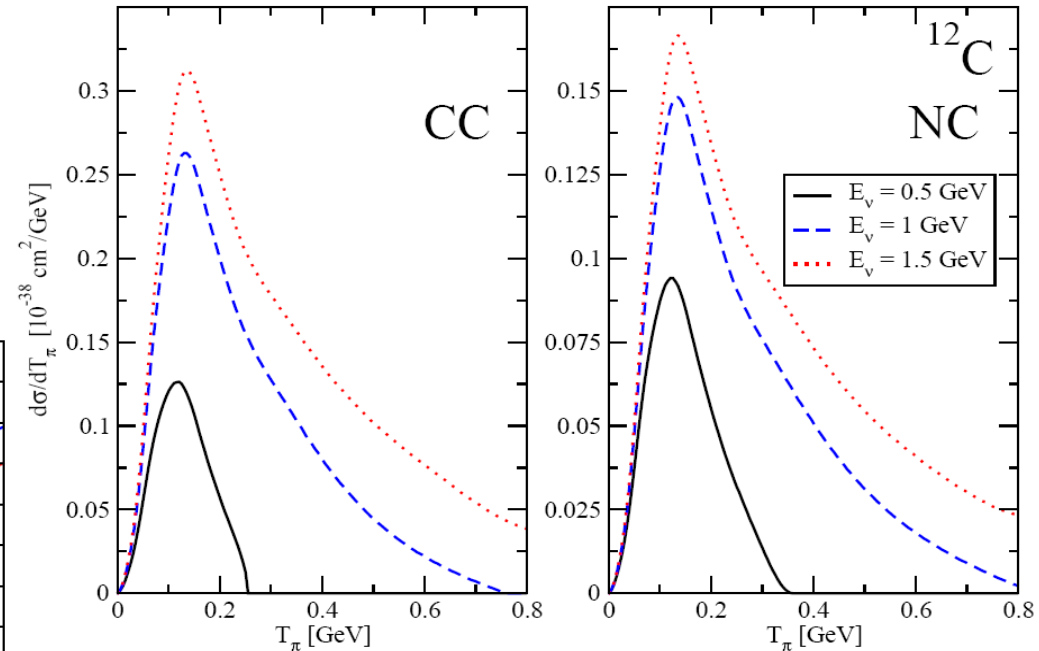
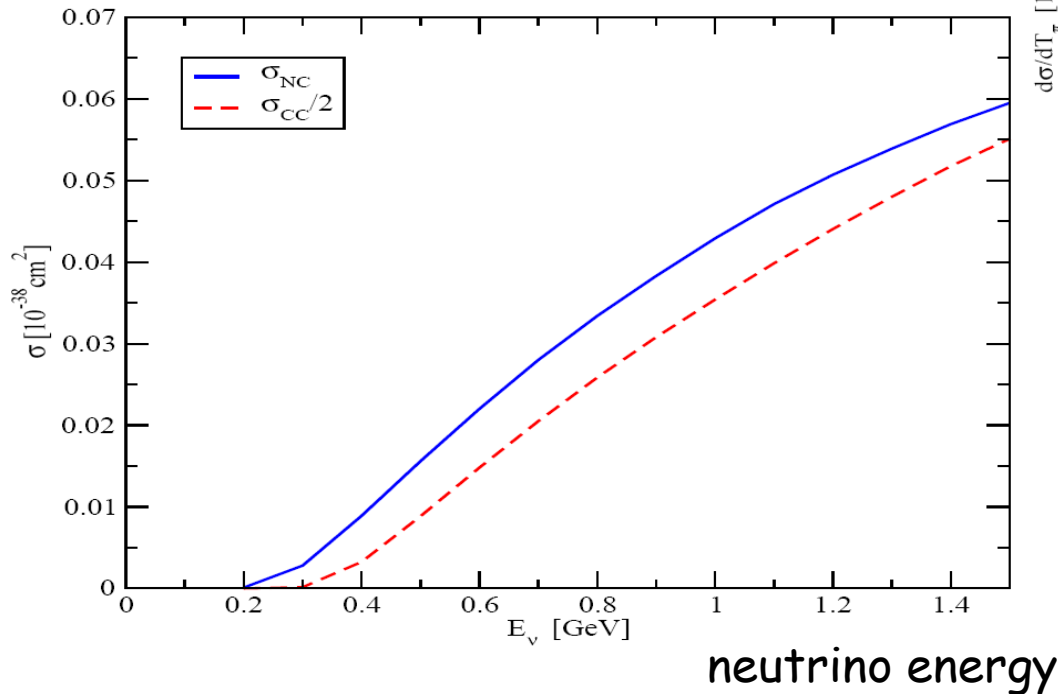
Theoretical models:

$$\frac{\pi^+ \text{ coh. CC}}{\pi^0 \text{ coh. NC}} = 1.5 \sim 2 !!$$

ν_μ induced coherent pion production off ^{12}C

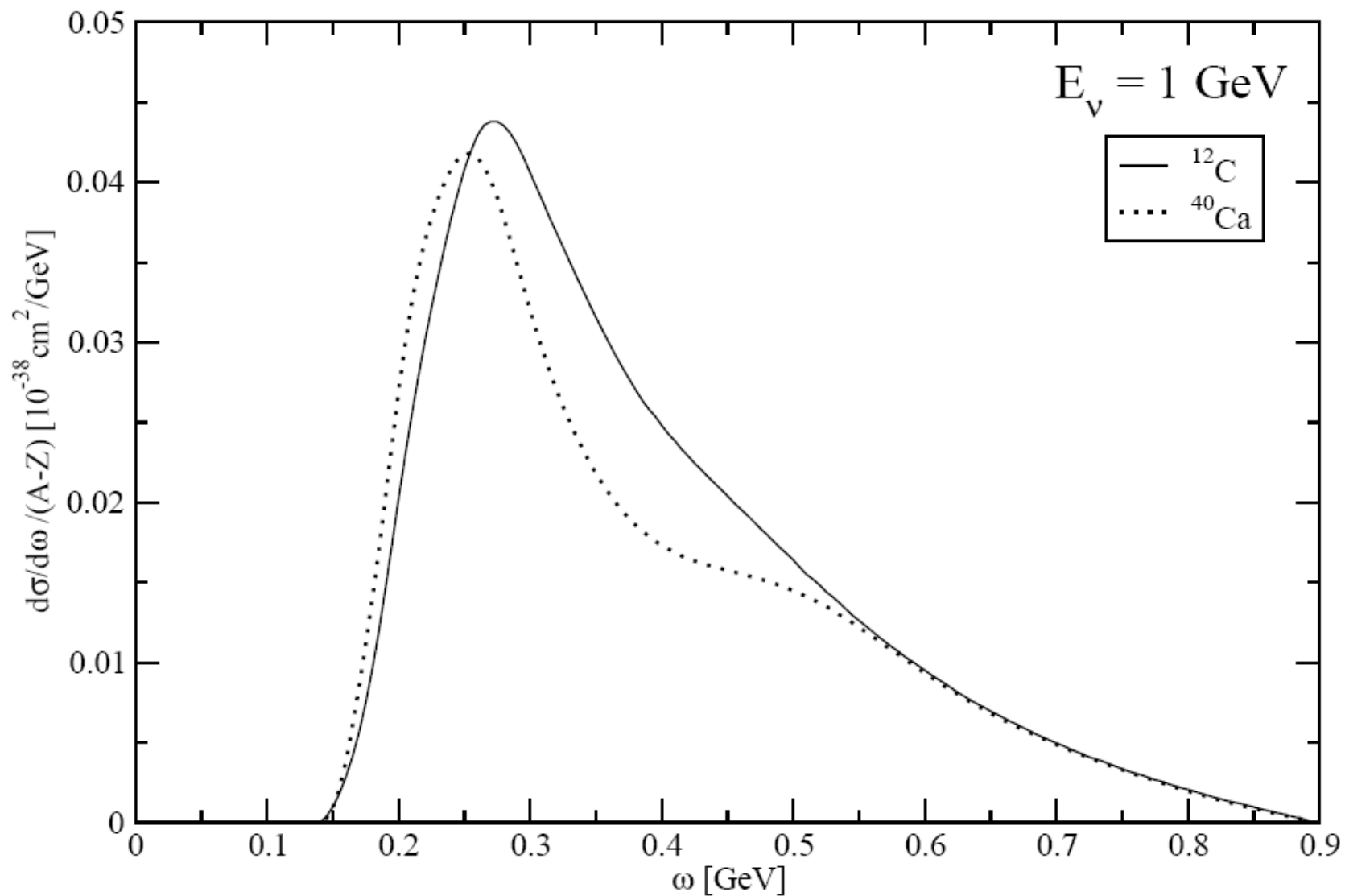
Differential cross section

Total cross section



pion kinetic energy

ν_μ induced coherent pion production



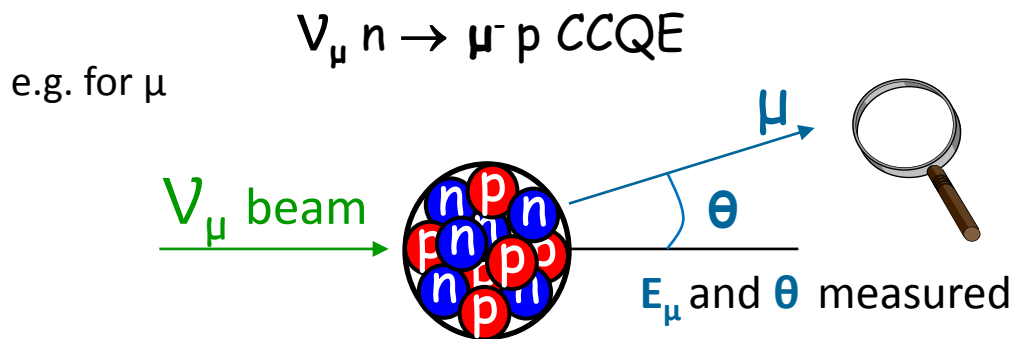
Neutrino energy reconstruction problems and neutrino oscillations

Towards the neutrino oscillation physics

Neutrino oscillation experiments require the determination of the neutrino energy which enters the expression of the oscillation probability.

The neutrino energy is unknown. We know only broad fluxes.

The determination of the neutrino energy is done through Charged Current QuasiElastic events.



Reconstructed neutrino energy

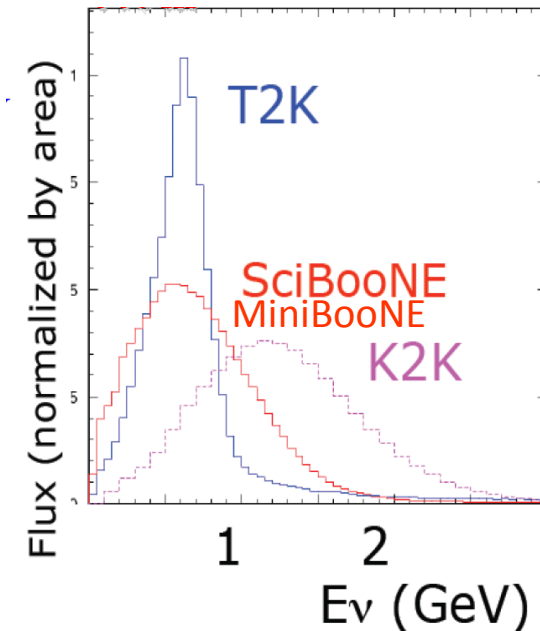
$$\overline{E_\nu} = \frac{E_\mu - m_\mu^2 / (2M)}{1 - (E_\mu - P_\mu \cos \theta) / M}$$

via two-body kinematics

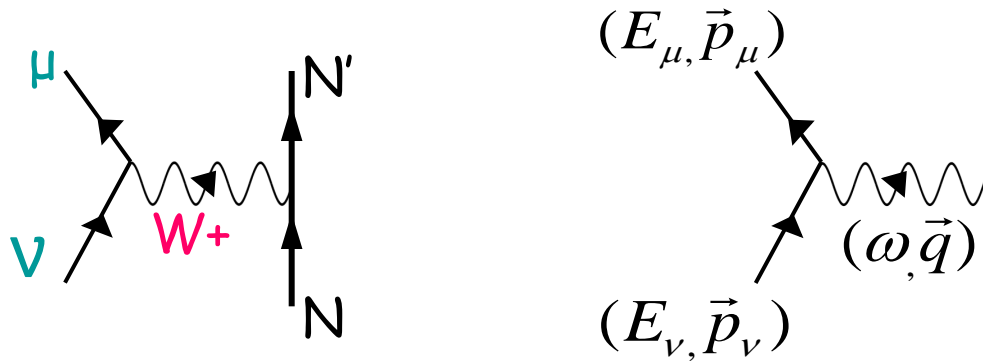
$\overline{E_\nu} = E_\nu$ is exact only for CCQE with free nucleon

reconstructed neutrino energy $\overline{E_\nu}$ \longleftrightarrow E_ν true neutrino energy

?



QE Scattering with free nucleon at rest: two-body kinematics



$$\omega = E_\nu - E_\mu$$

$$q^2 = E_\nu^2 + p_\mu^2 - 2E_\nu p_\mu \cos\theta$$

$$q^2 - \omega^2 = 4(E_\mu + \omega)E_\mu \sin^2\frac{\theta}{2} - m_\mu^2 + 2(E_\mu + \omega)(E_\mu - p_\mu) \cos\theta$$

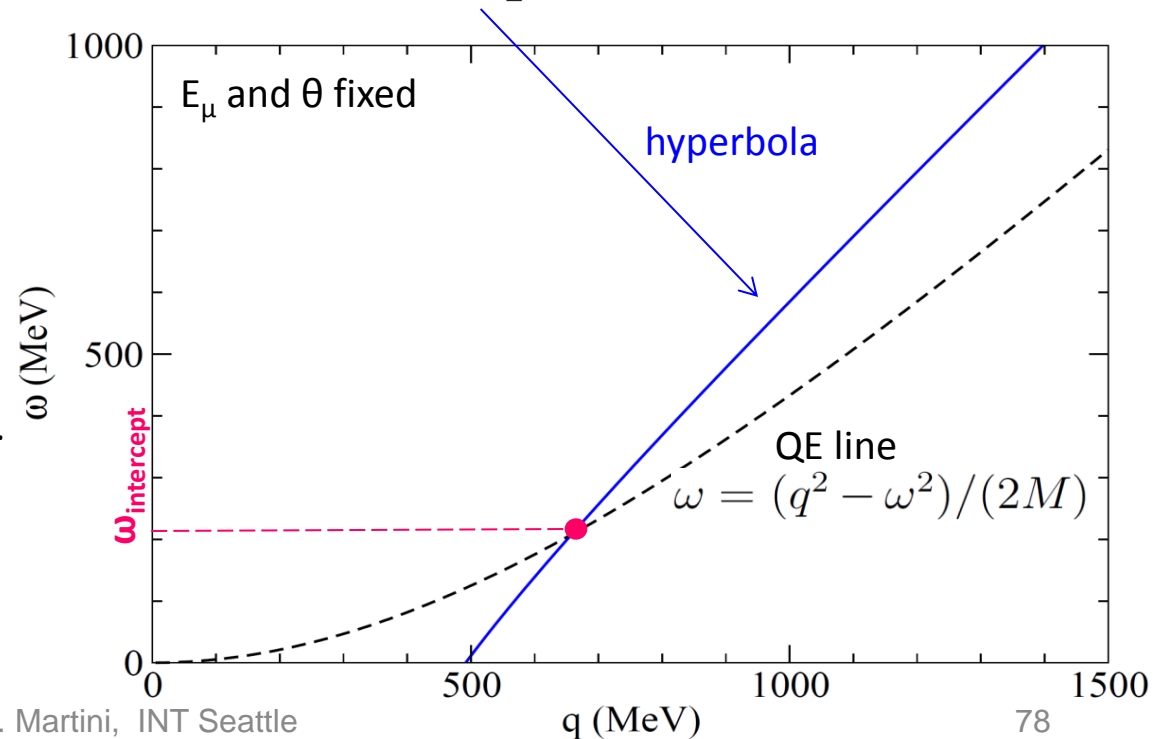
The nuclear response function is proportional to the delta distribution

$$\delta\left[\omega - \left(\sqrt{q^2 + M^2} - M\right)\right]$$

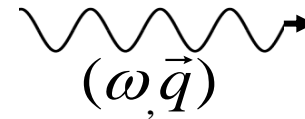
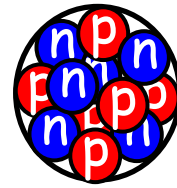
The intercept of the **hyperbola** with the **QE line** fixes the possible ω and q values for given E_μ and θ .

Hence the neutrino energy is determined

$$E_\nu = E_\mu + \omega_{\text{intercept}}$$

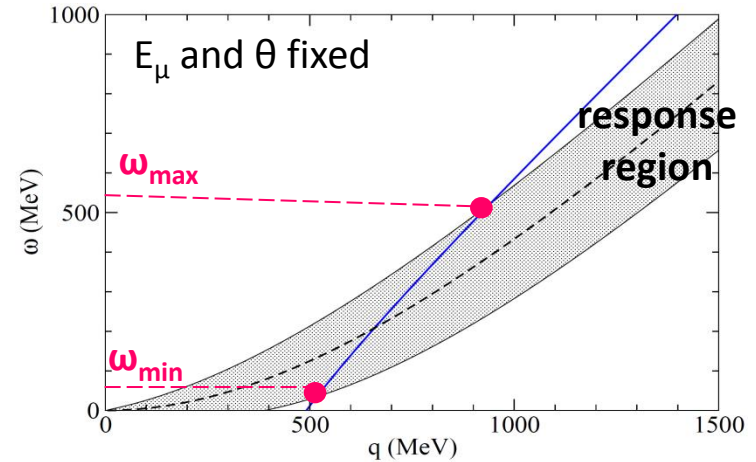
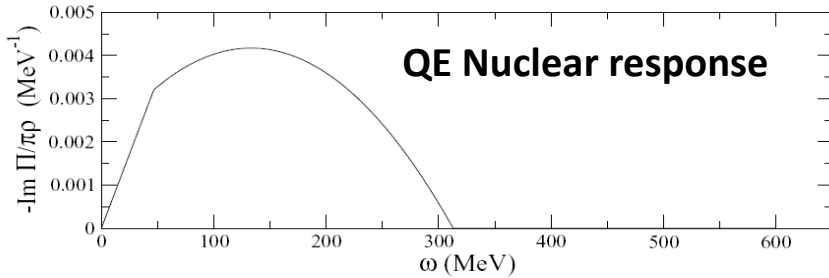


QE Scattering with nucleons inside the nucleus



1 particle- 1 hole (p-h) excitation

$$q < 2 k_F$$



Fermi motion spreads δ distribution (Fermi Gas)

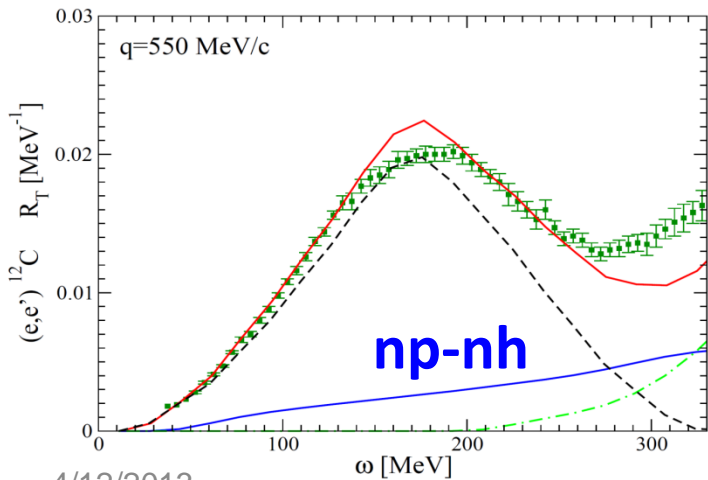
Pauli blocking cuts part of the low momentum nuclear response

RPA collective effects

Broadening of the neutrino energy

$$E_\nu = E_\mu + (\omega_{\min} \leq \omega \leq \omega_{\max})$$

np-nh excitations



- np-nh creates a high energy tail above the QE peak
- np-nh enlarges the region of response to the whole (ω, q) plane

no reason to fulfill the QE relation for E_ν reconstruction

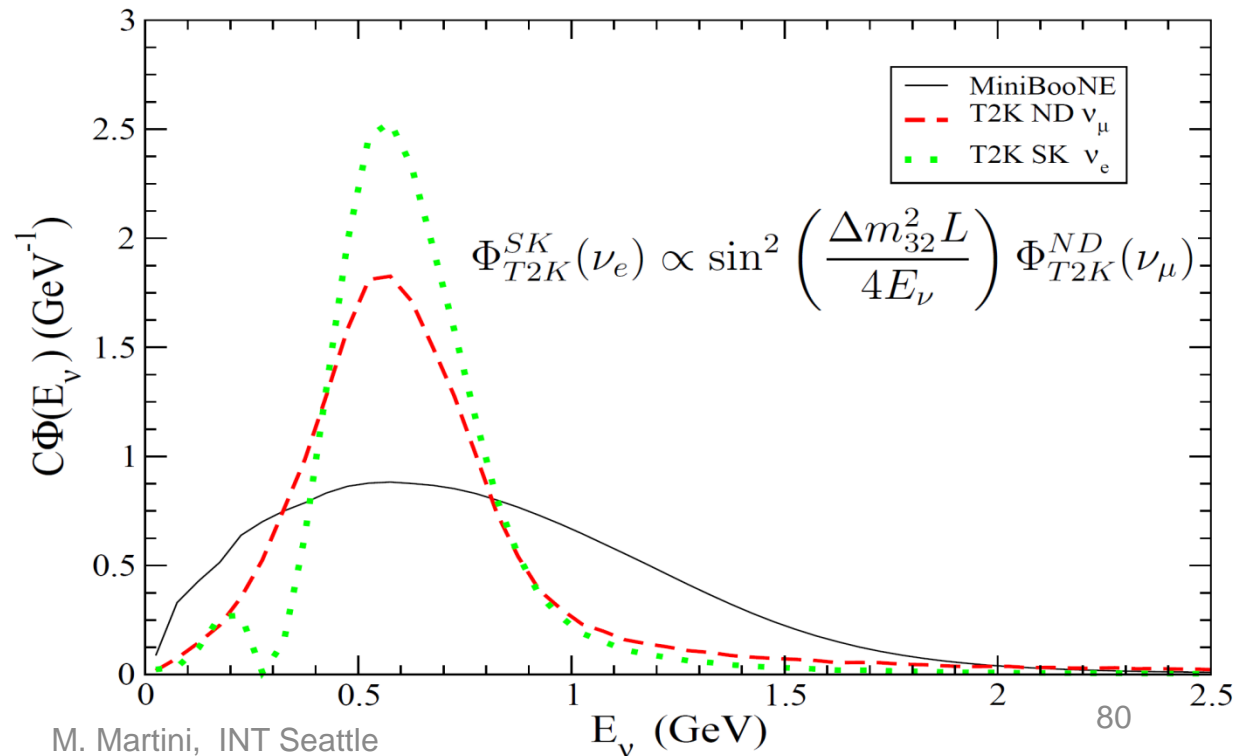
A. Neutrino energy distributions with fixed muon variables (E_μ and θ)

$$f(E_\nu, E_\mu, \theta)dE_\nu = C \left[\frac{d^2\sigma}{d\omega d\cos\theta} \right]_{\omega=E_\nu-E_\mu} \Phi(E_\nu)dE_\nu$$

Neutrino flux
3 cases for example

Neutrino-nucleus double differential cross section

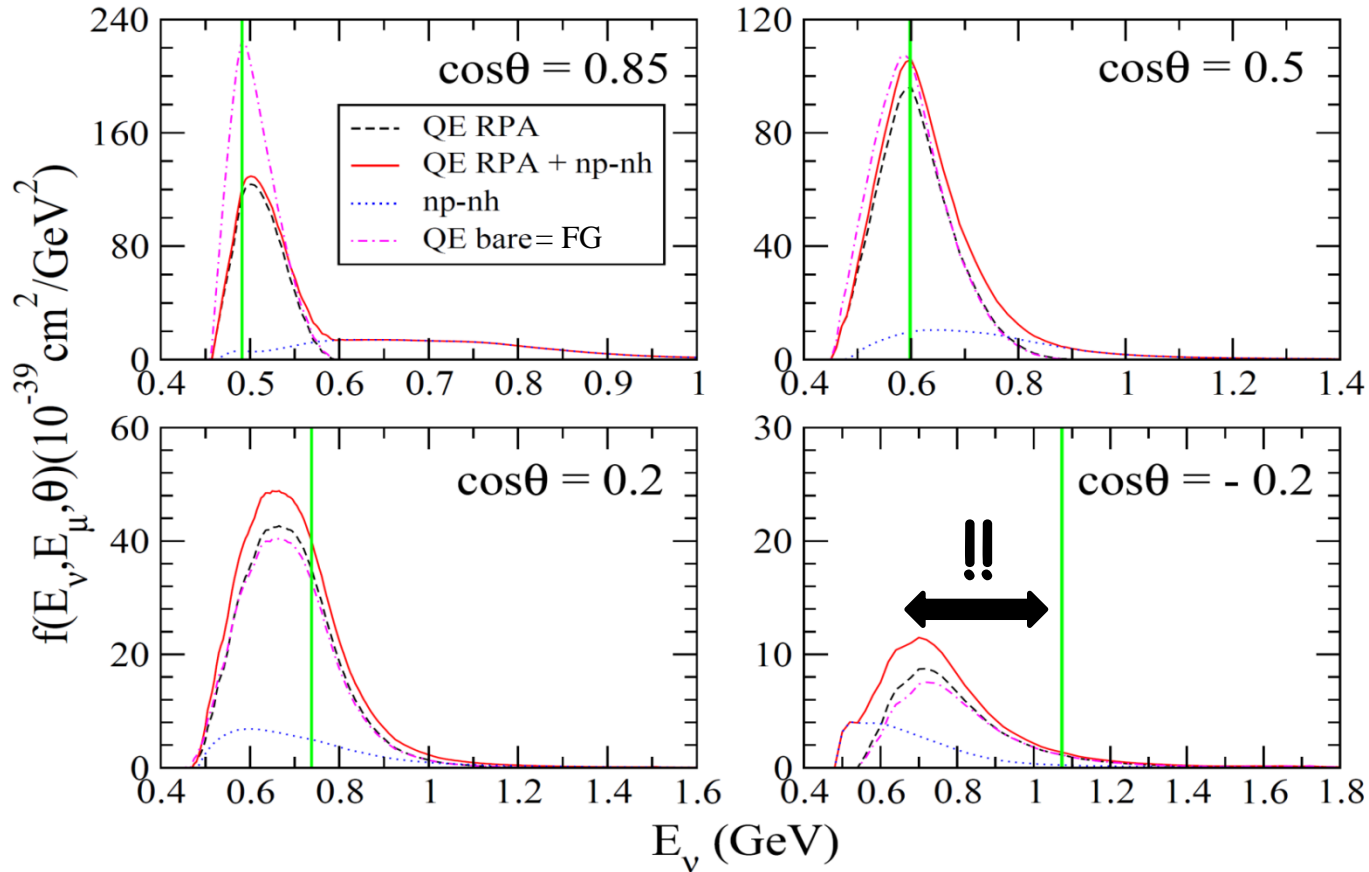
- Main ingredient of this calculation
- We consider our theoretical model which was quite successful in the reproduction of MiniBooNE QE data



Given E_μ and θ \Rightarrow Unique \bar{E}_ν reconstructed value (**vertical green line**) but a broadening of the true neutrino energy due to nuclear effects (Fermi motion, RPA, np-nh)

$T_\mu = 0.35$ GeV with T2K ND flux

Neutrino energy distributions



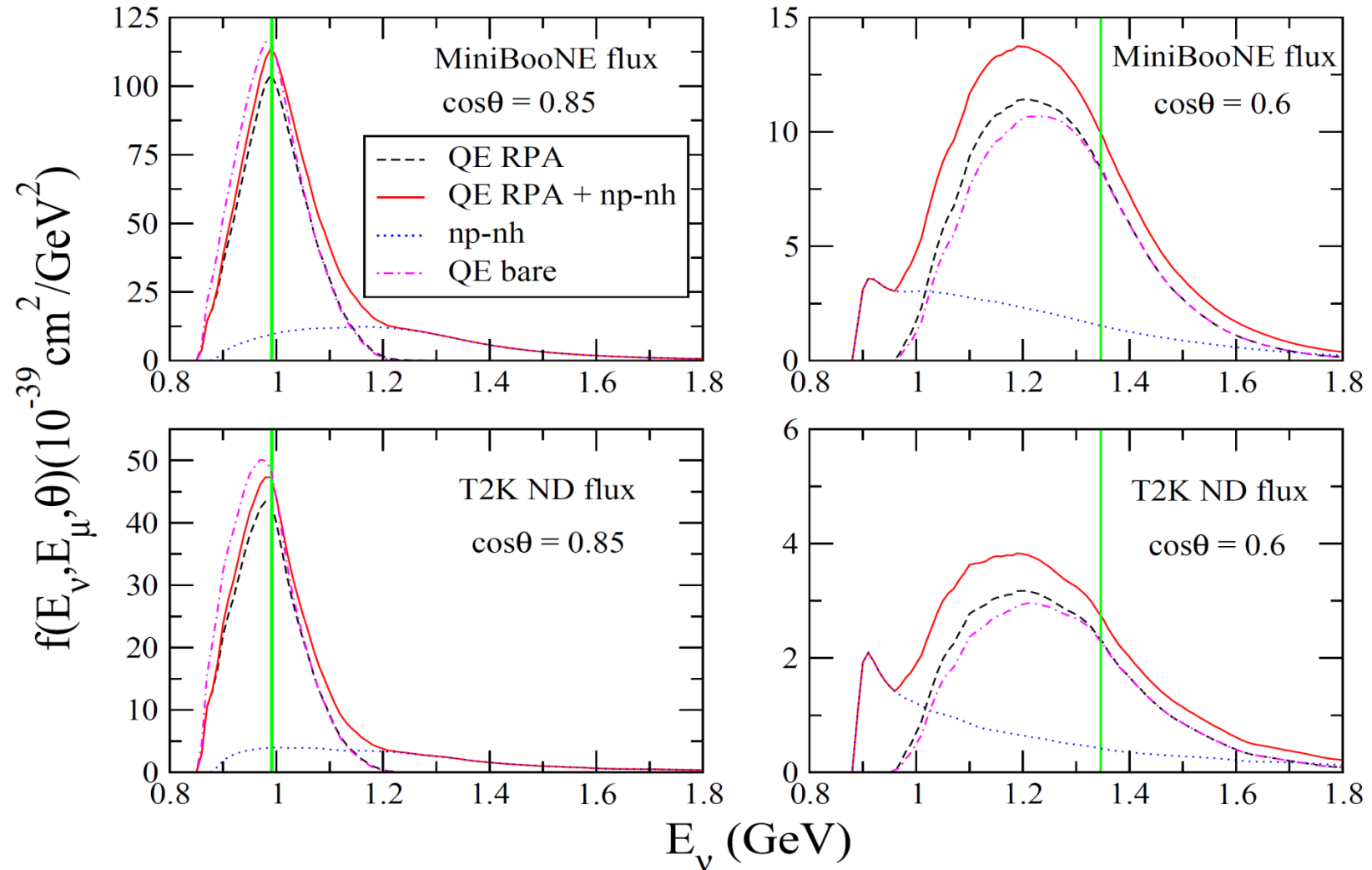
Sizeable deviations from the reconstructed energy

Hardening or softening depending on θ

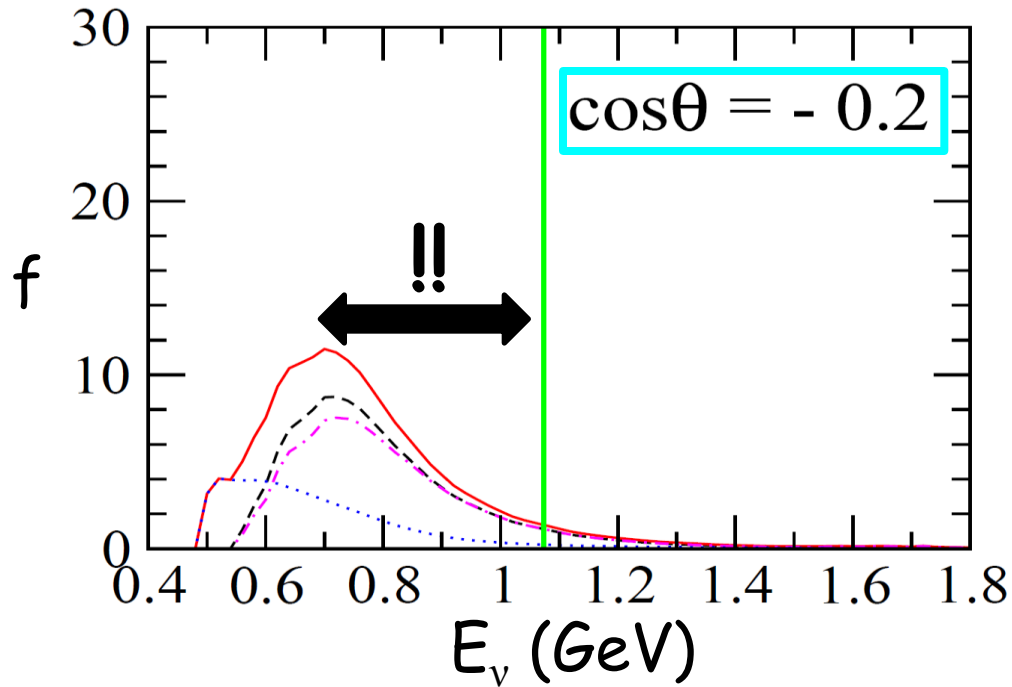
Tails created by the np-nh contribution

Neutrino energy distributions with fixed E_μ and θ

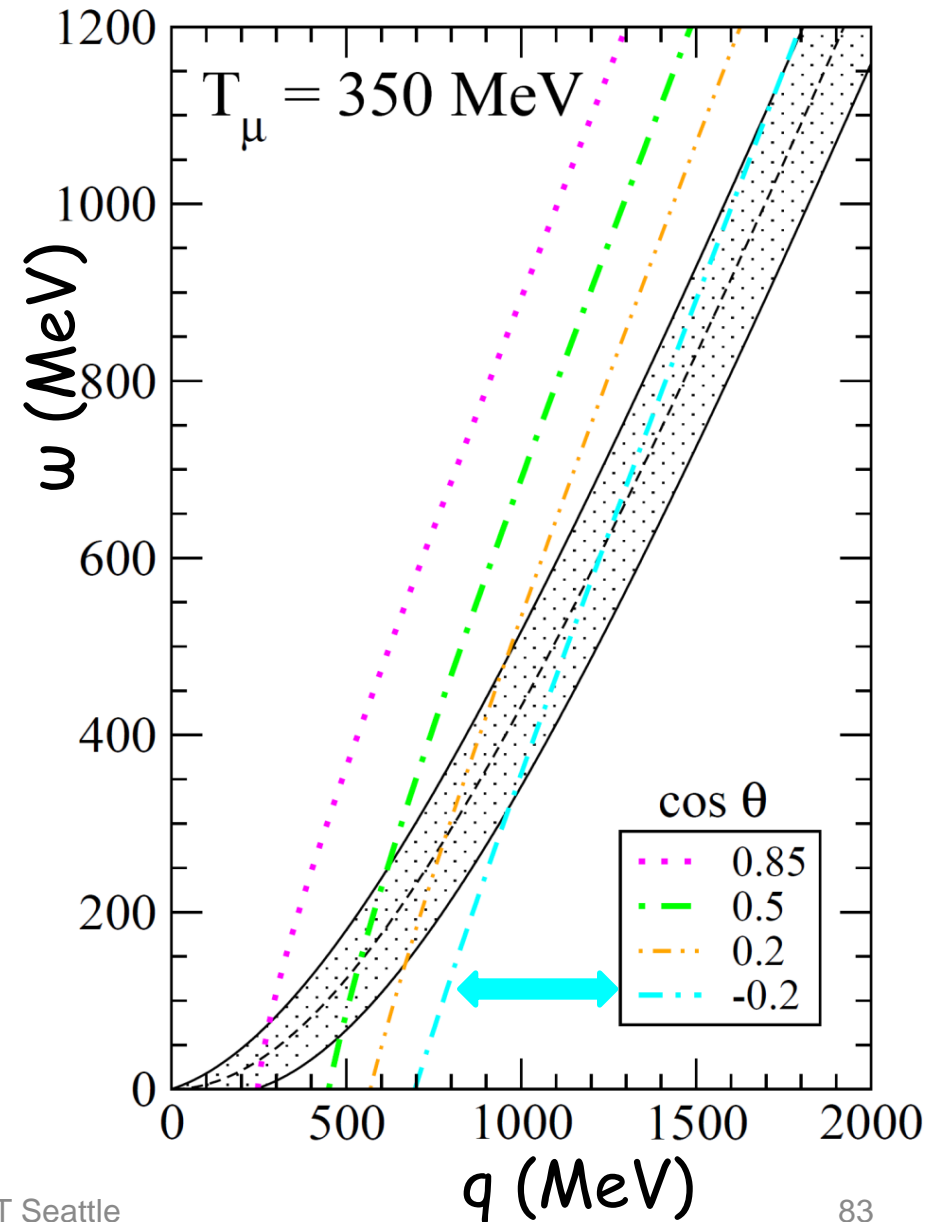
$$T_\mu = 0.75 \text{ GeV}$$



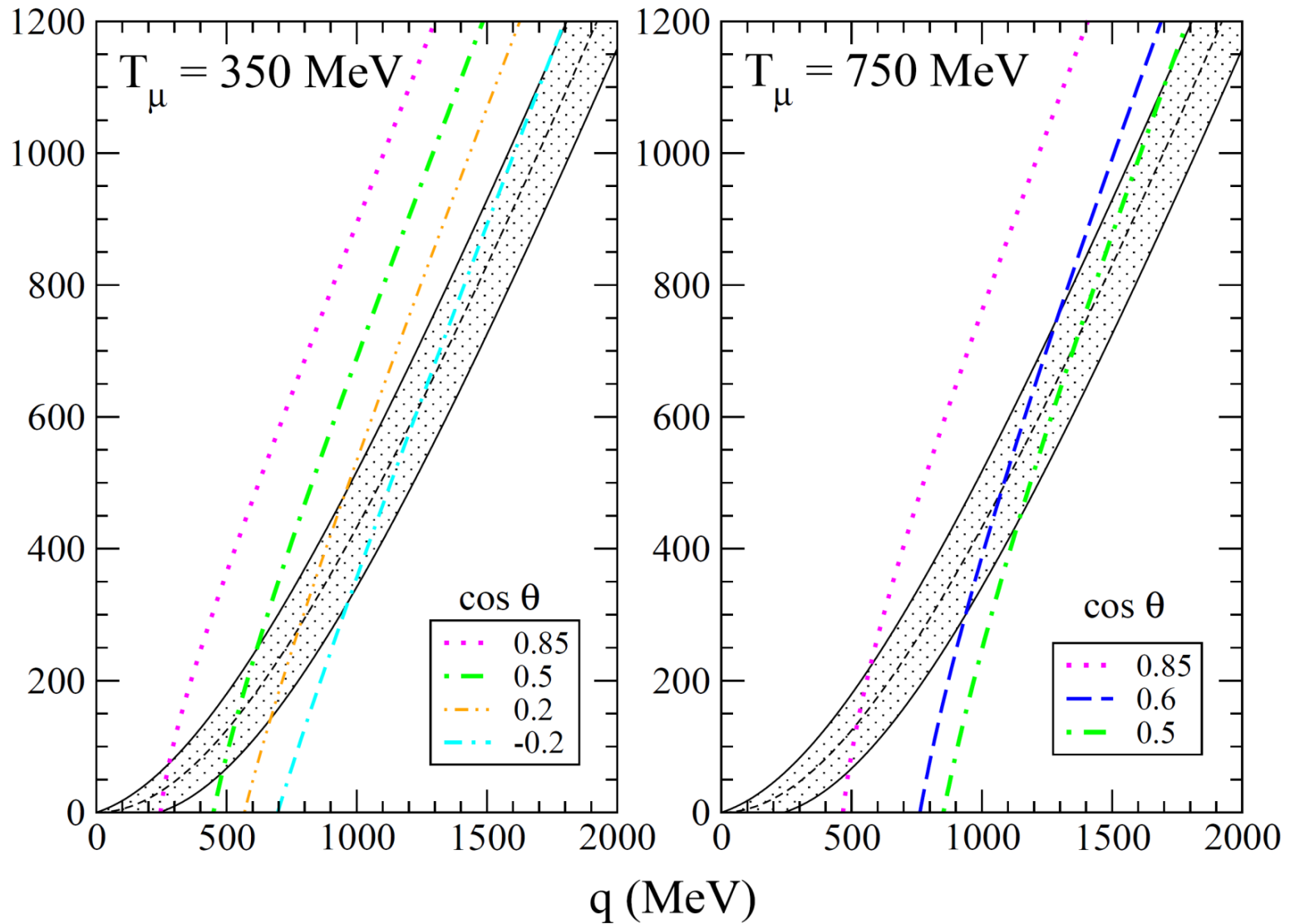
Remember: response region and hyperbolas



At large muon angle the portion of hyperbola inside the quasielastic region is very large



Response region and hyperbolas for several T_μ and θ



With increasing muon angle the portion of hyperbola inside the quasielastic region become very large

B. Neutrino energy distributions with no specification of lepton observables

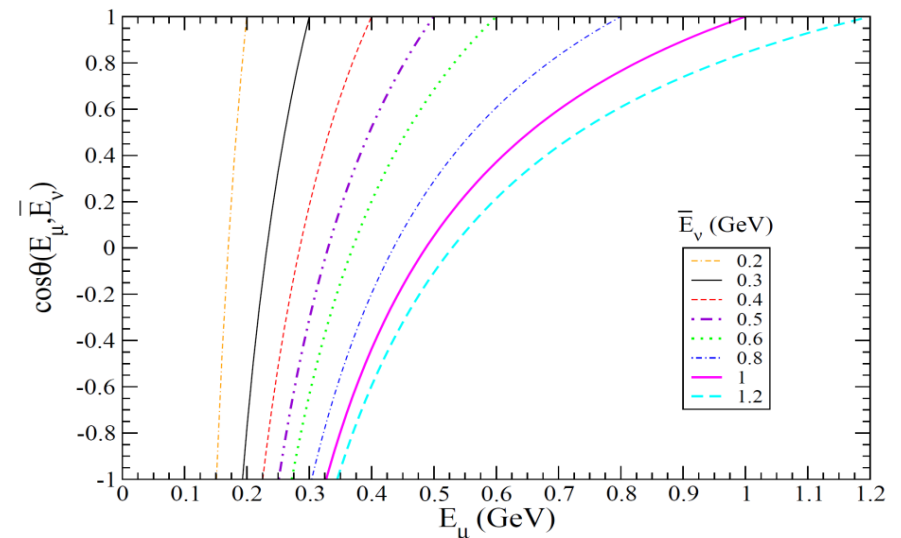
Neutrino reconstructed energy \bar{E}_ν is fixed

Many couples of E_μ and θ can lead to the same \bar{E}_ν

One has to sum over these couples

$$\bar{E}_\nu P_\mu \cos \theta + M(\bar{E}_\nu - E_\mu) - \bar{E}_\nu E_\mu + m_\mu^2/2 = 0 \iff \bar{E}_\nu = \frac{E_\mu - m_\mu^2/(2M)}{1 - (E_\mu - P_\mu \cos \theta)/M}$$

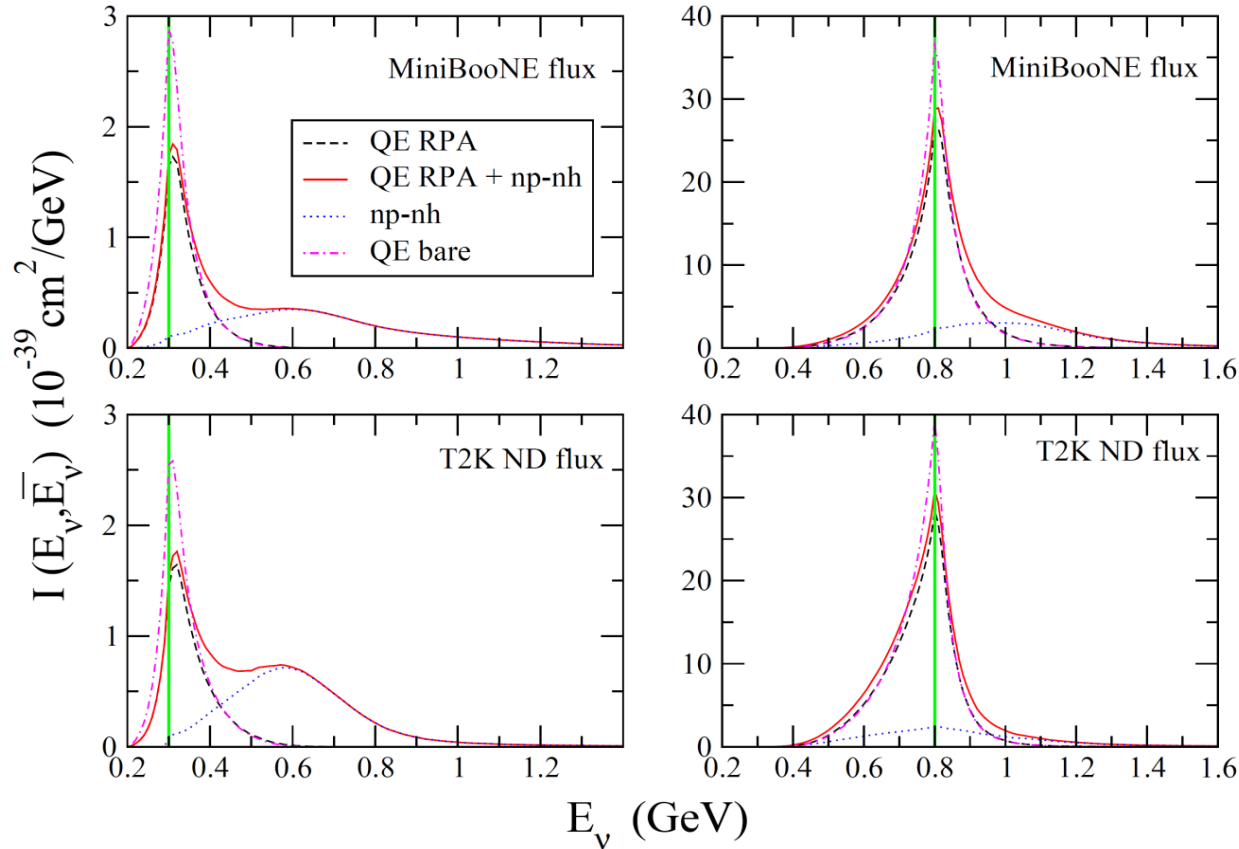
$\cos \theta(E_\mu, \bar{E}_\nu)$ is the cosine solution
of this equation for given \bar{E}_ν and E_μ



$$F(E_\nu, \bar{E}_\nu) = c \frac{\Phi(E_\nu)}{\int dE_\nu \Phi(E_\nu)} \int_{E_\mu^{\min}}^{E_\mu^{\max}} dE_\mu \left[\frac{d^2 \sigma}{d\omega d\cos\theta} \right]_{\omega=E_\nu-E_\mu, \cos\theta=\cos\theta(E_\mu, \bar{E}_\nu)}$$

Probability distribution before normalization

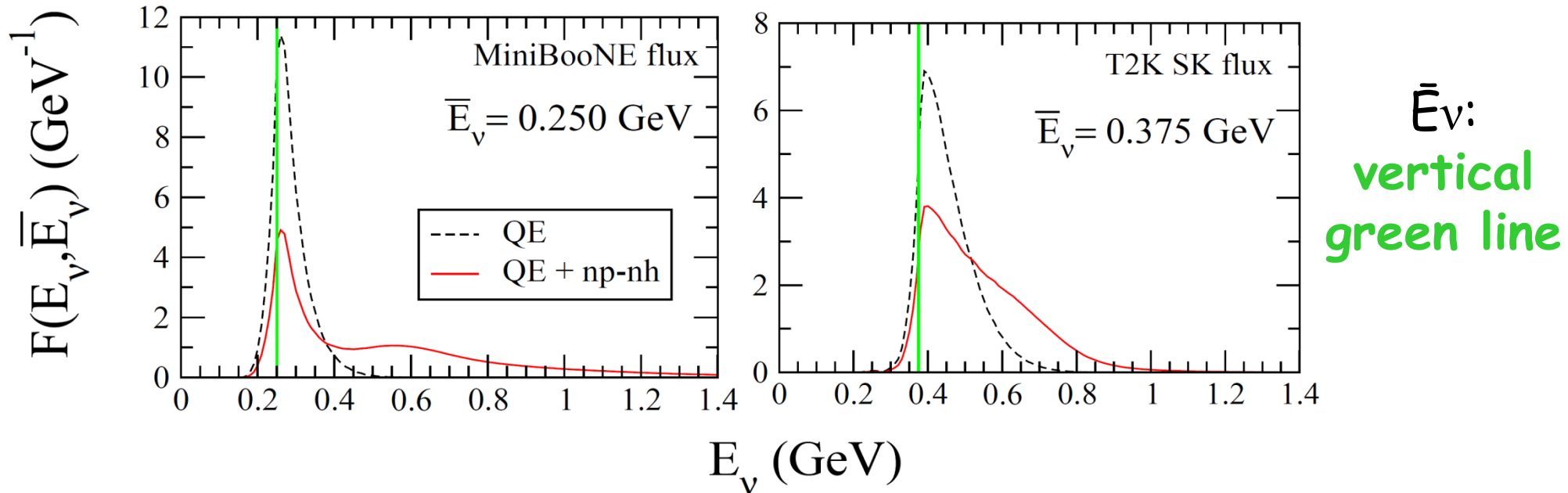
$$F(E_\nu, \overline{E}_\nu) = c \frac{\Phi(E_\nu)}{\int dE_\nu \Phi(E_\nu)} \int_{E_\mu^{\min}}^{E_\mu^{\max}} dE_\mu \left[\frac{d^2\sigma}{d\omega d\cos\theta} \right]_{\omega=E_\nu-E_\mu, \cos\theta=\cos\theta(E_\mu, \overline{E}_\nu)} = cI(E_\nu, \overline{E}_\nu)$$



Probability energy distributions with no specification of lepton observables

$$F(E_\nu, \bar{E}_\nu) = c \frac{\Phi(E_\nu)}{\int dE_\nu \Phi(E_\nu)} \int_{E_\mu^{\min}}^{E_\mu^{\max}} dE_\mu \left[\frac{d^2\sigma}{d\omega d\cos\theta} \right]_{\omega=E_\nu-E_\mu, \cos\theta=\cos\theta(E_\mu, \bar{E}_\nu)}$$

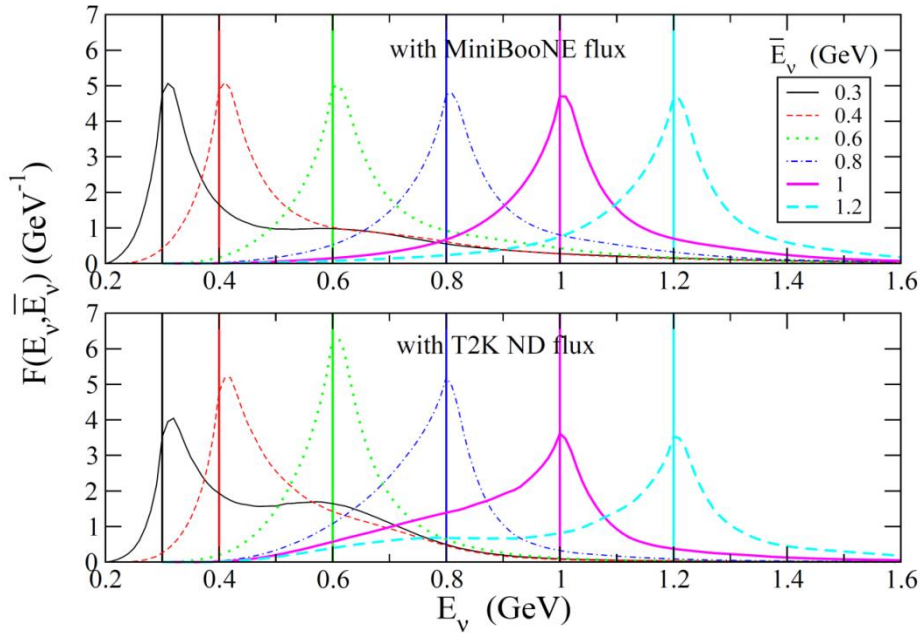
$$\int dE_\nu F(E_\nu, \bar{E}_\nu) = 1$$



High energy tail due to the np-nh contribution

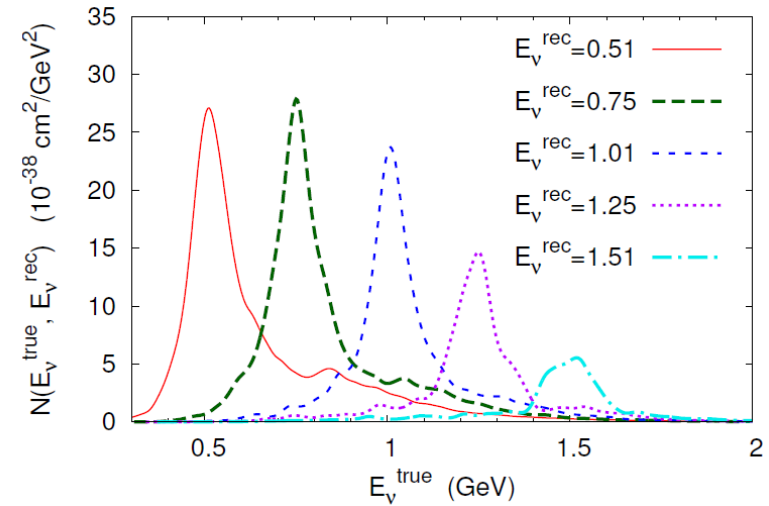
Distributions in terms of true E_ν for fixed values of reconstructed \bar{E}_ν

Martini, Ericson, Chanfray, PRD 85 093012 (2012)

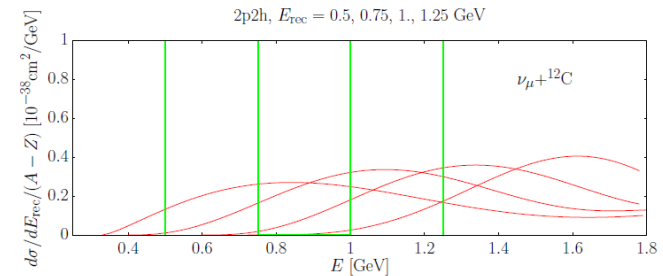
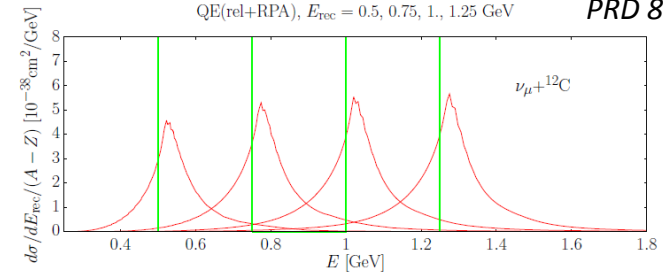


- The distributions are not symmetrical around \bar{E}_ν .
- The asymmetry favors higher energies at low \bar{E}_ν and smaller energies for large \bar{E}_ν .
- Crucial role of neutrino flux.

O. Lalakulich, U. Mosel, K. Gallmeister PRC 86 054606 (2012)



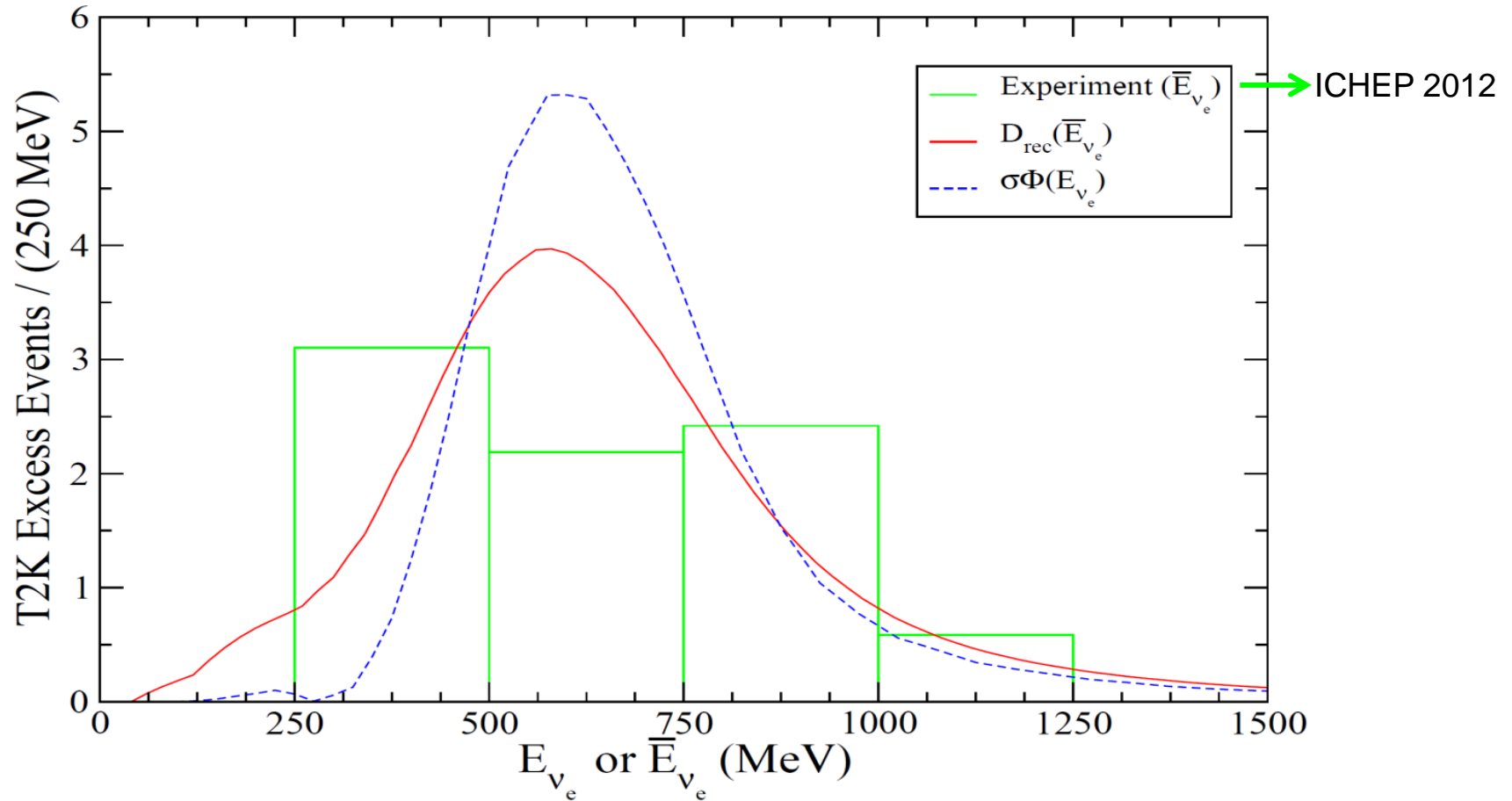
J. Nieves, F. Sanches, I. Ruiz Simo, M.J. Vicente Vacas
 QE(rel+RPA), $E_{rec} = 0.5, 0.75, 1., 1.25$ GeV
 PRD 85 113008 (2012)



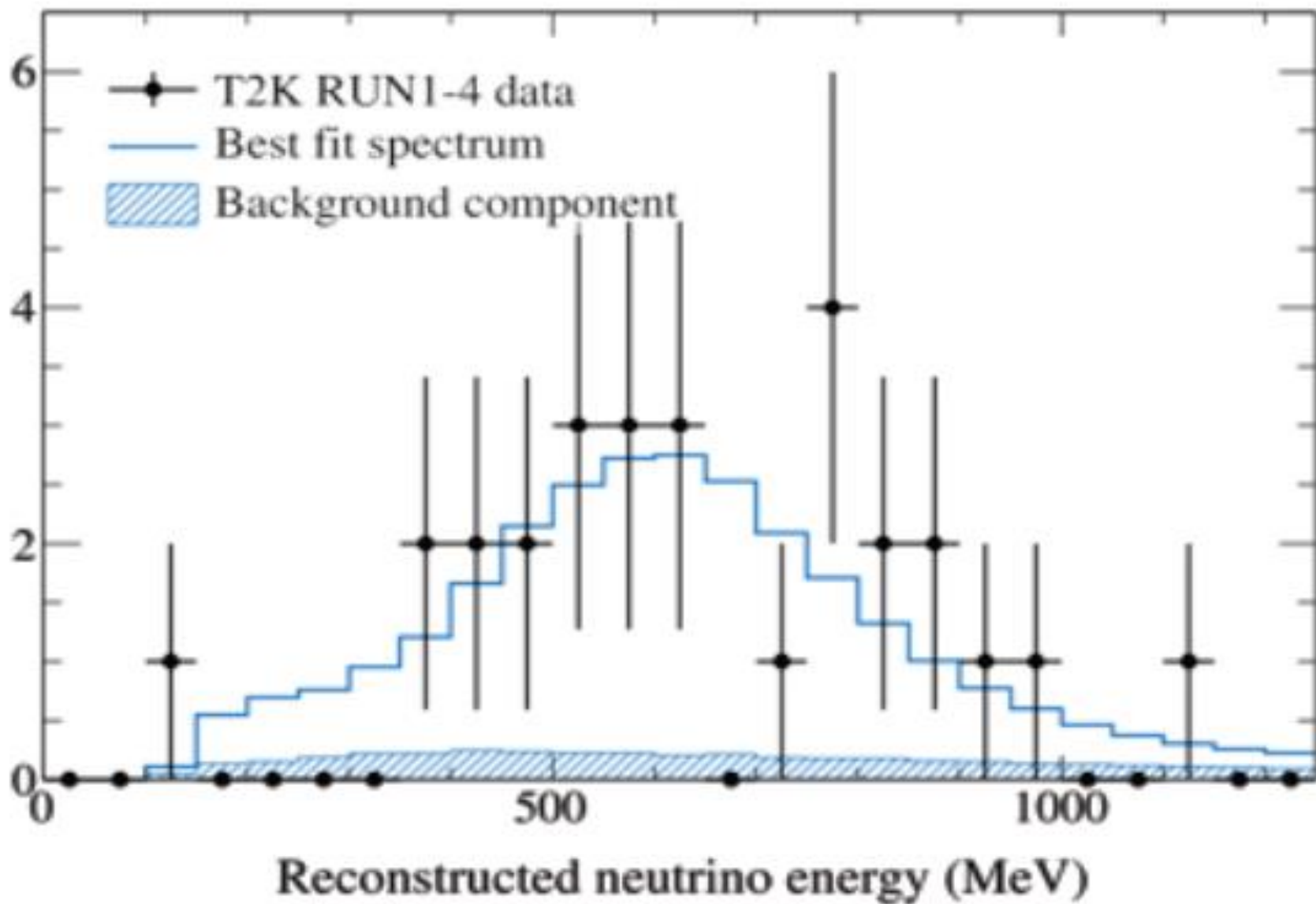
True neutrino energy



Reconstructed energy



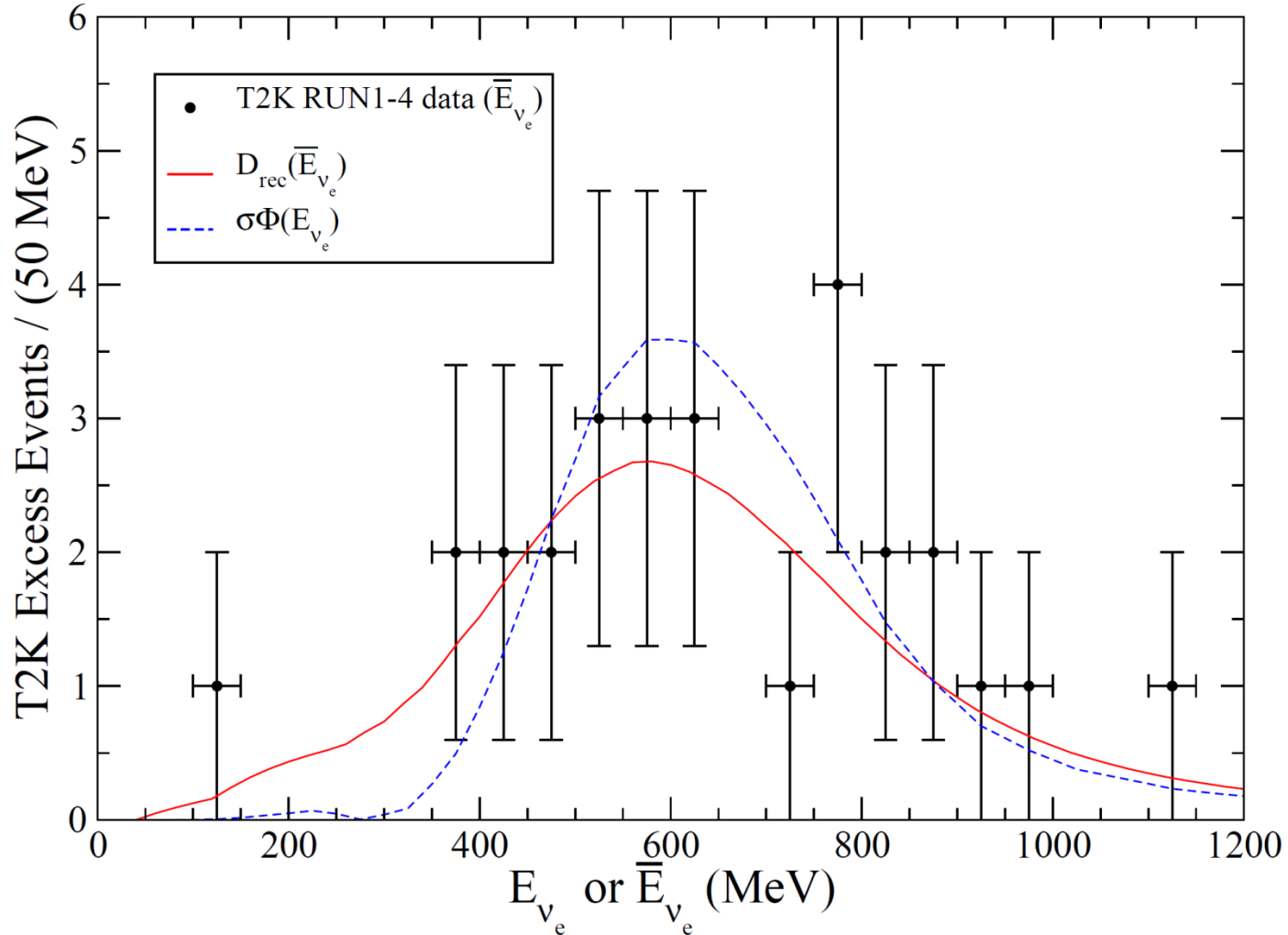
Number of ν_e candidate events / (50 MeV)



$\nu_{\mu} \rightarrow \nu_e$ T2K

PRL 107 (2011), PRD 88 (2013)

2013: 28 events

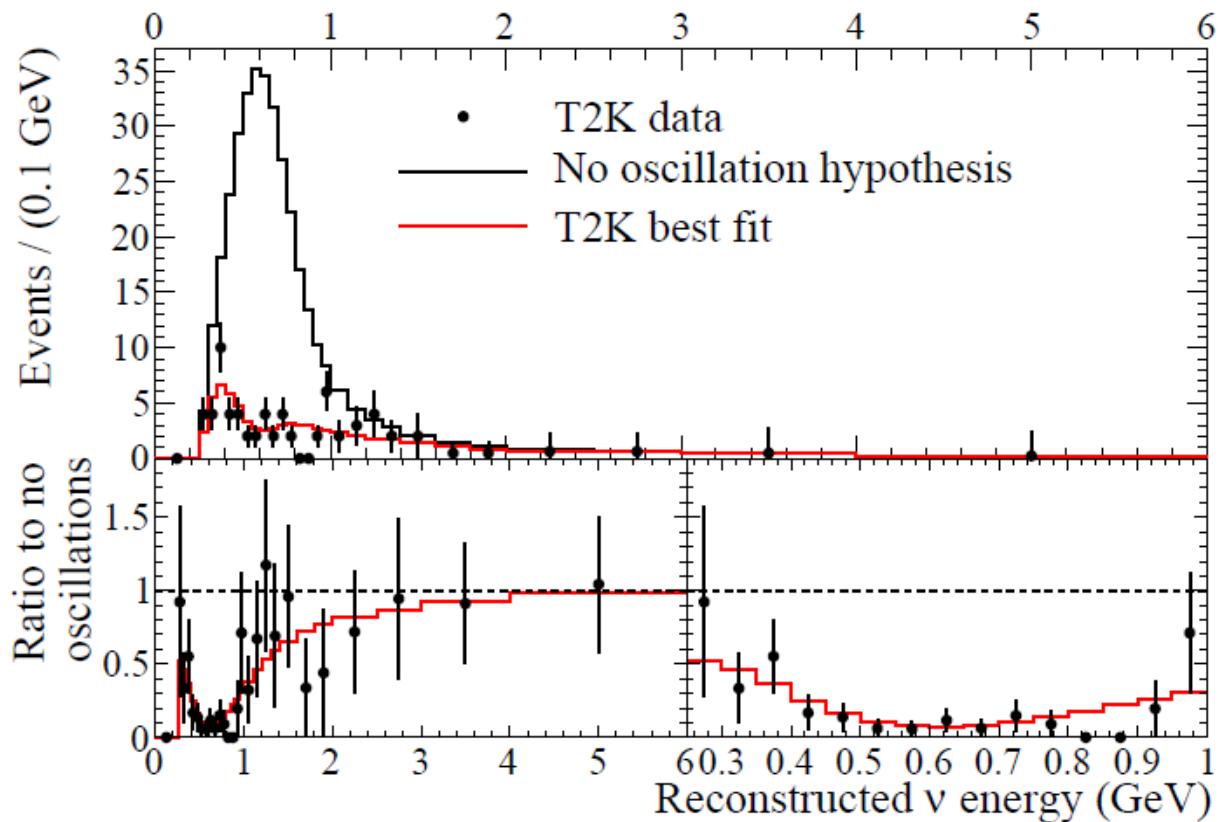
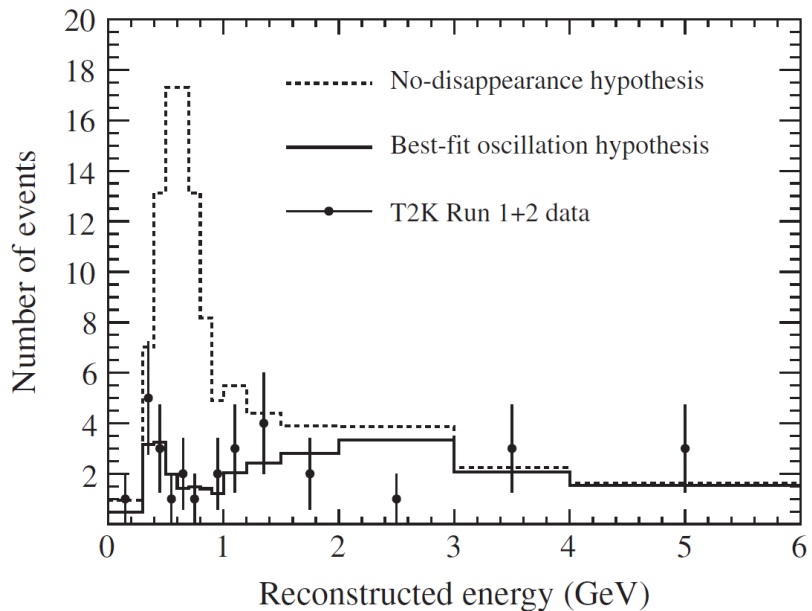


The reconstruction correction tends to make events leak outside the high flux region especially towards the low energy side, in agreement with the observed trend

T2K ν_μ disappearance

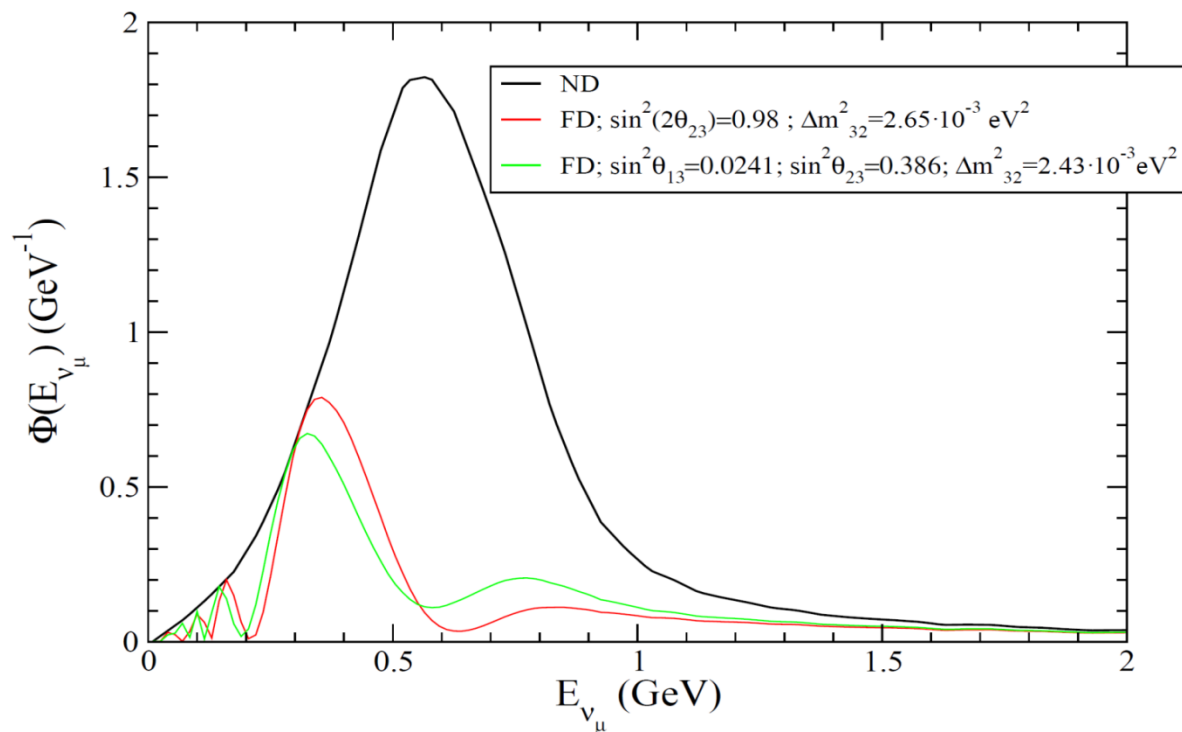
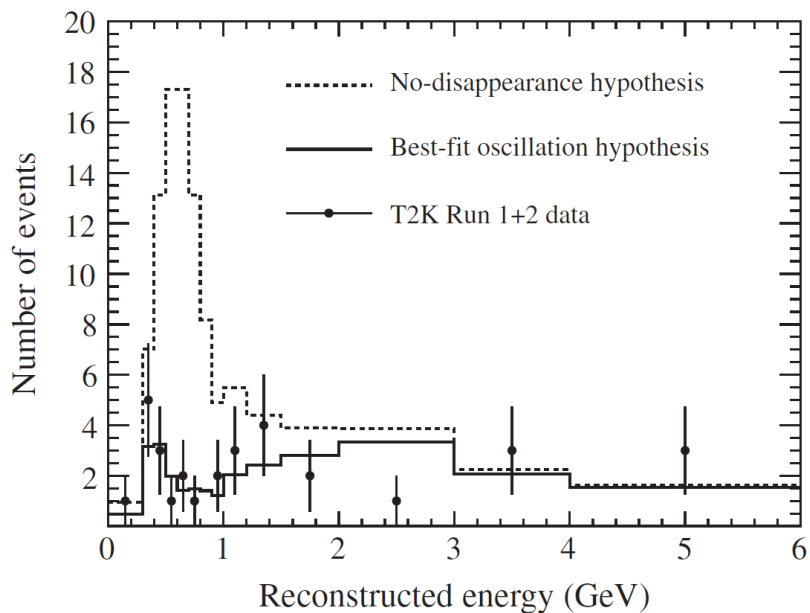
T2K arXiv 1308.0465 (2013)

T2K PRD 85, 031103 (2012)



T2K ν_μ disappearance

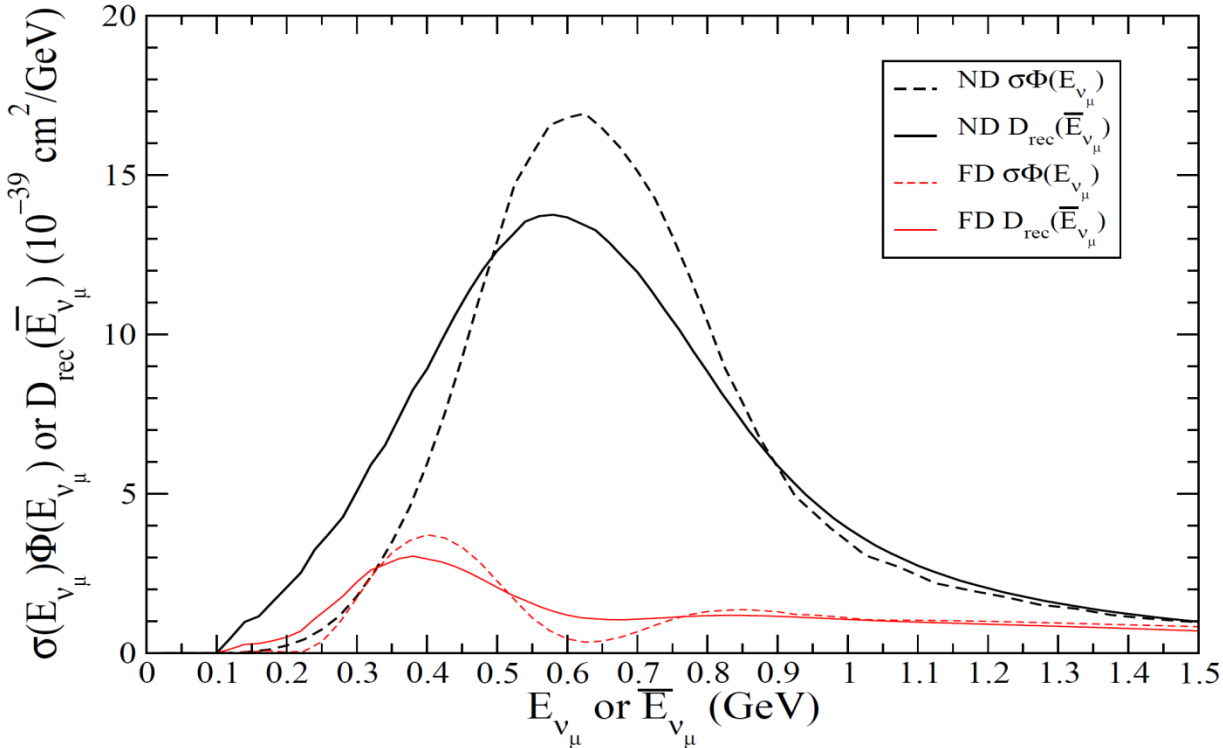
T2K PRD 85, 031103 (2012)



$$\Phi_{\nu_\mu}^{FD}(E_{\nu_\mu}) = \left[1 - 4 \cos^2 \theta_{13} \sin^2 \theta_{23} (1 - \cos^2 \theta_{13} \sin^2 \theta_{23}) \sin^2 \left(\frac{\Delta m_{32}^2 L}{4E_{\nu_\mu}} \right) \right] \Phi_{\nu_\mu}^{ND}(E_{\nu_\mu})$$

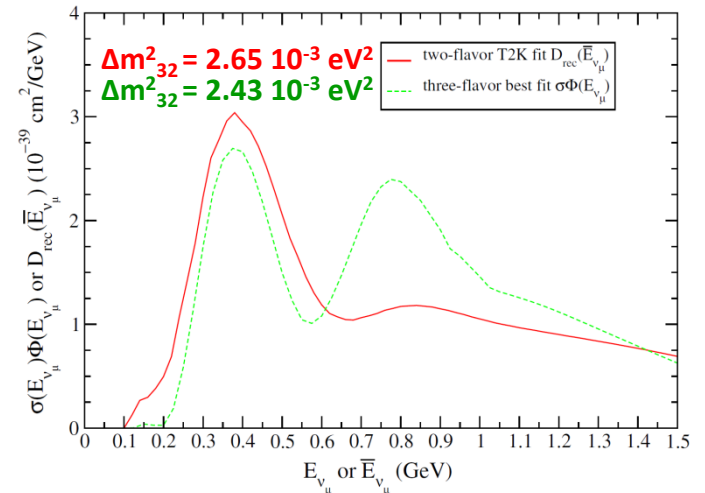
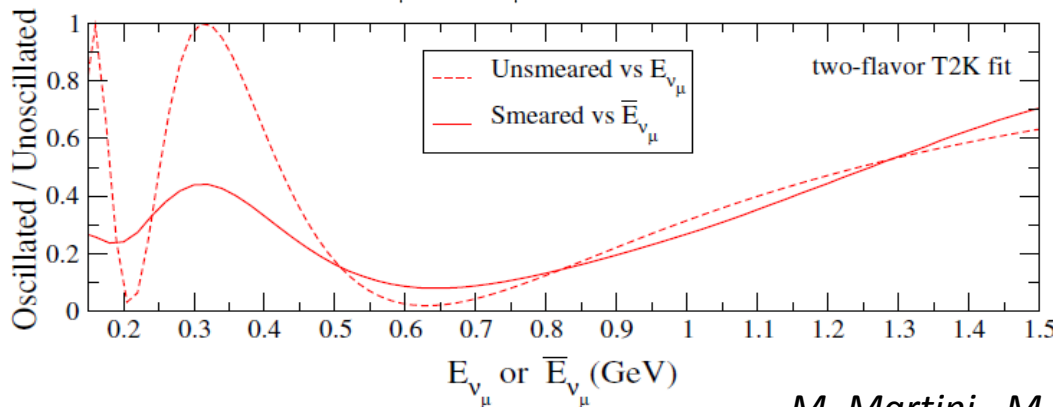
ν_μ disappearance T2K

PRD85 (2012); 1308.0465 (2013)



After reconstruction:

- Near Detector:
clear low energy enhancement
 - Far Detector:
low energy tail and
the middle hole is largely filled
- Effects largely due to np-nh

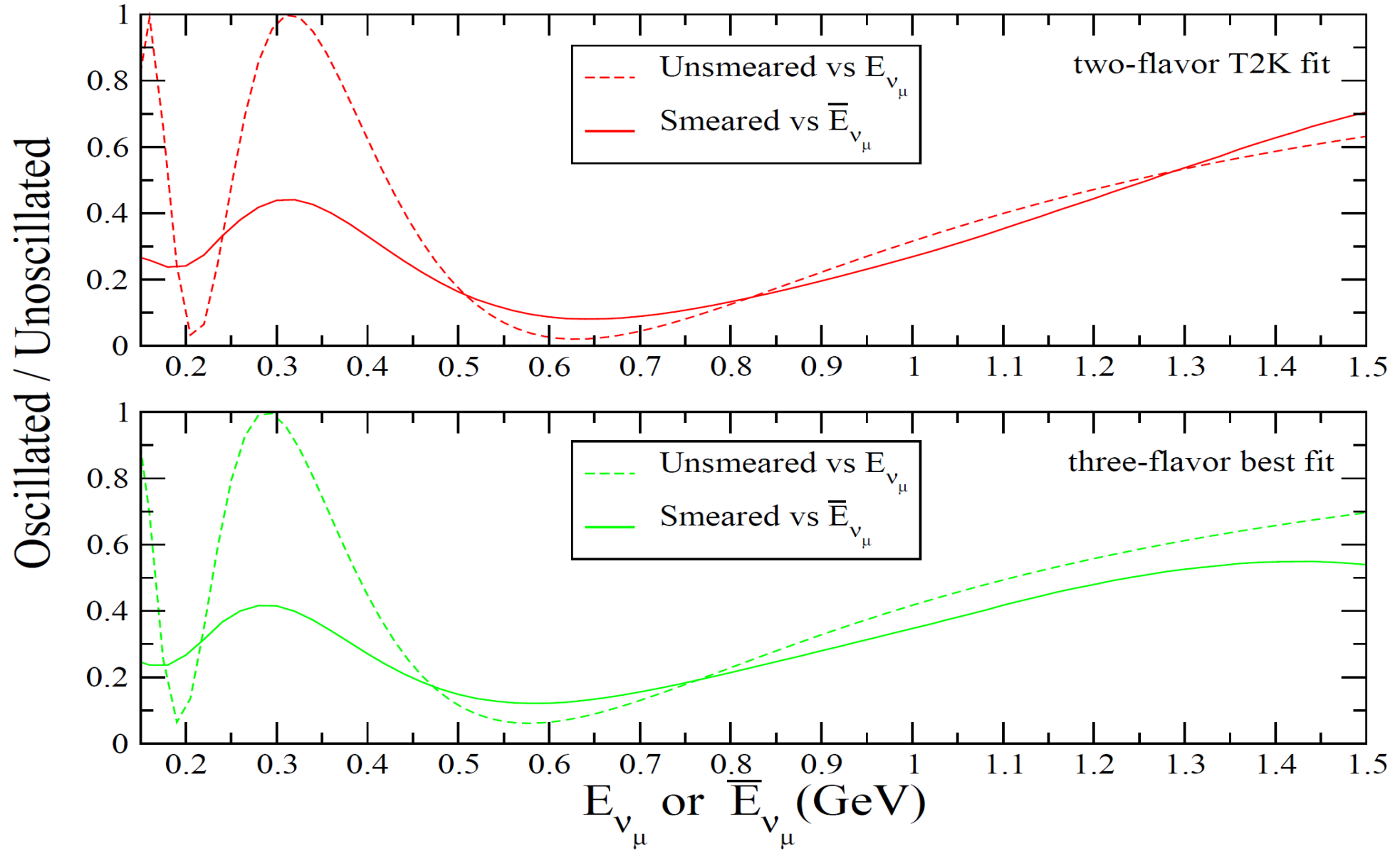


E_ν reconstruction leads to an increase of oscillation mass parameters

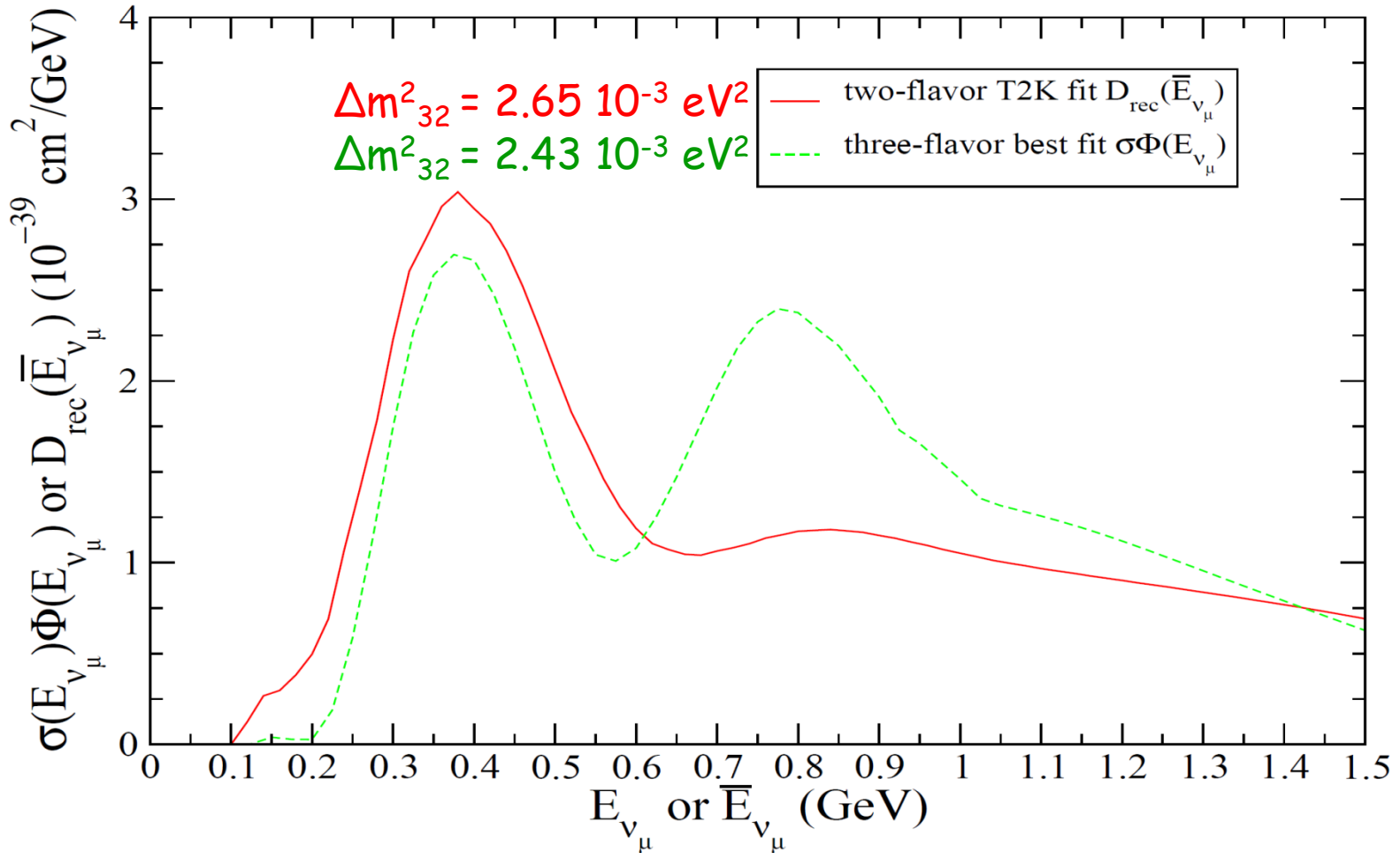
M. Martini, M. Ericson, G. Chanfray, PRD 87 013009 (2013)

Similar results in: O. Lalakulich, U. Mosel, K. Gallmeister, PRC 86 054606 (2012)

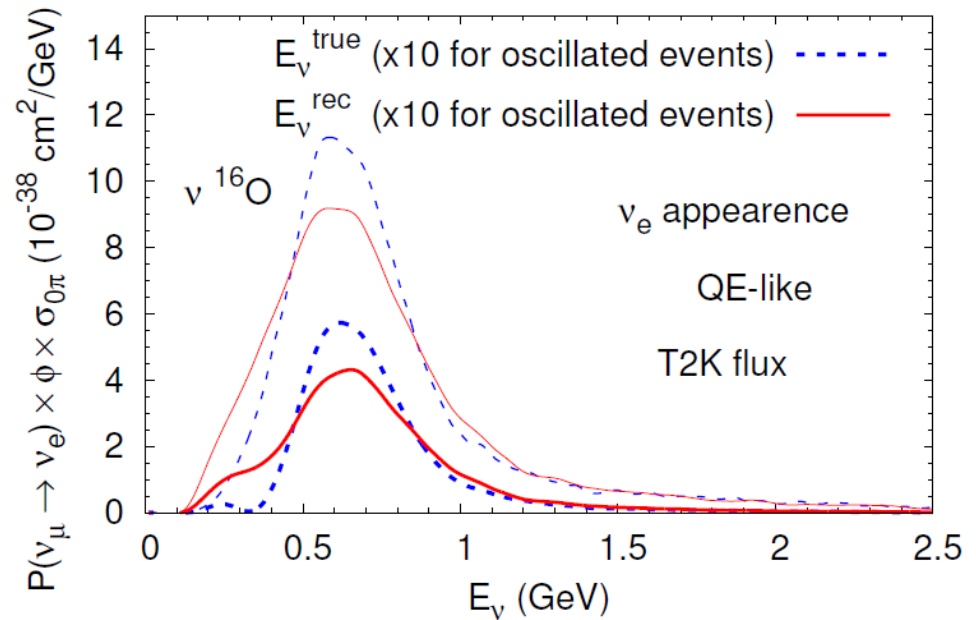
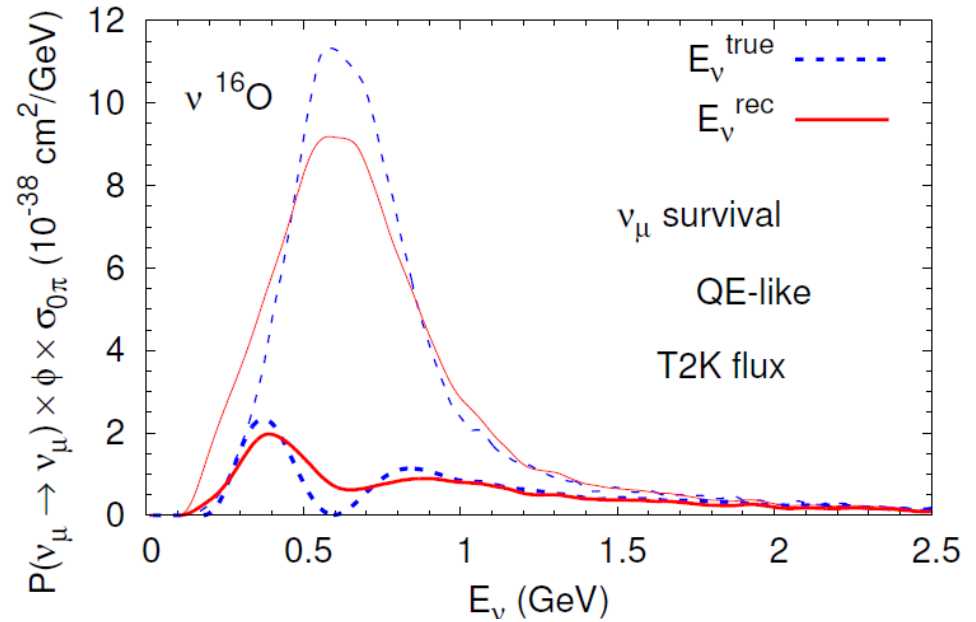
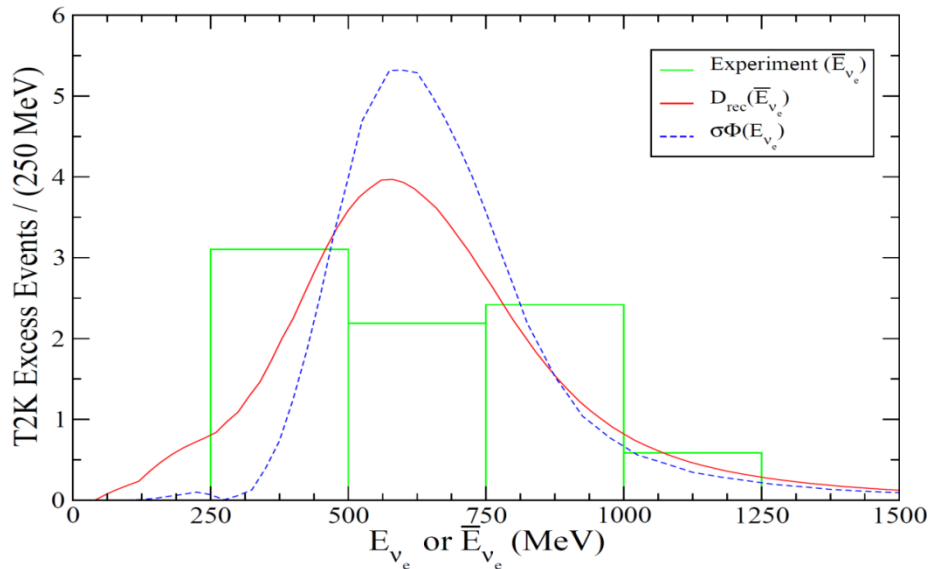
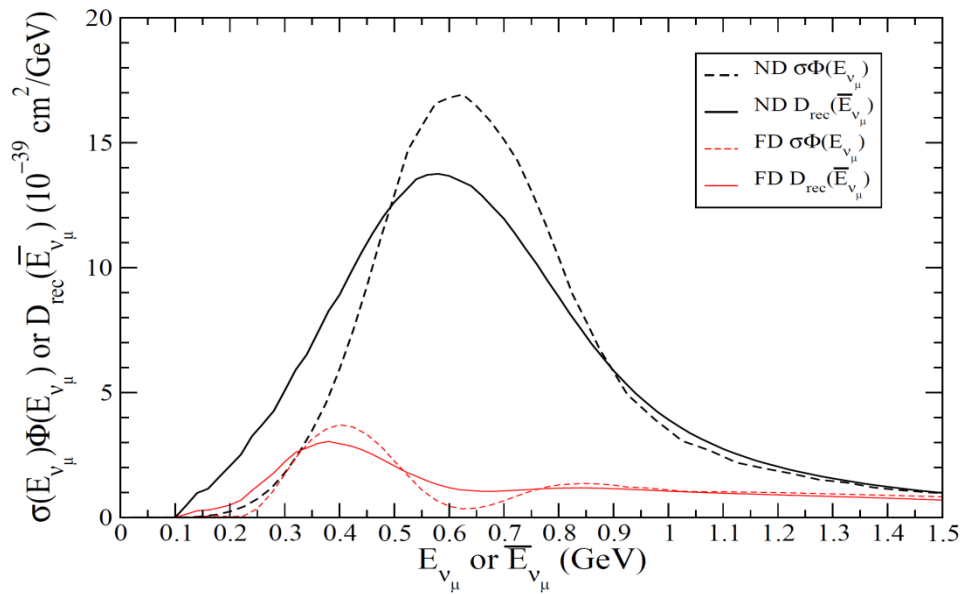
Ratio of the distributions FD/ND



T2K far detector

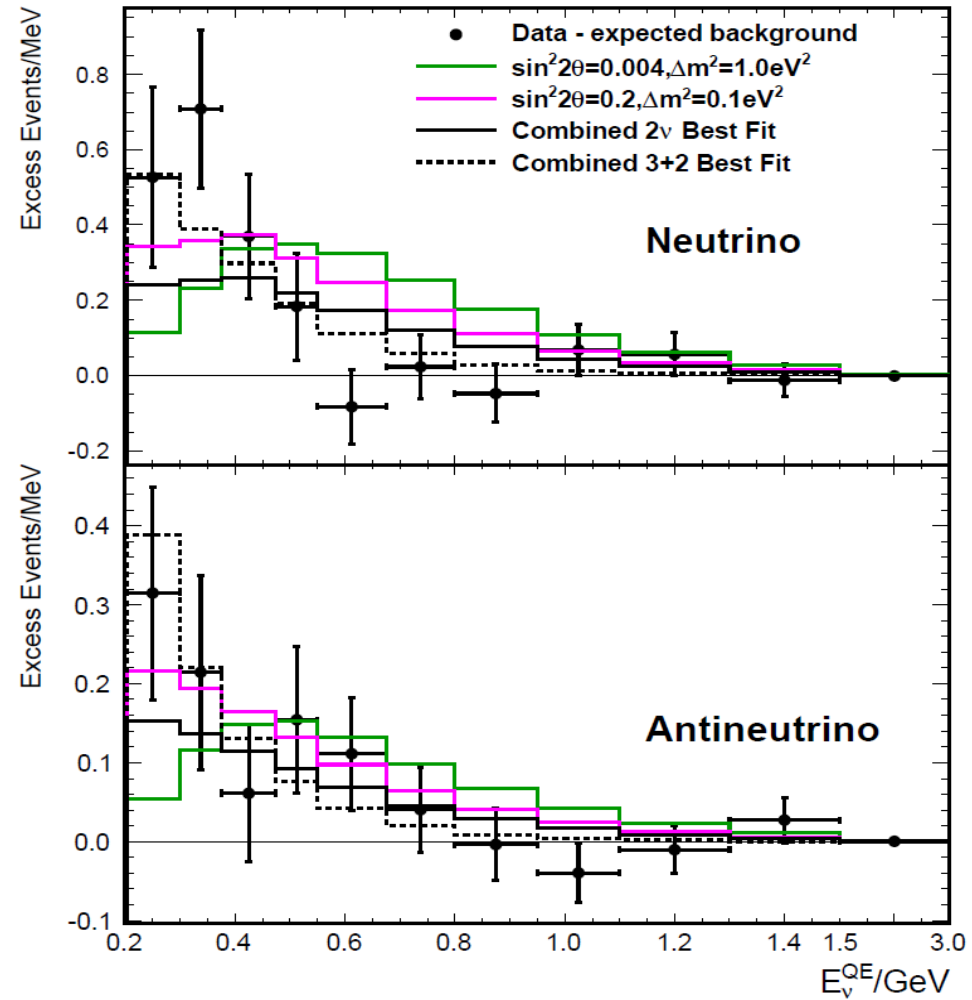
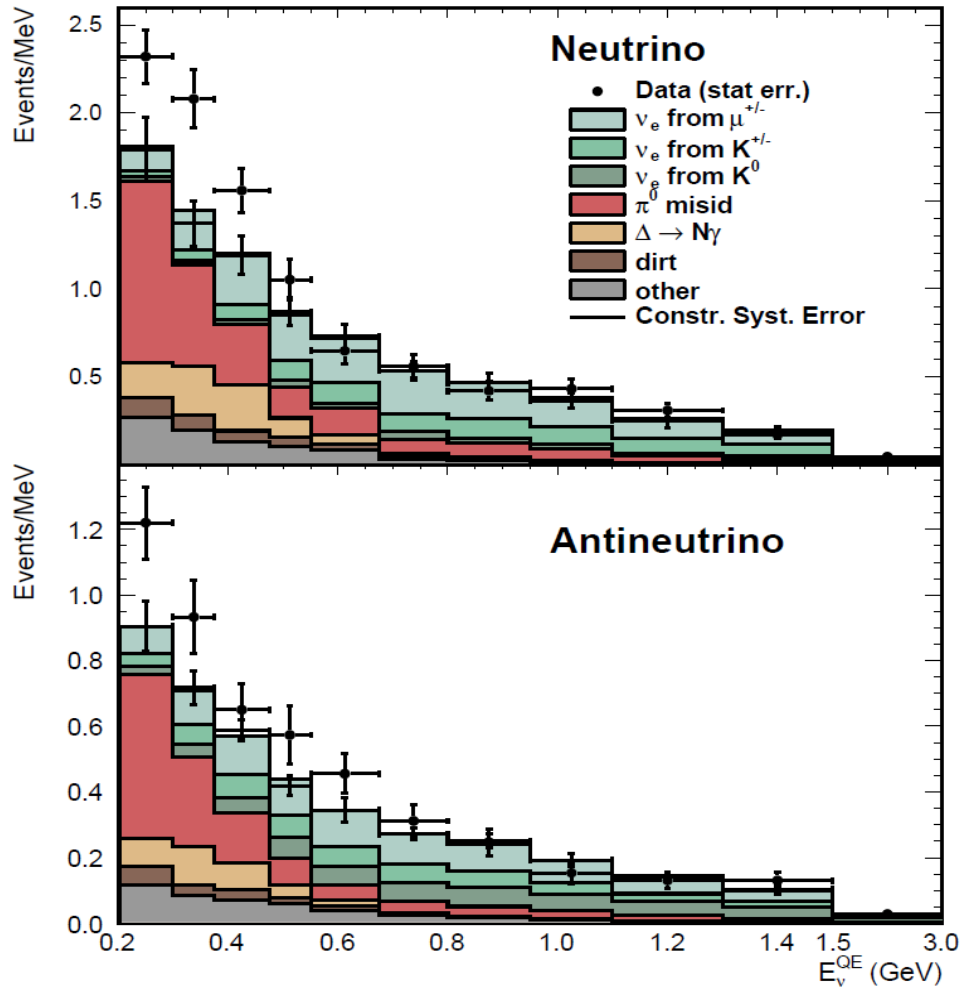


In the first peak region: the smeared curve can be reproduced in the unsmeared case with a lower value of the oscillation mass parameter



$\nu_\mu \rightarrow \nu_e$ MiniBooNE

PRL 98 (2007), PRL 102 (2009), PRL 105 (2010), PRL 110 (2013)

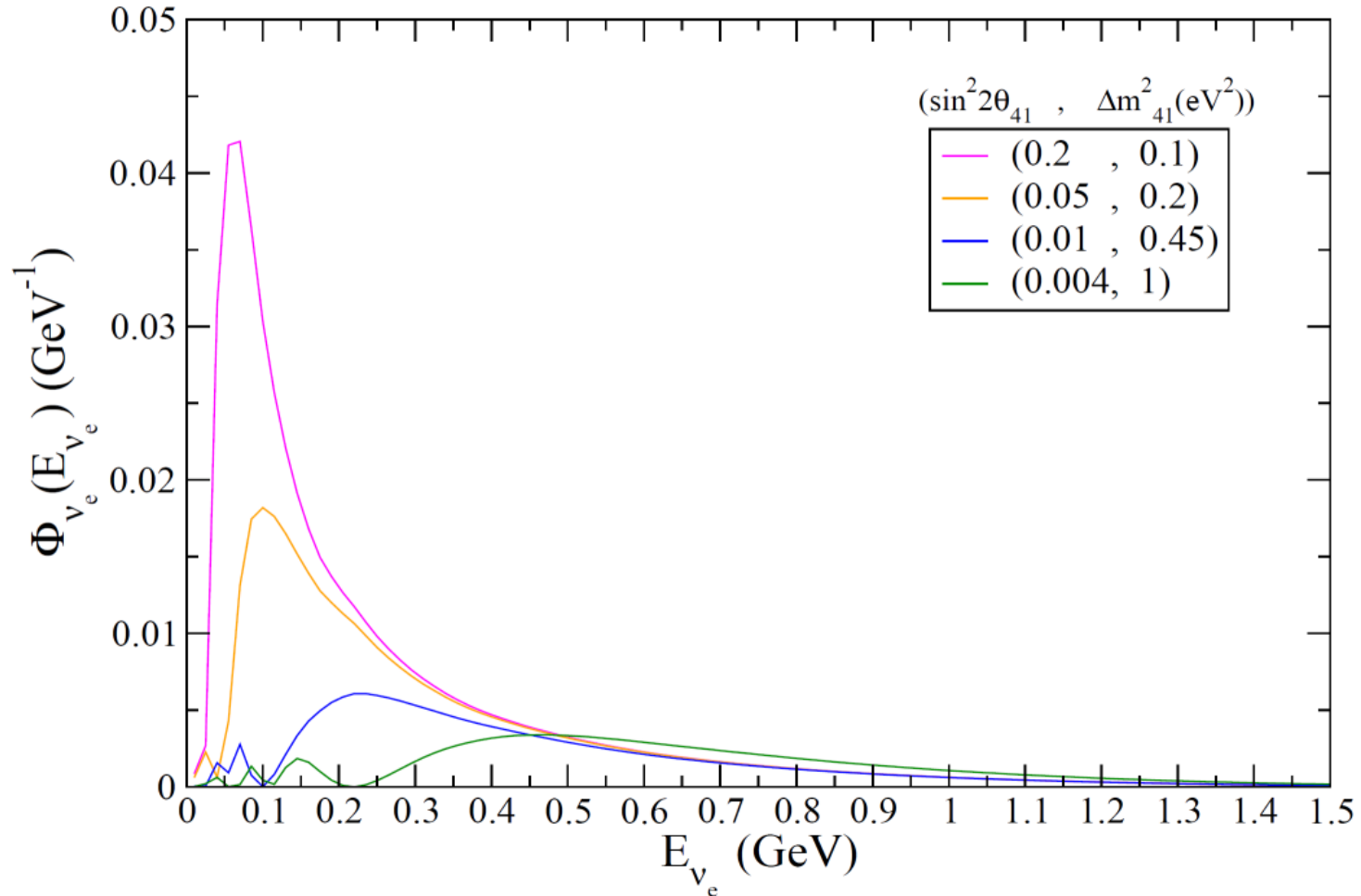


MiniBooNE Anomaly: Excess of events at low energies

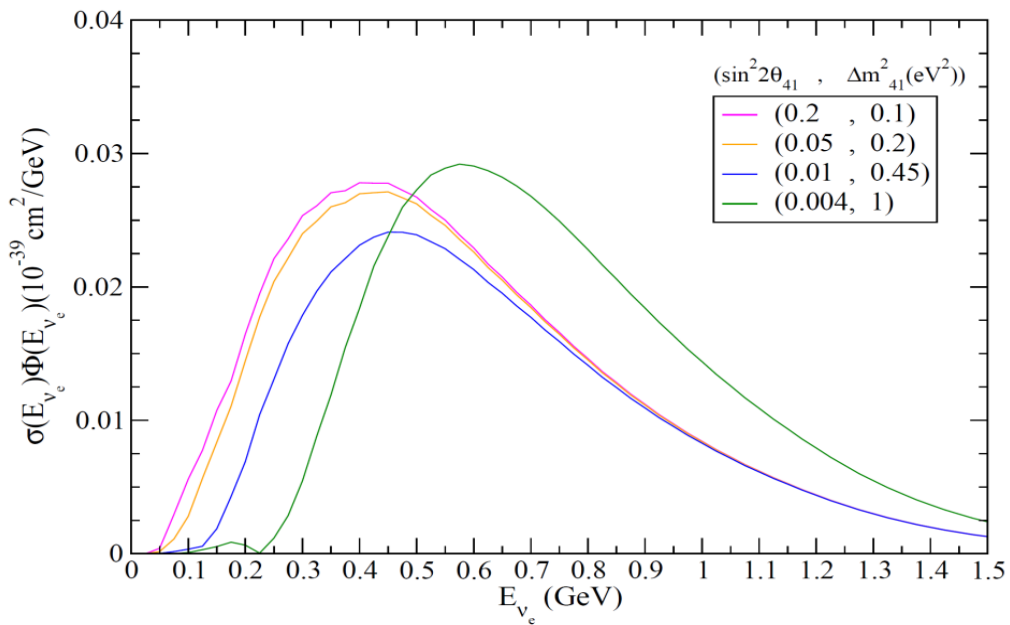
Sterile neutrino??

Oscillations induced by sterile neutrino; 3+1 hypothesis

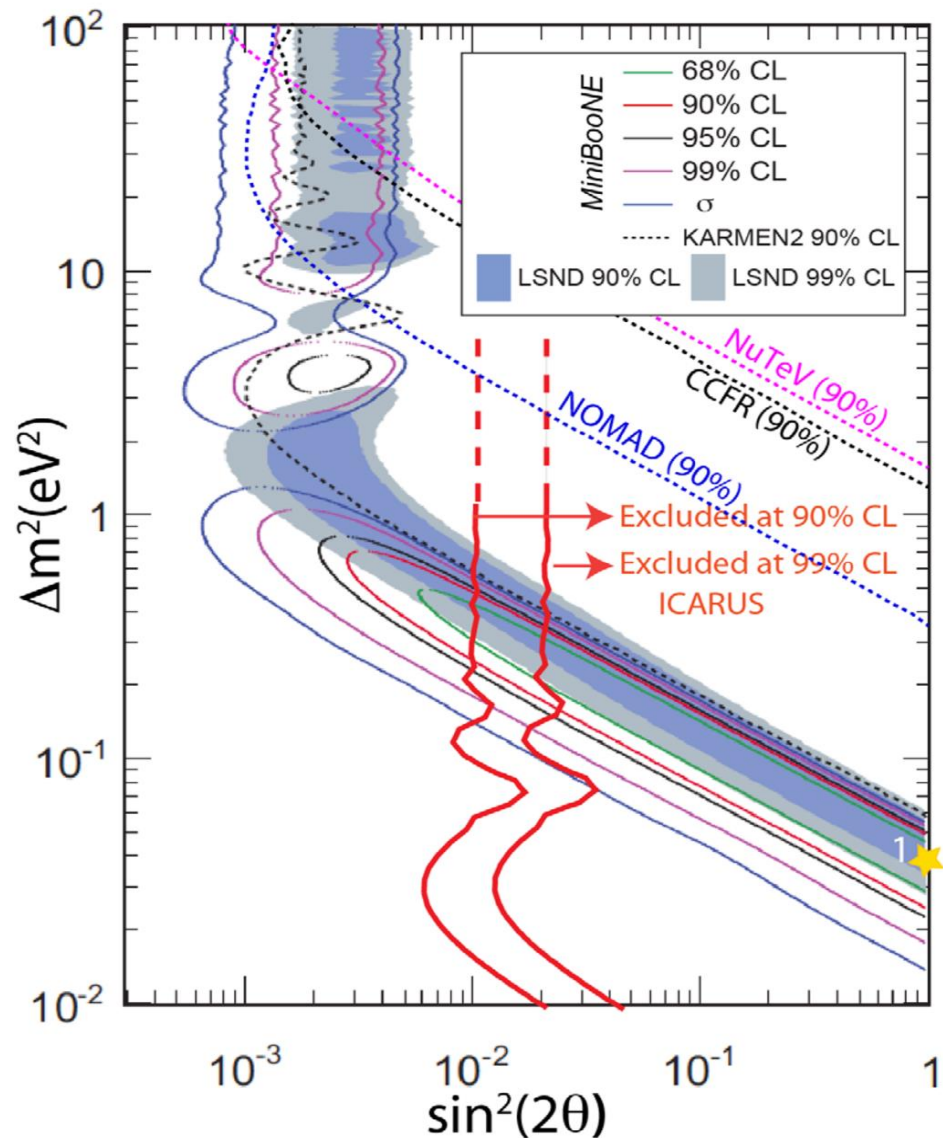
$$\Phi_{\nu_e}(E_{\nu_e}) = \Phi_{\nu_\mu}(E_{\nu_\mu}) \sin^2(2\theta_{41}) \sin^2\left(\frac{\Delta m_{41}^2 L}{4E_\nu}\right)$$



Some considerations on the oscillation parameters

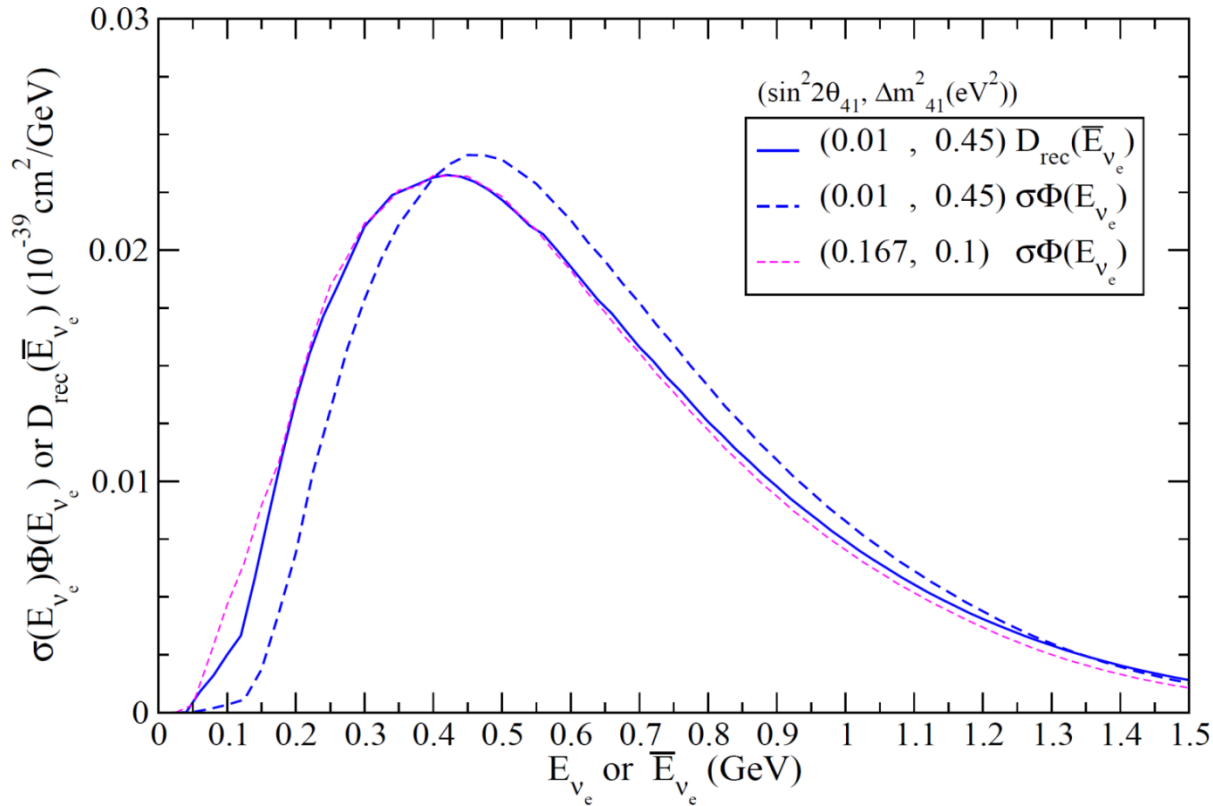


The low energy behavior of the MiniBooNE data favors small values of the mass parameter which concentrate the ν flux at low energies. But small values imply, in order to have enough events, large values of $\sin^2(2\theta)$ which are not compatible with the constraints from other sets of data.



Taking now into account the smearing procedure

M. Martini, M. Ericson, G. Chanfray, PRD 87 013009 (2013)



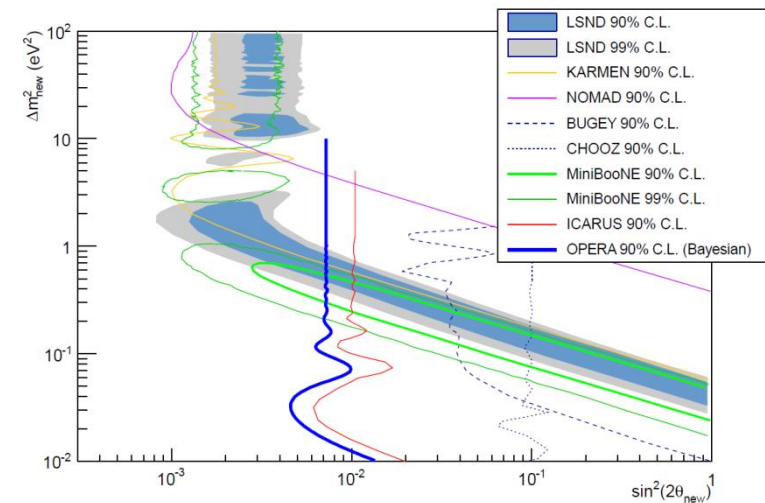
A large mass value allows the same quality of fit of data than is obtained in the unsmeared case with a much smaller mass.

The energy reconstruction leads to an increase of the oscillation mass parameters

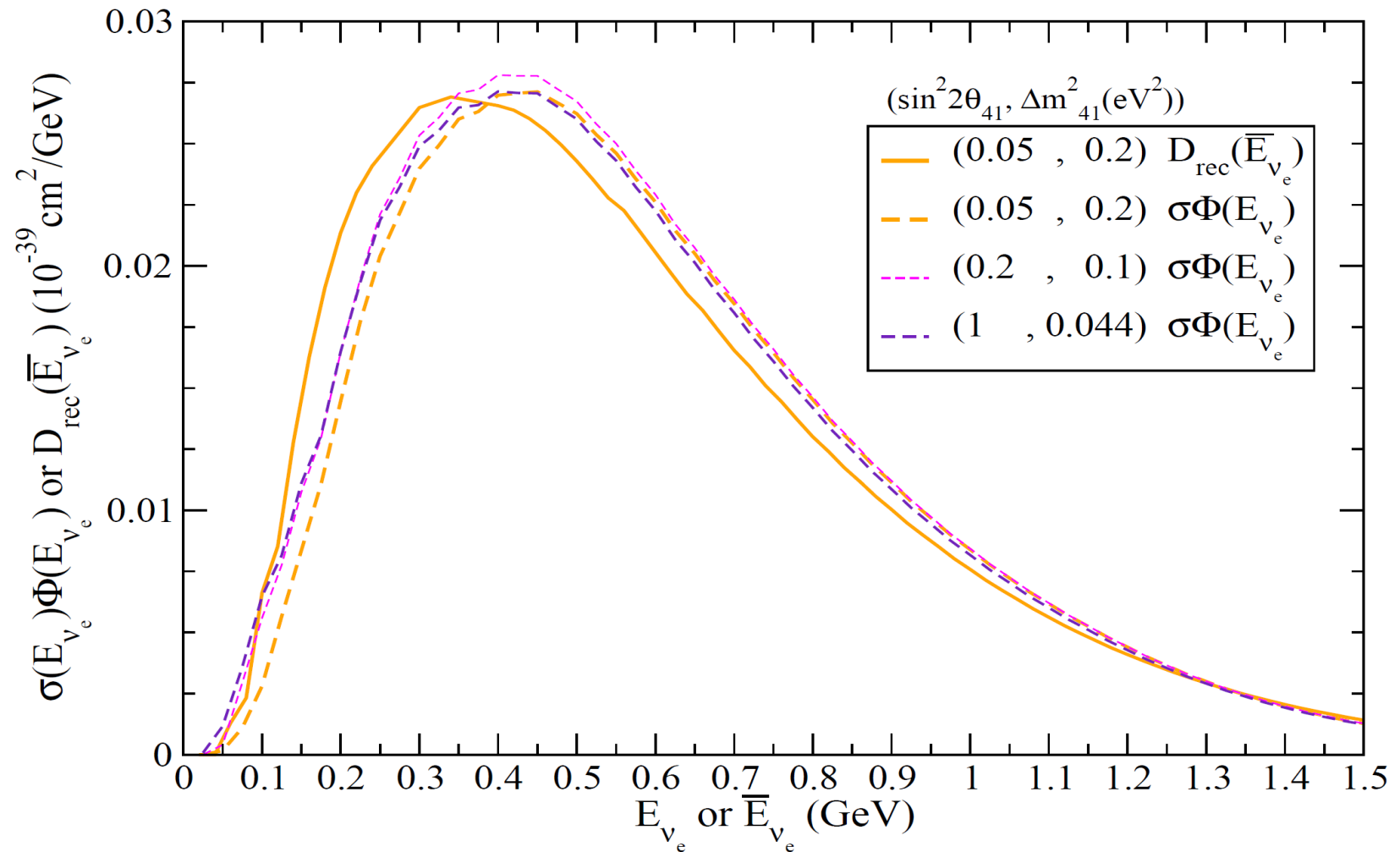


Gain for the compatibility with the existing constraints

OPERA, JHEP 1307 (2013) 004,
Addendum-ibid. 1307 (2013) 085



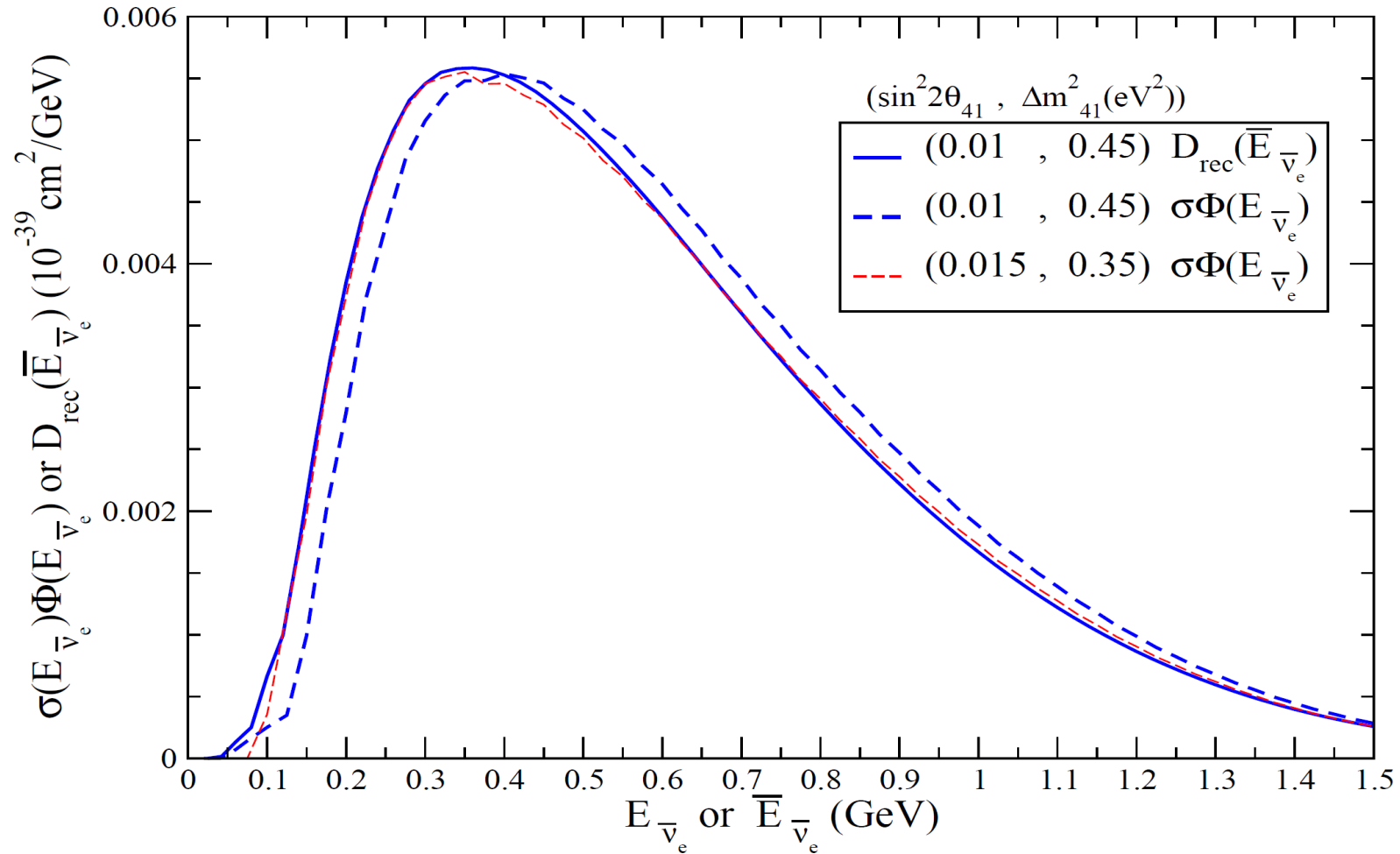
The case of smaller mass values



The smeared curve is shifted at lower energies (displacement of the peak ≈ 80 MeV).

It is impossible to reproduce the smeared curve with an unsmeared one even taking a very small mass.

Antineutrinos



Similar effects, although less pronounced, are present for antineutrinos.

$\nu_{\mu} \rightarrow \nu_e$ MiniBooNE

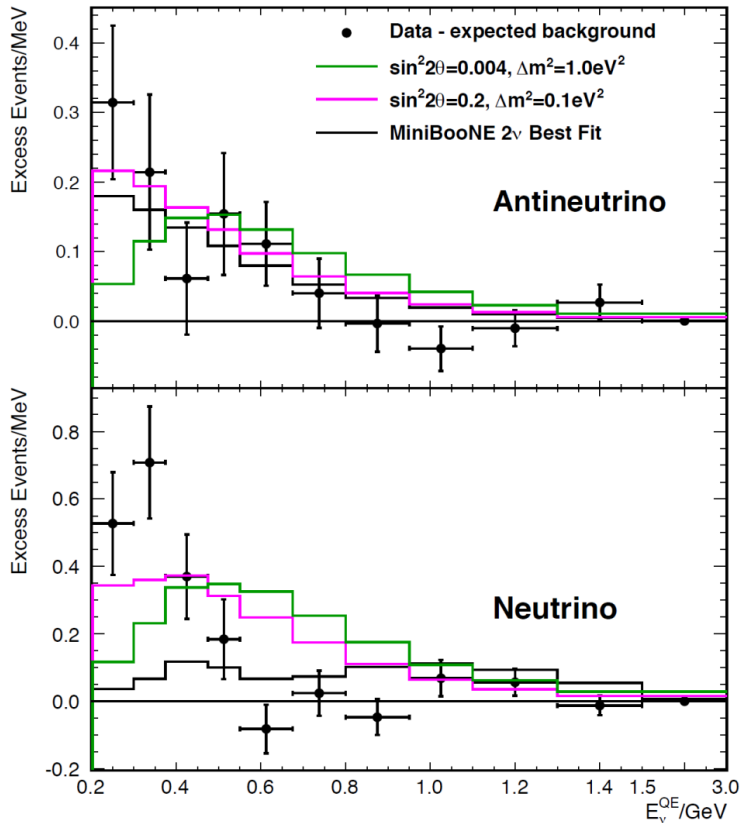
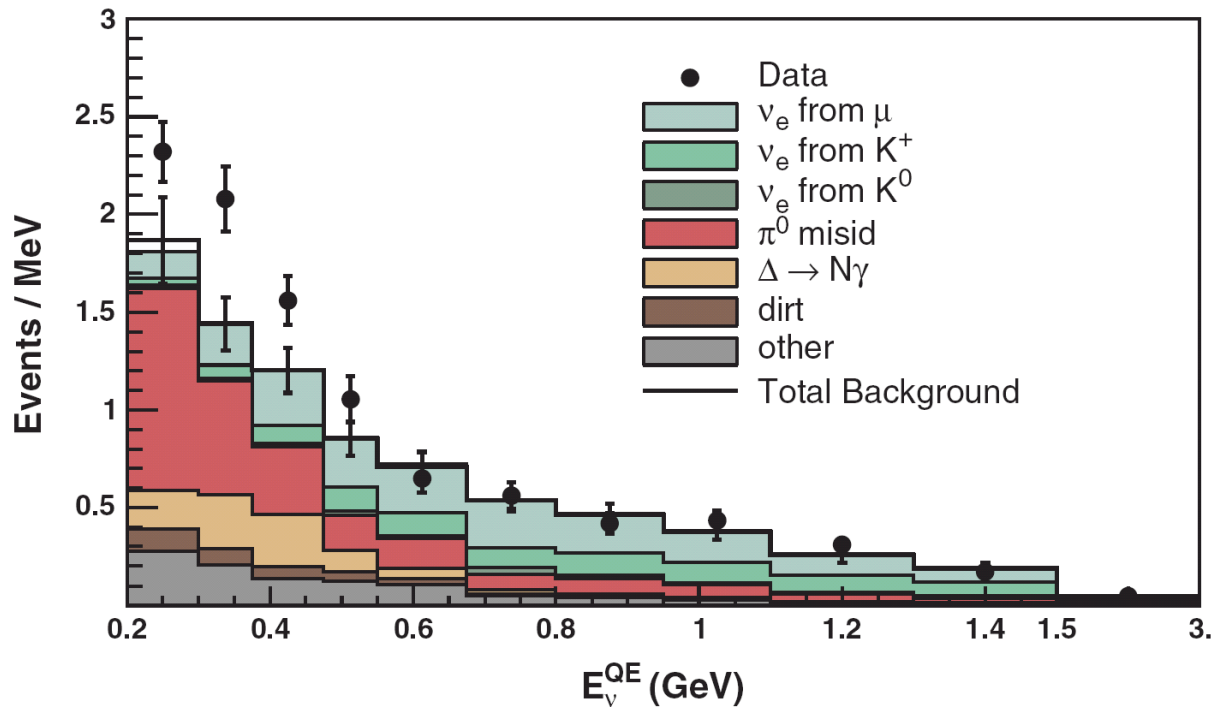


TABLE II: χ^2 values from oscillation fits to the antineutrino-mode data for different prediction models. The best fit $(\Delta m^2, \sin^2 2\theta)$ values are $(0.043 \text{ eV}^2, 0.88)$, $(0.059 \text{ eV}^2, 0.64)$, and $(0.177 \text{ eV}^2, 0.070)$ for the nominal, Martini, and disappearance models, respectively. The test point χ^2 values in the third column are for $\Delta m^2 = 0.5 \text{ eV}^2$ and $\sin^2 2\theta = 0.01$. The effective dof values are approximately 6.9 for best fits and 8.9 for the test points.

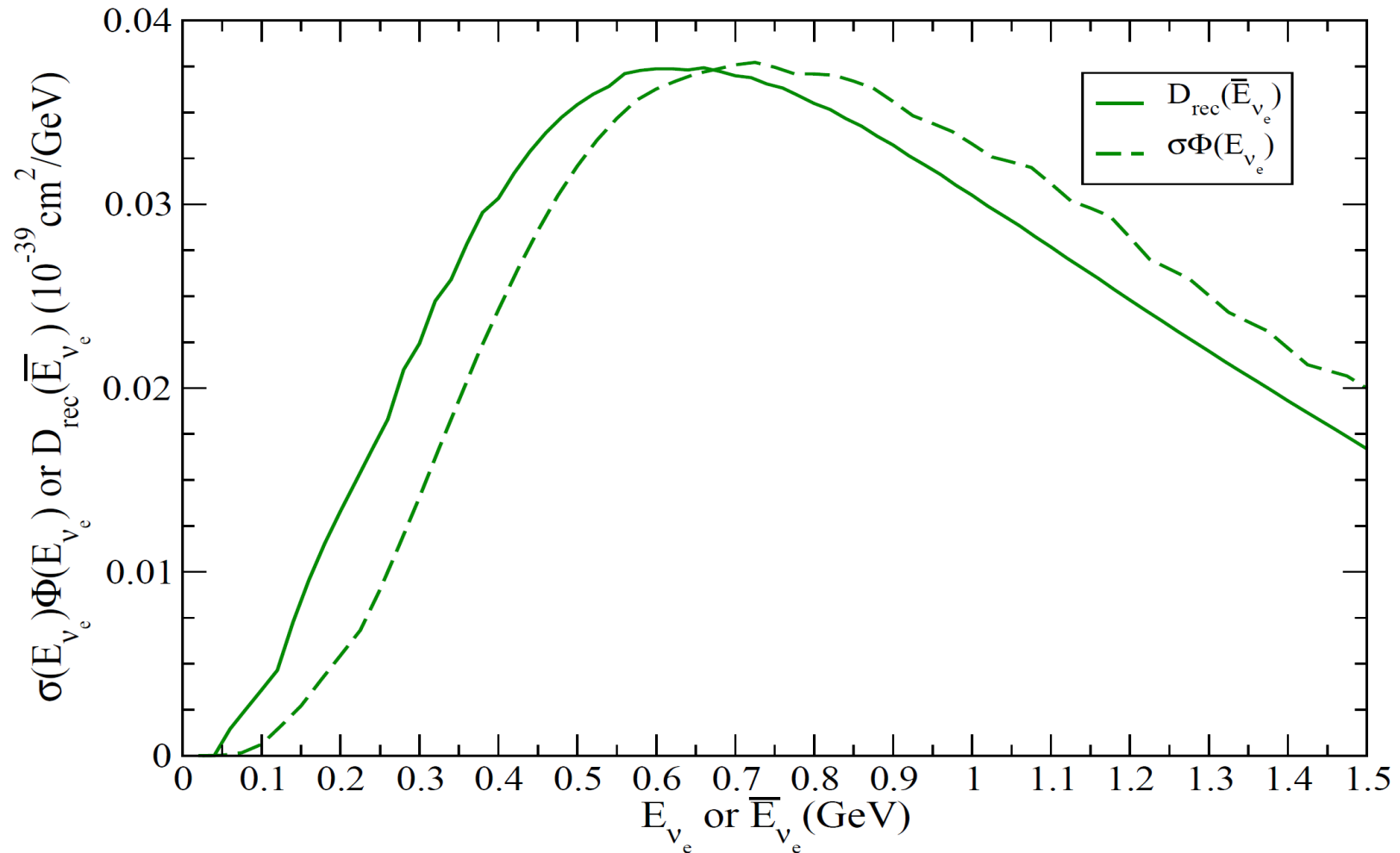
Prediction Model	χ^2 values	
	Best Fit	Test Pt.
Nominal $\bar{\nu}$ -mode Result	5.0	6.2
Martini <i>et al.</i> [25] Model	5.5	6.5
Model With Disapp. (see text)	5.4	6.7

arXiv: 1303.2588; PRL 110 (2013)

ν_e background and effective cross sections

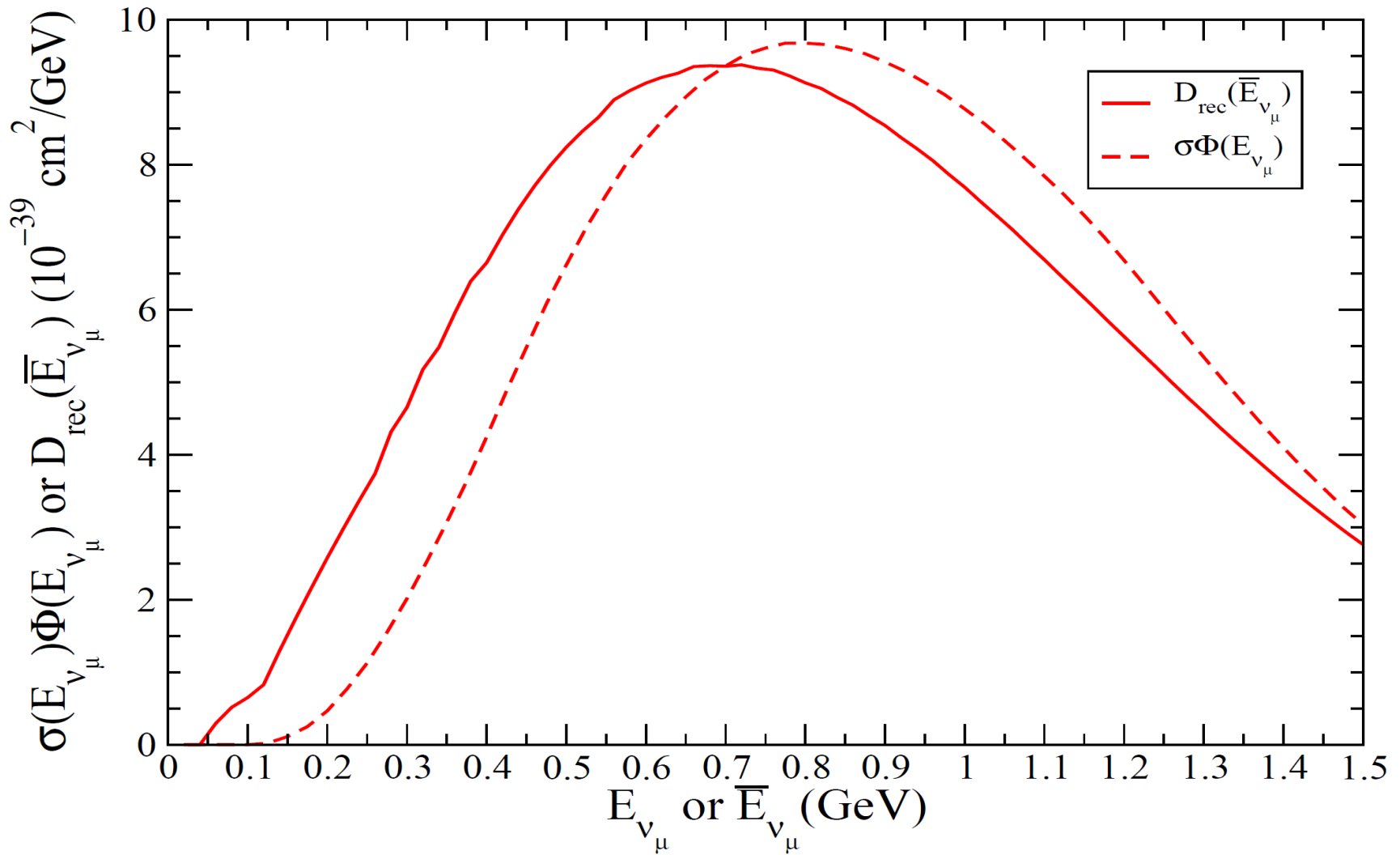


MiniBooNE electron events distribution for $\bar{\nu}_e$ background



The electron event background is underestimated for low reconstructed neutrino energies $E < 0.6$ GeV and overestimated for larger ones

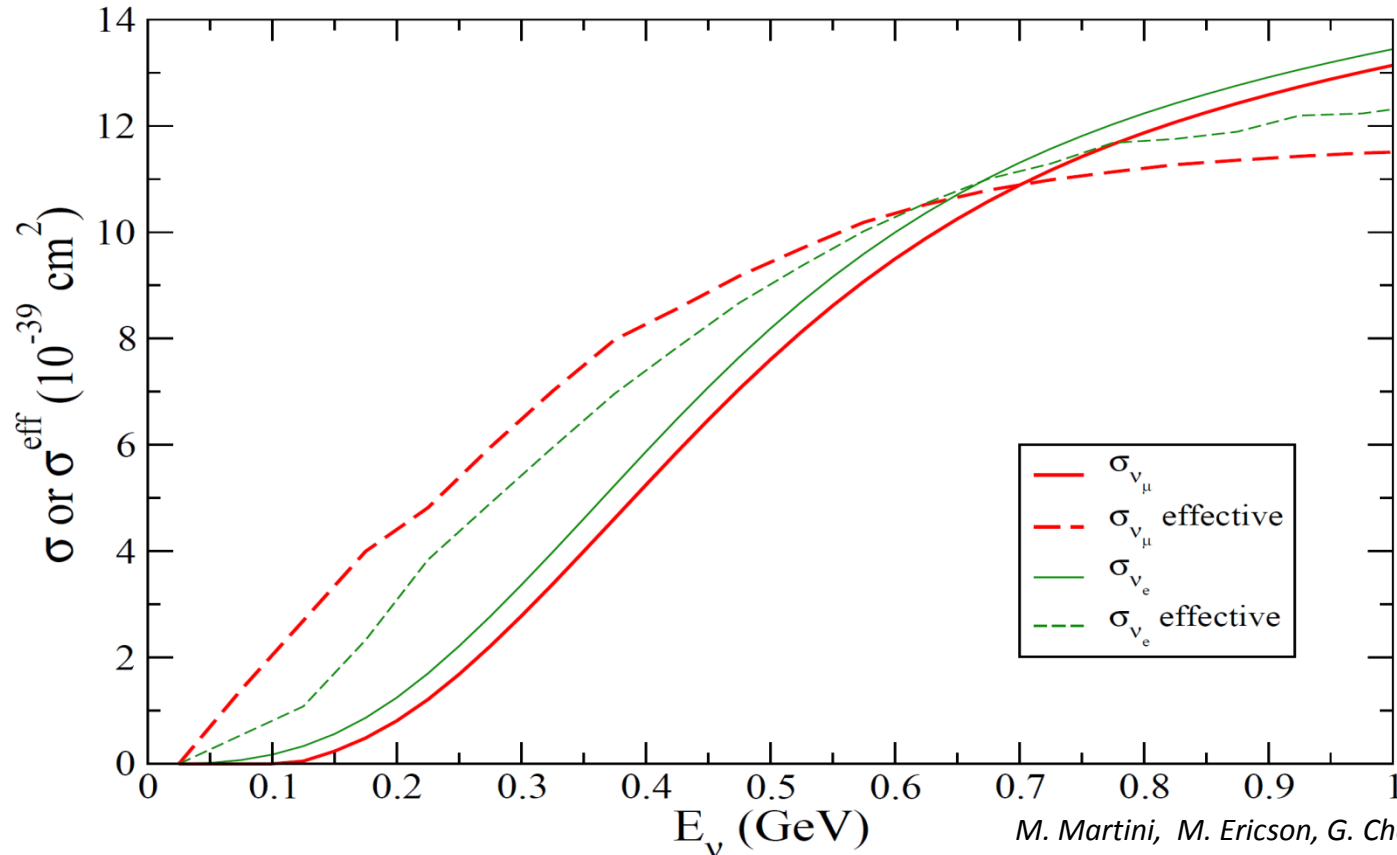
MiniBooNE muon events distribution



Real and effective cross sections for ν_μ and ν_e

Let's define the effective cross section through $D_{\text{rec}}(\bar{E}_\nu) = \sigma_\nu^{\text{eff}}(\bar{E}_\nu)\Phi(\bar{E}_\nu)$

Let's then ignore the difference between the true and reconstructed neutrino energies



The effective cross section is not universal but
it depends on the particular beam energy distribution
(here we used ν_μ and ν_e MiniBooNE fluxes)