

# NC photon production: nuclear models and pion photoproduction

**Xilin Zhang** (Indiana U.& Ohio U.),  
Brian Serot (Indiana U.)

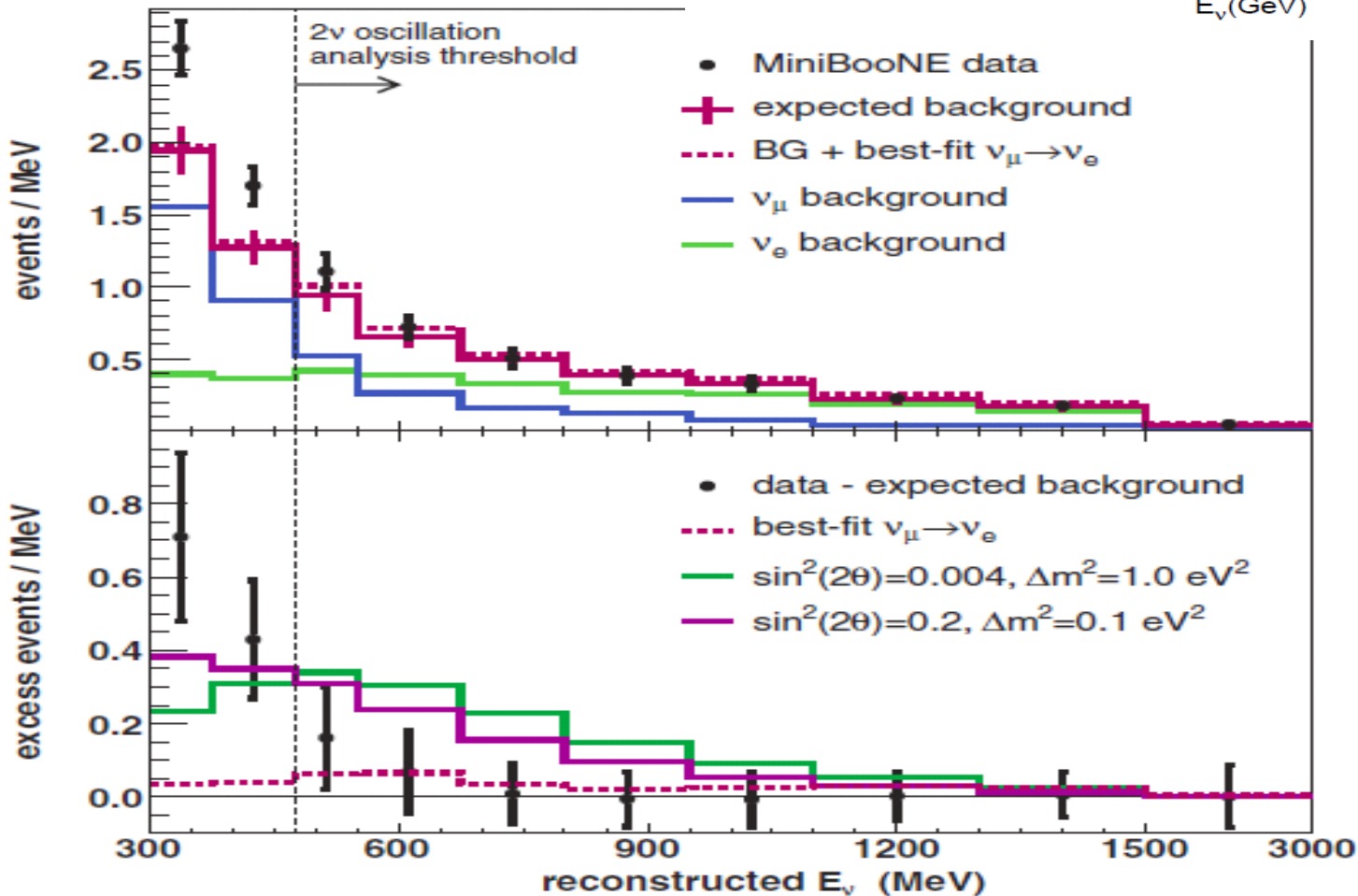
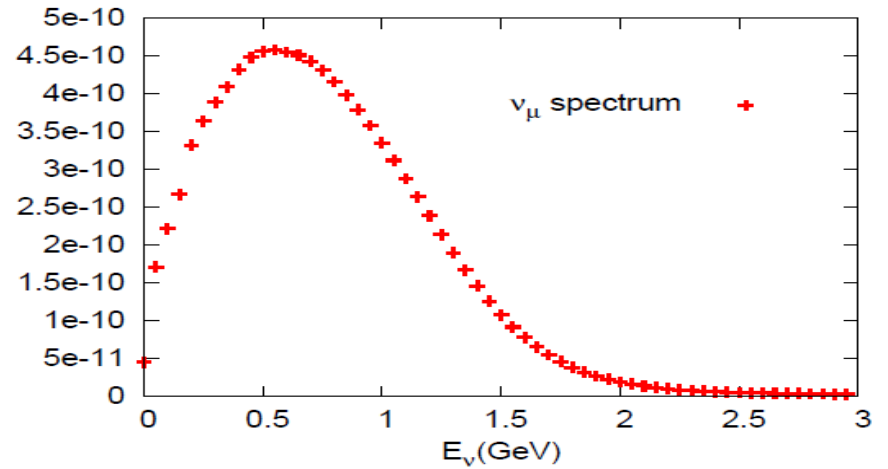
*Neutrino-Nucleus Interactions workshop, INT, Seattle, Dec. 2013*

# Outline

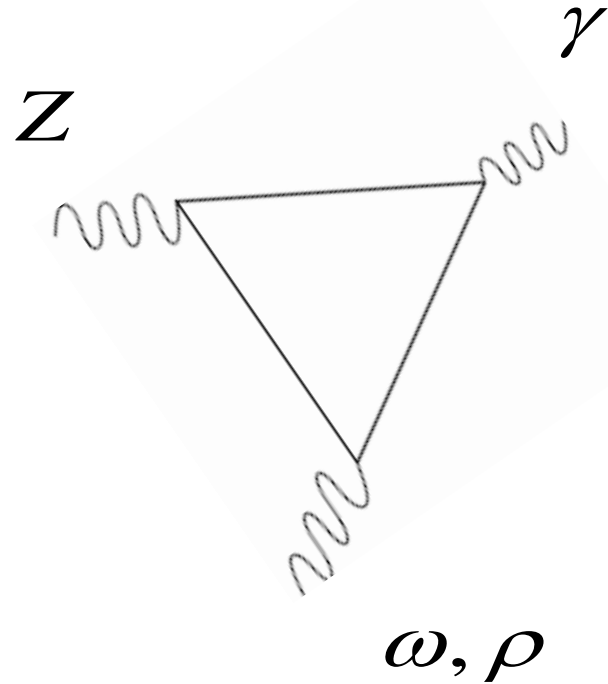
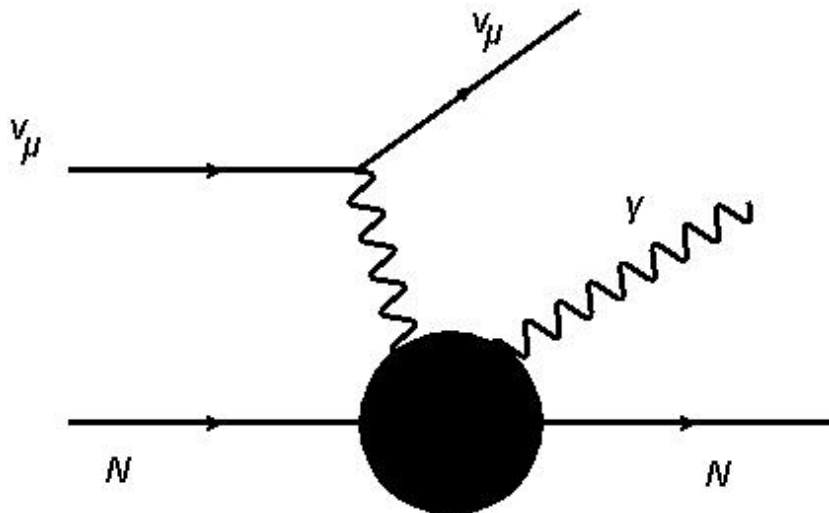
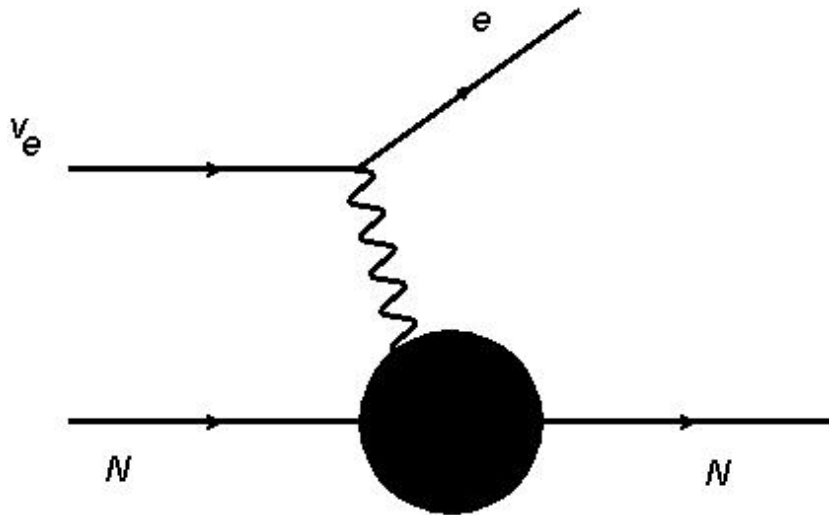
- Motivation: NC photon's role in MiniBooNE low energy excess
- Low energy region: Quantum Hadrodynamics (QHD, or Walecka model)
  - Pion and photon production from nucleon
  - Incoherent productions: reaction kernel modification in the medium
  - Coherent productions
- Extrapolation to GeV region, form factors
- MiniBooNE NC photon production
- Summary & questions

# MiniBooNE

$\Phi(E_\nu) \nu / \text{POT} / \text{GeV} / \text{cm}^2$



# New hadronic interactions?



*J.A. Harvey, C.T. Hill, R.J. Hill, Phys. Rev. Lett. **99**, 261601 (2007), Phys. Rev. D **77**, 085017(2008).  
R.J. Hill, Phys. Rev. D **81**, 013008 (2010), **84** 017501(2011).*

# Production of single photons in the exclusive neutrino process $\nu N \rightarrow \nu \gamma N$

S. S. Gershtein, Yu. Ya. Komachenko, and M. Yu. Khlopov

*Institute of High Energy Physics, Serpukhov*

(Submitted 16 January 1981)

*Yad. Fiz. 33, 1597–1604 (June 1981)*

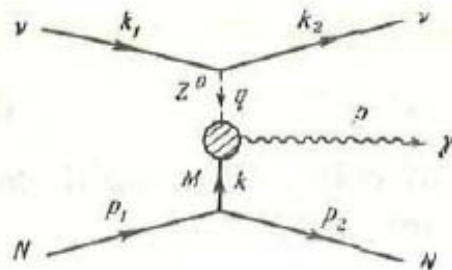
It is shown that the experimentally observed production of single photons in neutrino interactions involving neutral currents without visible accompaniment of other particles can be explained by the scattering of the neutrino by a virtual  $\omega$  meson with small momentum transfer to a nucleon and subsequent coherent enhancement of the process in the nucleus.

PACS numbers: 13.15. + g, 14.80.Kx

## 1. INTRODUCTION

In neutrino experiments performed at CERN using the chamber Gargamelle, more than ten events were detected in which it was observed that single photons with energy 1–10 GeV were produced without visible tracks of any other particles.<sup>1</sup> It can be assumed that the observed events correspond to the weak-electromagnetic process of single-photon production in the reaction

$$\nu N \rightarrow \nu \gamma N, \quad (1)$$



$$H_{\nu\nu} = \sum_M T^{(M)} P^{(M)} J_{\nu\nu}^{(M)}, \quad (3)$$

in which  $T^{(M)}$  is the vertex for emission of a virtual meson ( $M$ ) by the target nucleon,  $P^{(M)}$  is the meson propagator, and  $J_{\mu\nu}^{(M)} = \int \langle 0 | T(J_{\mu}^{W}(x), J_{\nu}^{EM}(y)) | M \rangle e^{i q x + i p y} d^4 x d^4 y$  is the weak-electromagnetic  $Z^0 M \gamma$  vertex. The notation for the particle momenta is given in Fig. 2.

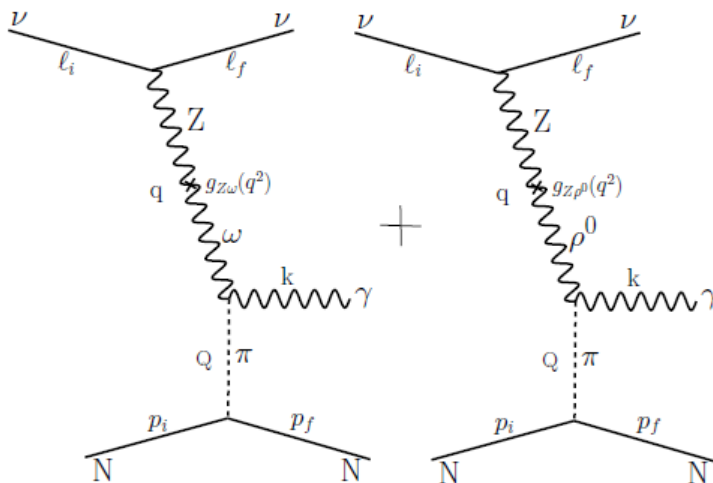
In accordance with the estimates of Ref. 3, we shall take into account the contributions to the diagram of Fig.

# A phenomenological study of photon production in low energy neutrino nucleon scattering

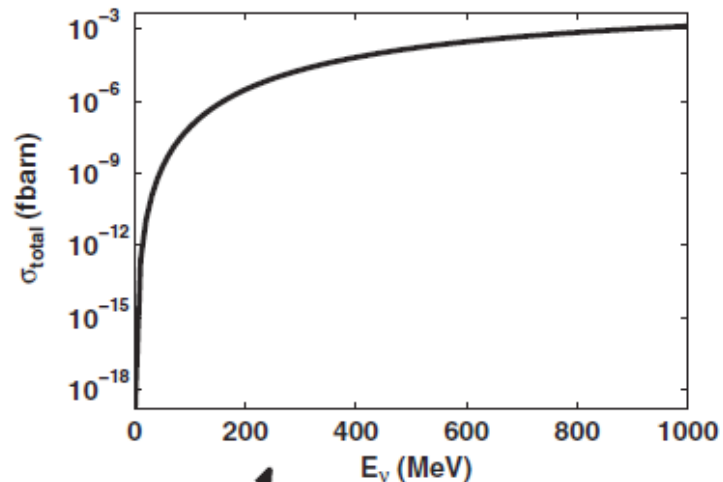
James Jenkins and T. Goldman

*Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM 87545*

Low energy photon production is an important background to many current and future precision neutrino experiments. We present a phenomenological study of  $t$ -channel radiative corrections to neutral current neutrino nucleus scattering. After introducing the relevant processes and phenomenological coupling constants, we will explore the derived energy and angular distributions as well as total cross-section predictions along their estimated uncertainties. This is supplemented throughout with comments on possible experimental signatures and implications. We conclude with a general discussion of the analysis in the context of complimentary methodologies.



Too small at  $E_{\nu} \sim 1$  GeV?





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A search for single photon events in neutrino interactions

*NOMAD exp.*

C.T. Kullenberg<sup>s</sup>, S.R. Mishra<sup>s,\*</sup>, D. Dimmery<sup>s</sup>, X.C. Tian<sup>s</sup>, D. Autiero<sup>h</sup>, S. Gninenko<sup>h,l</sup>, A. Rubbia<sup>h,x</sup>, S. Alekhin<sup>y</sup>, P. Astier<sup>n</sup>, A. Baldisseri<sup>r</sup>, M. Baldo-Ceolin<sup>m</sup>, M. Banner<sup>n</sup>, G. Bassompierre<sup>a</sup>, K. Benslama<sup>i</sup>, N. Besson<sup>r</sup>, I. Bird<sup>h,i</sup>, B. Blumenfeld<sup>b</sup>, F. Bobisut<sup>m</sup>, J. Bouchez<sup>r</sup>, S. Boyd<sup>t,1</sup>, A. Bueno<sup>c,x</sup>, S. Bunyatov<sup>f</sup>, L. Camilleri<sup>h</sup>, A. Cardini<sup>j</sup>, P.W. Cattaneo<sup>o</sup>, V. Cavasinni<sup>p</sup>, A. Cervera-Villanueva<sup>h,v</sup>, R. Challis<sup>k</sup>, A. Chalkley<sup>f</sup>, C. Collard<sup>m</sup>, C. C. C. Costa<sup>h,2</sup>, C. Costa<sup>o</sup>, M. C. C. C. Costa<sup>m</sup>, P. C. C. C. Costa<sup>i</sup>, M. D. D. Costa<sup>i</sup>

Letter of Intent: A new investigation of  $\nu_\mu \rightarrow \nu_e$  oscillations with improved sensitivity in an enhanced MiniBooNE experiment

**MiniBooNE Collaboration**

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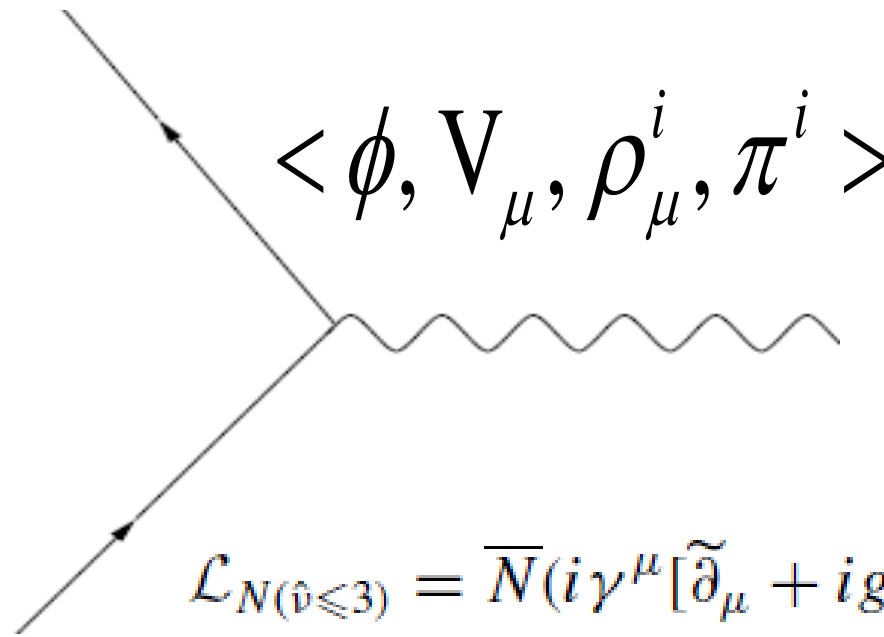
*Fermi National Accelerator Laboratory, Batavia, IL 60510*

J. Grange, J. Mousseau, B. Osmanov, & H. Ray

ep-ex] 8 Oct 2012

# QHD

- NN interactions (**relativistic** field theory since 1970):



$\langle \phi, V_\mu, \rho_\mu^i, \pi^i \rangle$

$$\mathcal{L}_{N(\hat{v} \leq 3)} = \bar{N}(i\gamma^\mu [\tilde{\partial}_\mu + ig_\rho \rho_\mu + ig_v V_\mu] + g_A \gamma^\mu \gamma^5 \tilde{a}_\mu - M + g_s \phi)N$$

*B. Serot and J. Walecka, Adv. Nucl. Phys. 16, 1 (1986)*



# QHD

- NN interactions (**relativistic** field theory since 1970).
- Mean-field approximation (RMF): works for nuclear matter and mid-heavy nuclei; meson fields develop expectation values; nucleon spin-orbital coupling...

$$h_{\text{LS,T}} = \left[ \frac{1}{4\bar{M}^2} \frac{1}{r} \left( \frac{d\Phi}{dr} + \frac{dW}{dr} \right) + \frac{f_v}{2M\bar{M}} \frac{1}{r} \frac{dW}{dr} + O(v^4) \right] \boldsymbol{\sigma} \cdot \mathbf{L}$$

# QHD

- NN interactions (**relativistic** field theory since 1970).
- Mean-field approximation (RMF): works for nuclear matter and mid-heavy nuclei; meson fields develop expectation values; nucleon spin-orbital coupling...
- Symmetries and currents: Lorentz, EM gauge, Chiral (breaking) → **CVC and PCAC**

$$SU(2)_L \otimes SU(2)_R \otimes U(1)_B$$

# QHD

- NN interactions (**relativistic** field theory since 1970).
- Mean-field approximation (RMF): works for nuclear matter and mid-heavy nuclei; meson fields develop expectation values; nucleon spin-orbital coupling...
- Symmetries and currents.
- Introduce Delta resonance, Delta medium modifications.

$$\mathcal{L}_\Delta = \frac{-i}{2} \overline{\Delta}_\mu^a \{ \sigma^{\mu\nu}, (i \tilde{\not{D}} - h_\rho \not{\rho} - h_\nu \not{Y} - m + h_s \phi) \}_a^b \Delta_{b\nu} \quad ?$$

*B. Serot and X.Z., Advances in QFT (InTech, 2012) (arXiv:1110.2760)*

- Pion dynamics (optical potential)

$$\mathcal{L}_{\Delta, N, \pi} = h_A \overline{\Delta}^{a\mu} T_a^{\dagger iA} \tilde{a}_{i\mu} N_A + \text{c.c.},$$

*Loop calculation: Y. Hu, J. McIntire, and B. Serot NPA 794, 187 (2007)*

# A quick look:

- Chiral symmetry in QCD:  $SU(2)_L \otimes SU(2)_R \otimes U(1)_B$

$$\mathcal{L} = \mathcal{L}_{\text{QCD}} + \bar{q} \gamma_\mu (v^\mu + B v_{(s)}^\mu + \gamma_5 a^\mu) q - \bar{q} (s - i \gamma_5 p) q$$

$$q_{LA} \rightarrow \exp \left[ -i \frac{\theta(x)}{3} \right] \left( \exp \left[ -i \theta_{Li}(x) \frac{\tau^i}{2} \right] \right)_A^B q_{LB} \equiv \exp \left[ -i \frac{\theta(x)}{3} \right] (L)_A^B q_{LB},$$

$$q_R \rightarrow \exp \left[ -i \frac{\theta(x)}{3} \right] \exp \left[ -i \theta_{Ri}(x) \frac{\tau^i}{2} \right] q_R \equiv \exp \left[ -i \frac{\theta(x)}{3} \right] R q_R,$$

# A quick look:

- Chiral symmetry in QCD:
- Its nonlinear realization at low energy EFT:

$$U \equiv \exp \left[ 2i \frac{\pi_i(x)}{f_\pi} t^i \right] \rightarrow LUR^\dagger,$$

$$\xi \equiv \sqrt{U} = \exp \left[ i \frac{\pi_i}{f_\pi} t^i \right] \rightarrow L\xi h^\dagger = h \xi R^\dagger,$$

$$\tilde{v}_\mu \equiv \frac{-i}{2} [\xi^\dagger (\partial_\mu - il_\mu) \xi + \xi (\partial_\mu - ir_\mu) \xi^\dagger] \equiv \tilde{v}_{i\mu} t^i \rightarrow h \tilde{v}_\mu h^\dagger - ih \partial_\mu h^\dagger,$$

$$\tilde{a}_\mu \equiv \frac{-i}{\gamma} [\xi^\dagger (\partial_\mu - il_\mu) \xi - \xi (\partial_\mu - ir_\mu) \xi^\dagger] \equiv \tilde{a}_{i\mu} t^i \rightarrow h \tilde{a}_\mu h^\dagger,$$

# A quick look:

- Chiral symmetry in QCD:
- Its nonlinear realization at low energy EFT:

$$(\tilde{\partial}_\mu \psi)_\alpha \equiv (\partial_\mu + i \tilde{v}_\mu - i v_{(s)\mu} B)_\alpha^\beta \psi_\beta \rightarrow \exp[-i\theta(x)B] h_\alpha^\beta (\tilde{\partial}_\mu \psi)_\beta ,$$

$$\tilde{v}_{\mu\nu} \equiv -i[\tilde{a}_\mu, \tilde{a}_\nu] \rightarrow h \tilde{v}_{\mu\nu} h^\dagger ,$$

$$F_{\mu\nu}^{(+)} \equiv \tilde{\zeta}^\dagger f_{L\mu\nu} \tilde{\zeta} + \tilde{\zeta} f_{R\mu\nu} \tilde{\zeta}^\dagger \rightarrow h F_{\mu\nu}^{(+)} h^\dagger ,$$

$$F_{\mu\nu}^{(-)} \equiv \tilde{\zeta}^\dagger f_{L\mu\nu} \tilde{\zeta} - \tilde{\zeta} f_{R\mu\nu} \tilde{\zeta}^\dagger \rightarrow h F_{\mu\nu}^{(-)} h^\dagger ,$$

# A quick look:

- The lagrangian, baryon section:

$$\mathcal{L}_{N(\hat{v} \leq 3)} = \bar{N}(i\gamma^\mu[\tilde{\partial}_\mu + ig_\rho \rho_\mu + ig_v V_\mu] + g_A \gamma^\mu \gamma^5 \tilde{a}_\mu - M + g_s \phi)N$$

$$\mathcal{L}_\Delta = \frac{-i}{2} \bar{\Delta}_\mu^a \{ \sigma^{\mu\nu}, (i \tilde{\not{\partial}} - h_\rho \not{\rho} - h_v \not{V} - m + h_s \phi) \}_a^b \Delta_{b\nu}$$

$$\mathcal{L}_{\Delta, N, \pi} = h_A \bar{\Delta}^{a\mu} T_a^{\dagger iA} \tilde{a}_{i\mu} N_A + \text{c.c.},$$

# A quick look:

- The lagrangian, baryon section.
- The lagrangian, meson section:

$$\begin{aligned}\mathcal{L}_{\text{meson}(\hat{d}\leq 4)} = & \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{4} f_\pi^2 \text{Tr}[\tilde{\partial}_\mu U (\tilde{\partial}^\mu U)^\dagger] \\ & + \frac{1}{4} f_\pi^2 m_\pi^2 \text{Tr}(U + U^\dagger - 2) \\ & - \frac{1}{2} \text{Tr}(\rho_{\mu\nu} \rho^{\mu\nu}) - \frac{1}{4} V^{\mu\nu} V_{\mu\nu} \\ & + \frac{1}{2g_\gamma} \left( \text{Tr}(F^{(+)\mu\nu} \rho_{\mu\nu}) + \frac{1}{3} f_s^{\mu\nu} V_{\mu\nu} \right).\end{aligned}$$

*Vector  
meson  
dominance (VMD)*



# A quick look:

- Electroweak (EW) interactions:

$$l_\mu = -e \frac{\tau^0}{2} A_\mu + \frac{g}{\cos \theta_w} \sin^2 \theta_w \frac{\tau^0}{2} Z_\mu - \frac{g}{\cos \theta_w} \frac{\tau^0}{2} Z_\mu - g V_{ud} \left( W_\mu^{+1} \frac{\tau_{+1}}{2} + W_\mu^{-1} \frac{\tau_{-1}}{2} \right),$$

$$r_\mu = -e \frac{\tau^0}{2} A_\mu + \frac{g}{\cos \theta_w} \sin^2 \theta_w \frac{\tau^0}{2} Z_\mu,$$

$$v_{(s)\mu} = -e \frac{1}{2} A_\mu + \frac{g}{\cos \theta_w} \sin^2 \theta_w \frac{1}{2} Z_\mu.$$

# A quick look:

- Electroweak (EW) interactions:

$$\begin{aligned} \mathcal{L}_{\Delta,N,\text{bg}} = & \frac{ic_{1\Delta}}{M} \bar{\Delta}_\mu^a \gamma_\nu \gamma^5 T_a^{\dagger iA} F_i^{(+)\mu\nu} N_A + \frac{ic_{3\Delta}}{M^2} \bar{\Delta}_\mu^a i\gamma^5 T_a^{\dagger iA} (\tilde{\partial}_\nu F^{(+)\mu\nu})_i N_A + \frac{c_{6\Delta}}{M^2} \bar{\Delta}_\lambda^a \sigma_{\mu\nu} T_a^{\dagger iA} (\tilde{\partial}^\lambda \bar{F}^{(+)\mu\nu})_i N_A \\ & - \frac{d_{2\Delta}}{M^2} \bar{\Delta}_\mu^a T_a^{\dagger iA} (\tilde{\partial}_\nu F^{(-)\mu\nu})_i N_A - \frac{id_{4\Delta}}{M} \bar{\Delta}_\mu^a \gamma_\nu T_a^{\dagger iA} F_i^{(-)\mu\nu} N_A - \frac{id_{7\Delta}}{M^2} \bar{\Delta}_\lambda^a \sigma_{\mu\nu} T_a^{\dagger iA} (\tilde{\partial}^\lambda F^{(-)\mu\nu})_i N_A + \text{c.c.}, \end{aligned}$$

$$\mathcal{L}_{\Delta,N,\rho} = \frac{ic_{1\Delta\rho}}{M} \bar{\Delta}_\mu^a \gamma_\nu \gamma^5 T_a^{\dagger iA} \rho_i^{\mu\nu} N_A + \frac{ic_{3\Delta\rho}}{M^2} \bar{\Delta}_\mu^a i\gamma^5 T_a^{\dagger iA} (\tilde{\partial}_\nu \rho^{\mu\nu})_i N_A + \frac{c_{6\Delta\rho}}{M^2} \bar{\Delta}_\lambda^a \sigma_{\mu\nu} T_a^{\dagger iA} (\tilde{\partial}^\lambda \bar{\rho}^{\mu\nu})_i N_A + \text{c.c.}.$$

# A quick look (recap)

- Chiral symmetry
- The lagrangian
- Electroweak interactions (CVC and PCAC)
- Two body currents....

## Axial-vector current in nuclear many-body physics

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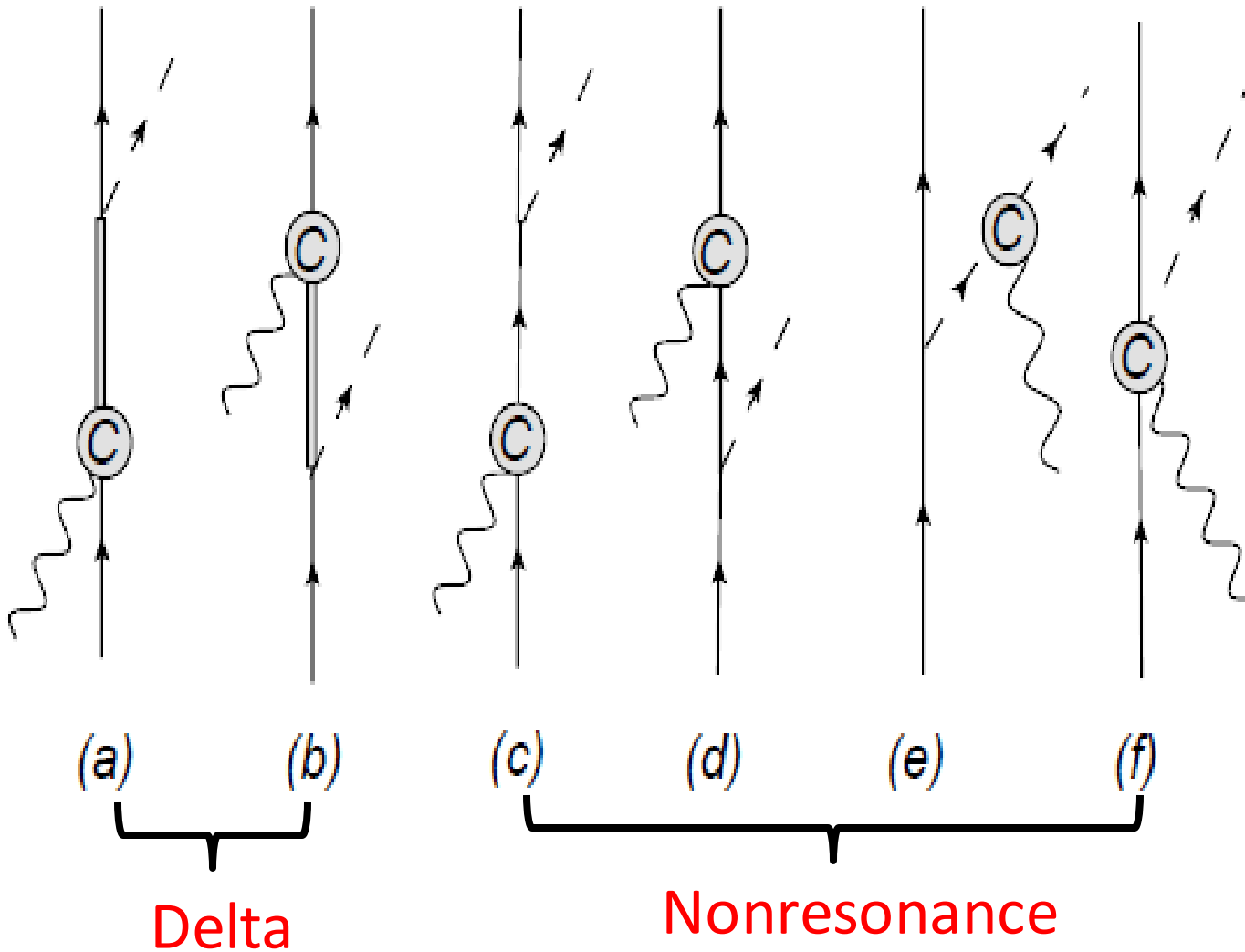
John Dirk Walecka‡

*Department of Physics, The College of William and Mary, Williamsburg, Virginia 23187*

(Received 10 July 2002; published 27 November 2002)

Weak-interaction currents are studied in a recently proposed effective field theory of the nuclear many-body problem. The Lorentz-invariant effective field theory contains nucleons, pions, as well as isoscalar, scalar ( $\sigma$ ) and vector ( $\omega$ ) fields, and isovector, vector ( $\rho$ ) fields. The theory exhibits a nonlinear realization of  $SU(2)_L \times SU(2)_R$  chiral symmetry and has three desirable features: it uses the same degrees of freedom to describe the axial-vector current and the strong-interaction dynamics, it satisfies the symmetries of the underlying theory of quantum chromodynamics, and its parameters can be calibrated using strong-interaction phenomena, like hadron scattering or the empirical properties of finite nuclei. Moreover, it has recently been verified that for normal nuclear systems, it is possible to systematically expand the effective Lagrangian in powers of the meson fields (and their derivatives) and to reliably truncate the expansion after the first few orders. Here it is shown that the expressions for the axial-vector current, evaluated through the first few orders in the field expansion, satisfy both PCAC and the Goldberger-Treiman relation, and it is verified that the corresponding vector and axial-vector charges satisfy the familiar chiral charge algebra. Explicit results are derived for the Lorentz-covariant, axial-vector, two-nucleon amplitudes, from which axial-vector meson-exchange currents can be deduced.

# Pion production off the nucleon



# Nucleon current form factors

$$\langle N, B | V_\mu^i | N, A \rangle = \langle B | \frac{\tau^i}{2} | A \rangle \bar{u}_f \left( \gamma_\mu + 2\delta F_1^{V,md} \frac{q^2 \gamma_\mu - \not{q} q_\mu}{q^2} + 2F_2^{V,md} \frac{\sigma_{\mu\nu} i q^\nu}{2M} \right) u_i$$

$$\begin{aligned} \langle N, B; \pi, j, k_\pi | A_\mu^i | N, A \rangle &= -\frac{\epsilon_{ijk}^i}{f_\pi} \langle B | \frac{\tau^k}{2} | A \rangle \bar{u}_f \gamma^\nu u_i \left[ g_{\mu\nu} + 2\delta F_1^{V,md} ((q - k_\pi)^2) \frac{q \cdot (q - k_\pi) g_{\mu\nu} - (q - k_\pi)_\mu q_\nu}{(q - k_\pi)^2} \right] \\ &\quad - \frac{\epsilon_{ijk}^i}{f_\pi} \langle B | \frac{\tau^k}{2} | A \rangle \bar{u}_f \frac{\sigma_{\mu\nu} i q^\nu}{2M} u_i \left[ 2\lambda^{(1)} + 2\delta F_2^{V,md} ((q - k_\pi)^2) \frac{q \cdot (q - k_\pi)}{(q - k_\pi)^2} \right] \end{aligned}$$

*VMD. This relation will be used in high energy extrapolation.*

$$\langle N, B | A_\mu^i | N, A \rangle = -G_A^{md}(q^2) \langle B | \frac{\tau^i}{2} | A \rangle \bar{u}_f \left( \gamma_\mu - \frac{q_\mu \not{q}}{q^2 - m_\pi^2} \right) \gamma^5 u_i \quad ?$$

$$G_A(q^2) = g_A / (1 - q^2 / M_A^2)^2 \quad ?$$

# Transition form factors

$$\begin{aligned} & \langle \Delta, a, p_\Delta | V^{i\mu}(A^{i\mu}) | N, A, p_N \rangle \\ & \equiv T_a^{\dagger iA} \bar{u}_{\Delta\alpha}(p_\Delta) \Gamma_{V(A)}^{\alpha\mu}(q) u_N(p_N) \end{aligned}$$

$$\begin{aligned} \Gamma_V^{\alpha\mu} = & \frac{2c_{1\Delta}(q^2)}{M} (q^\alpha \gamma^\mu - \not{q} g^{\alpha\mu}) \gamma^5 \\ & + \frac{2c_{3\Delta}(q^2)}{M^2} (q^\alpha q^\mu - g^{\alpha\mu} q^2) \gamma^5 \\ & - \frac{8c_{6\Delta}(q^2)}{M^2} q^\alpha \sigma^{\mu\nu} i q_\nu \gamma^5, \end{aligned}$$

$$\begin{aligned} \Gamma_A^{\alpha\mu} = & -h_A \left( g^{\alpha\mu} - \frac{q^\alpha q^\mu}{q^2 - m_\pi^2} \right) \\ & + \frac{2d_{2\Delta}}{M^2} (q^\alpha q^\mu - g^{\alpha\mu} q^2) \\ & - \frac{2d_{4\Delta}}{M} (q^\alpha \gamma^\mu - g^{\alpha\mu} \not{q}) - \frac{4d_{7\Delta}}{M^2} q^\alpha \sigma^{\mu\nu} i q_\nu \end{aligned}$$

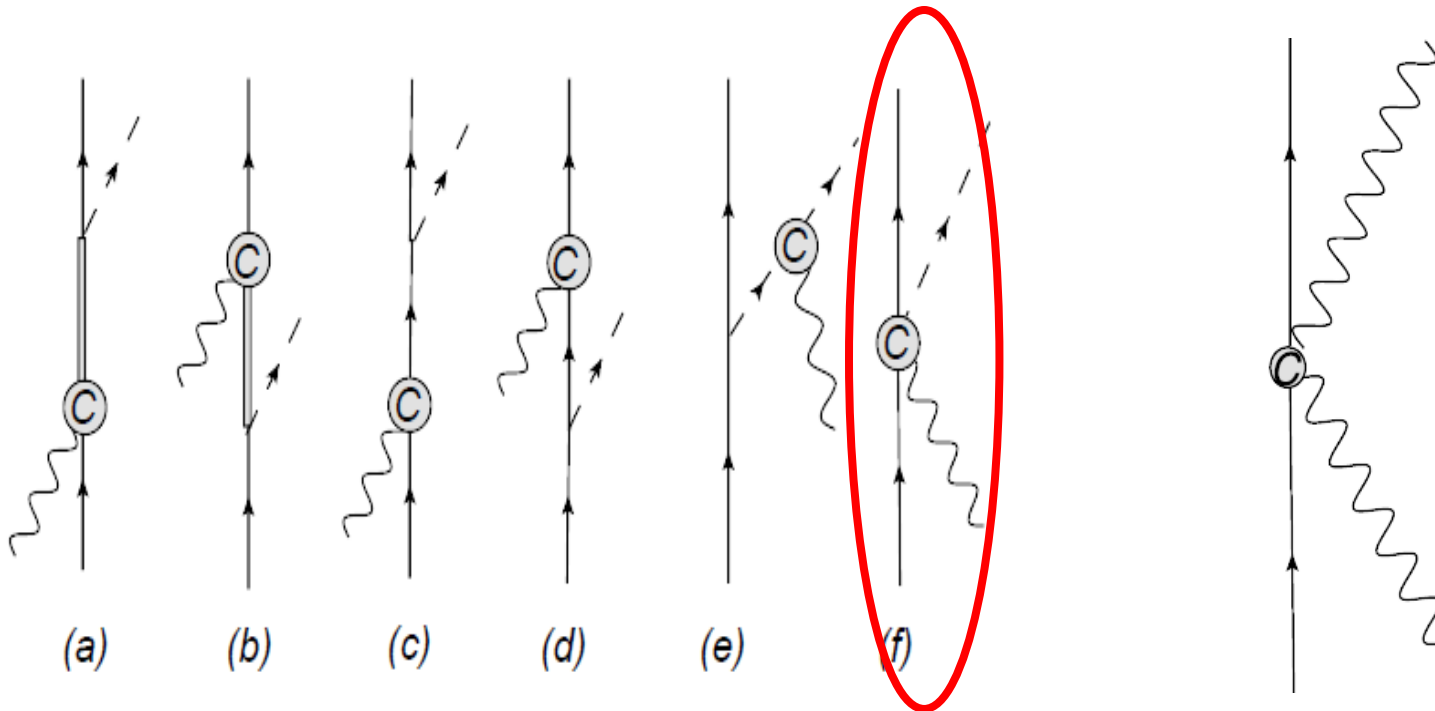
$$h_A(q^2) \equiv h_A + h_{\Delta a_1} \frac{q^2}{q^2 - m_{a_1}^2}, \quad d_{i\Delta}(q^2) \equiv d_{i\Delta} + d_{i\Delta a_1} \frac{q^2}{q^2 - m_{a_1}^2}, \quad \text{Meson dominance FF (MDFF)}$$

$$\begin{aligned} & \bar{u}_\alpha(p_\Delta) \left\{ \left[ \frac{C_3^V}{M} (g^{\alpha\mu} \not{q} - q^\alpha \gamma^\mu) + \frac{C_4^V}{M^2} (q \cdot p_\Delta g^{\alpha\mu} - q^\alpha p_\Delta^\mu) + \frac{C_5^V}{M^2} (q \cdot p_N g^{\alpha\mu} - q^\alpha p_N^\mu) \right] \gamma^5 \right. \\ & \left. + \left[ \frac{C_3^A}{M} (g^{\alpha\mu} \not{q} - q^\alpha \gamma^\mu) + \frac{C_4^A}{M^2} (q \cdot p_\Delta g^{\alpha\mu} - q^\alpha p_\Delta^\mu) + C_5^A g^{\alpha\mu} + \frac{C_6^A}{M^2} q^\mu q^\alpha \right] \right\} u(p_N). \quad ? \end{aligned}$$

K. Graczyk, D. Kietczewska, P. Przewłocki, and J. Sobczyk, PRD **80**, 093001 (2009). (CFF)

E. Hern´andez, J. Nieves, and M. Valverde, PRD **76**, 033005 (2007). (HFF with C5a reduced)

# NC photon production off the nucleon



$$\frac{c_1}{M^2} \bar{N} \gamma^\mu N \text{Tr}(\tilde{a}^\nu \bar{F}_{\mu\nu}^{(+)}), \quad \frac{e_1}{M^2} \bar{N} \gamma^\mu \tilde{a}^\nu N \bar{f}_{s\mu\nu}$$

*Also related to pion  
electro(photo)-production*



# Production of single photons in the exclusive neutrino process $\nu N \rightarrow \nu \gamma N$

S. S. Gershtein, Yu. Ya. Komachenko, and M. Yu. Khlopov

*Institute of High Energy Physics, Serpukhov*

(Submitted 16 January 1981)

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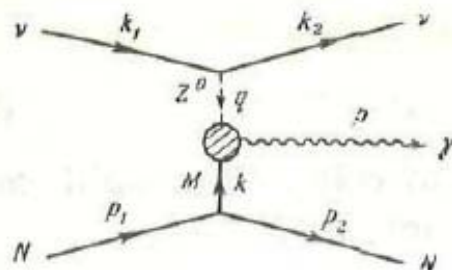
It is shown that the experimentally observed production of single photons in neutrino interactions involving neutral currents without visible accompaniment of other particles can be explained by the scattering of the neutrino by a virtual  $\omega$  meson with small momentum transfer to a nucleon and subsequent coherent enhancement of the process in the nucleus.

PACS numbers: 13.15. + g, 14.80.Kx

## 1. INTRODUCTION

In neutrino experiments performed at CERN using the chamber Gargamelle, more than ten events were detected in which it was observed that single photons with energy 1-10 GeV were produced without visible tracks of any other particles.<sup>1</sup> It can be assumed that the observed events correspond to the weak-electromagnetic process of single-photon production in the reaction

$$\nu N \rightarrow \nu \gamma N, \quad (1)$$



$$H_{\nu\nu} = \sum_M T^{(M)} P^{(M)} J_{\nu\nu}^{(M)}, \quad (3)$$

in which  $T^{(M)}$  is the vertex for emission of a virtual meson ( $M$ ) by the target nucleon,  $P^{(M)}$  is the meson propagator, and  $J_{\mu\nu}^{(M)} = \int \langle 0 | T(J_{\mu}^{W}(x), J_{\nu}^{EM}(y)) | M \rangle e^{i q x + i p y} d^4 x d^4 y$  is the weak-electromagnetic  $Z^0 M \gamma$  vertex. The notation for the particle momenta is given in Fig. 2.

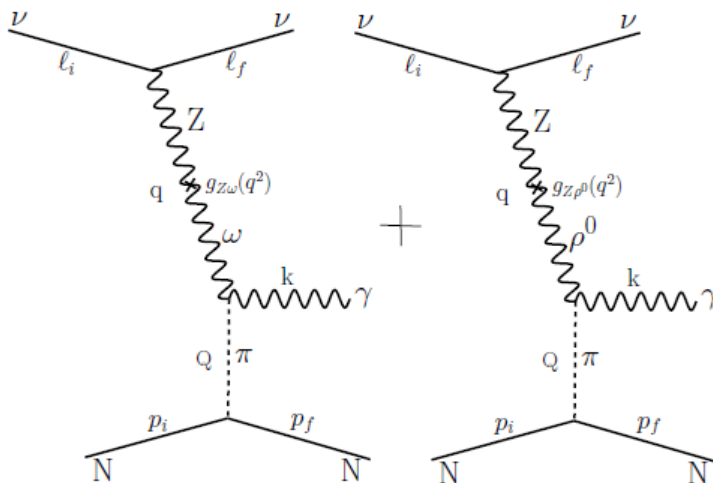
In accordance with the estimates of Ref. 3, we shall take into account the contributions to the diagram of Fig.

# A phenomenological study of photon production in low energy neutrino nucleon scattering

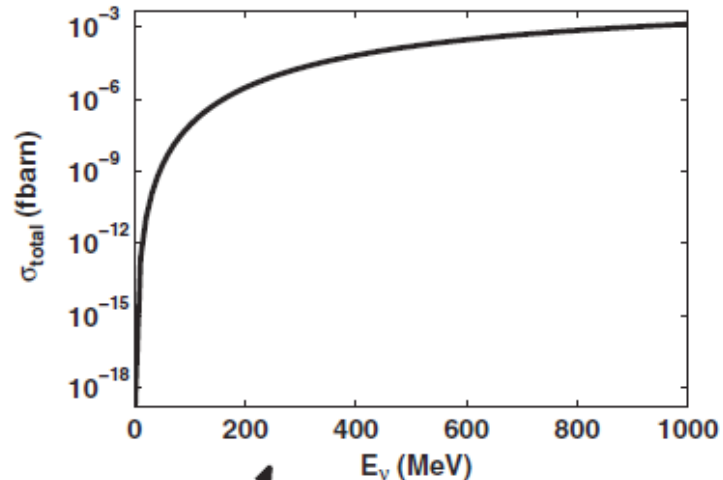
James Jenkins and T. Goldman

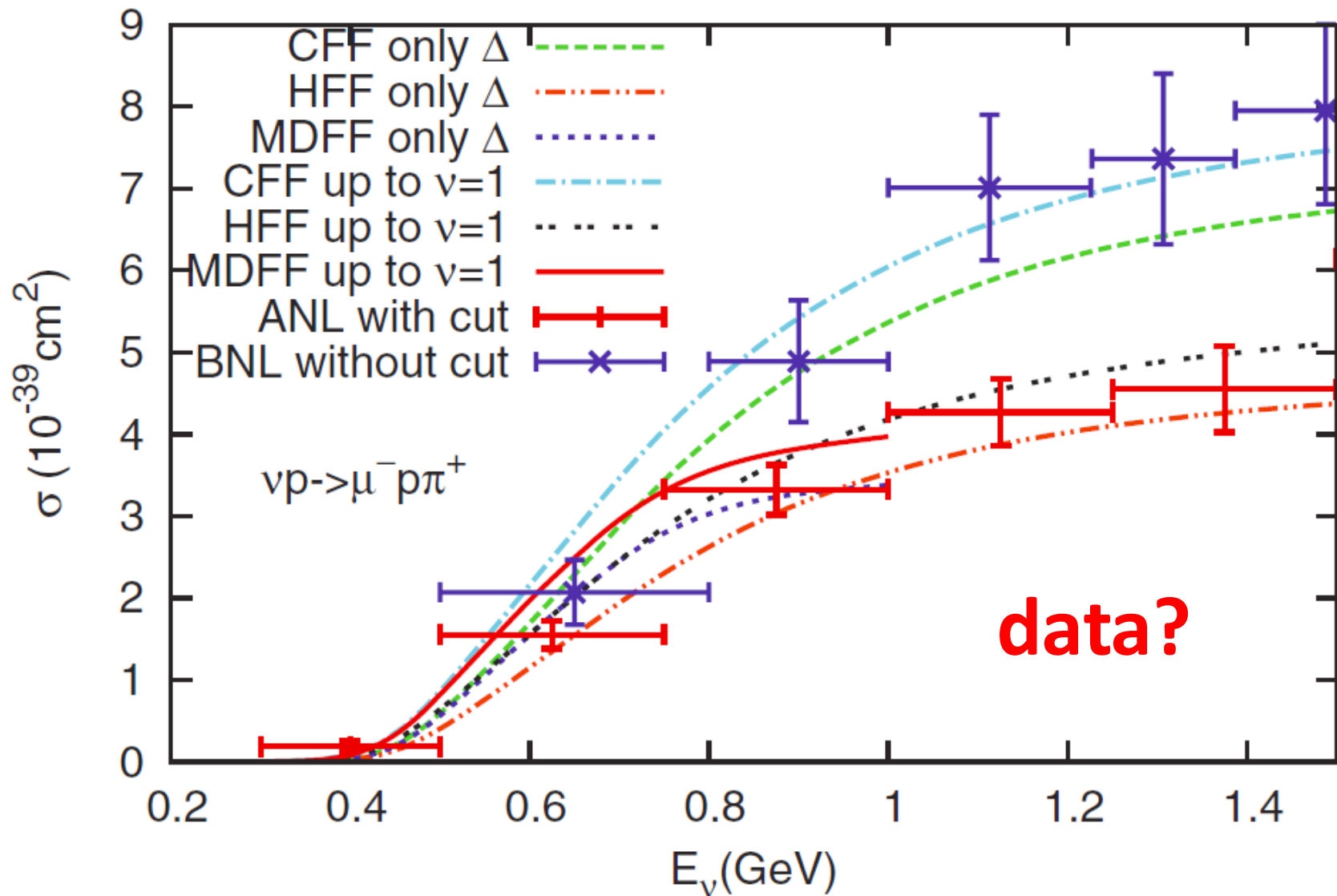
*Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM 87545*

Low energy photon production is an important background to many current and future precision neutrino experiments. We present a phenomenological study of  $t$ -channel radiative corrections to neutral current neutrino nucleus scattering. After introducing the relevant processes and phenomenological coupling constants, we will explore the derived energy and angular distributions as well as total cross-section predictions along their estimated uncertainties. This is supplemented throughout with comments on possible experimental signatures and implications. We conclude with a general discussion of the analysis in the context of complimentary methodologies.

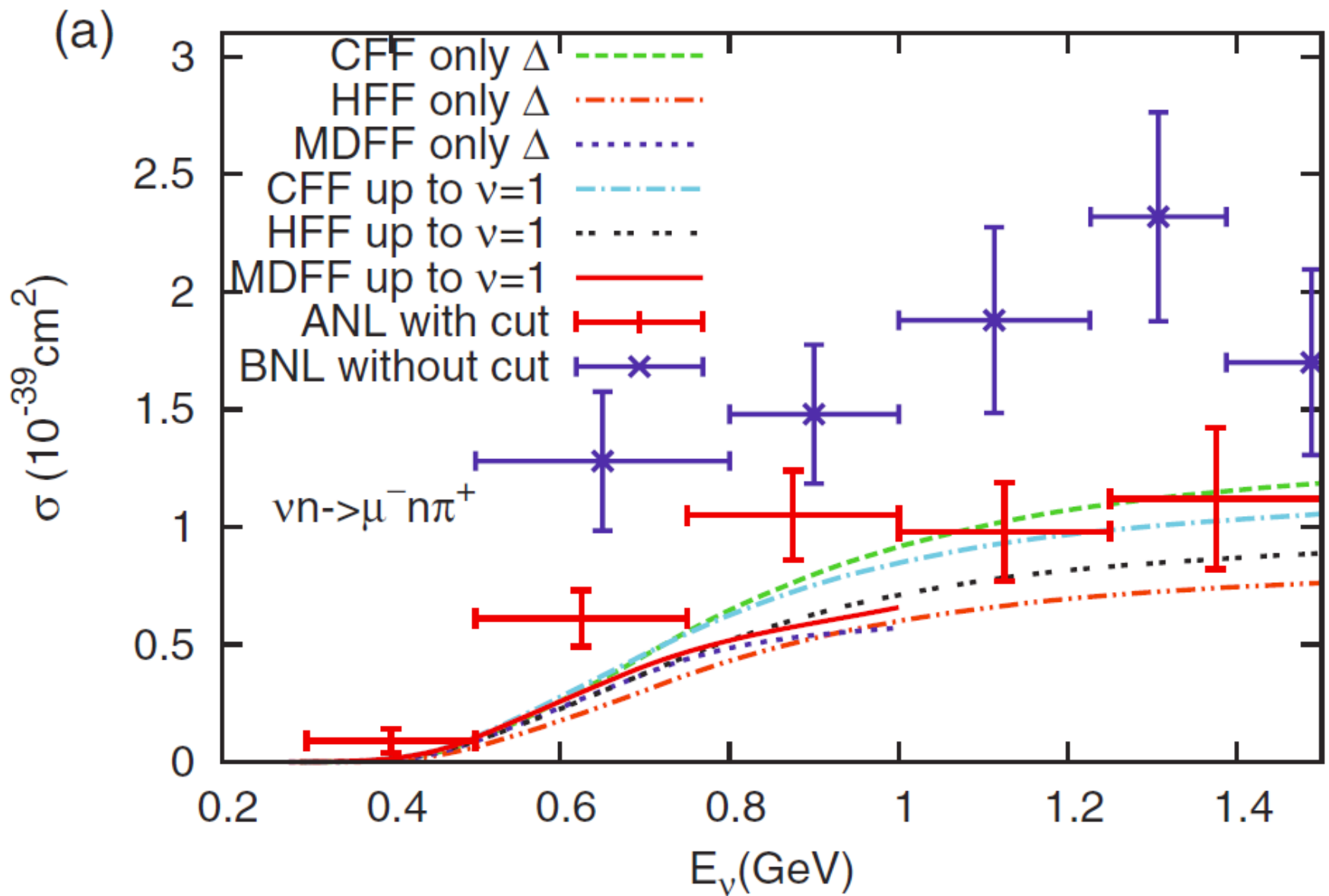


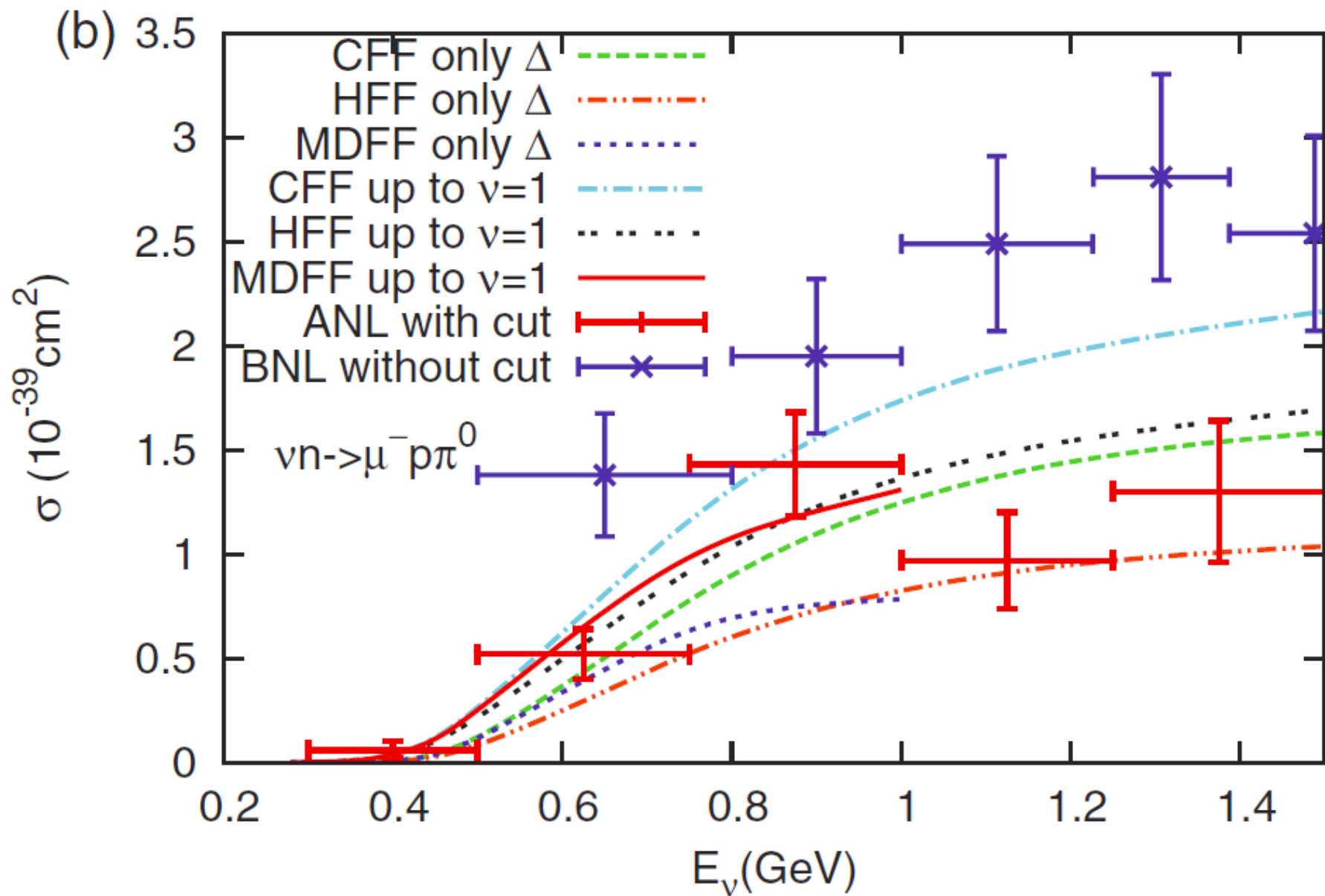
Too small at  $E_{\nu} \sim 1$  GeV?

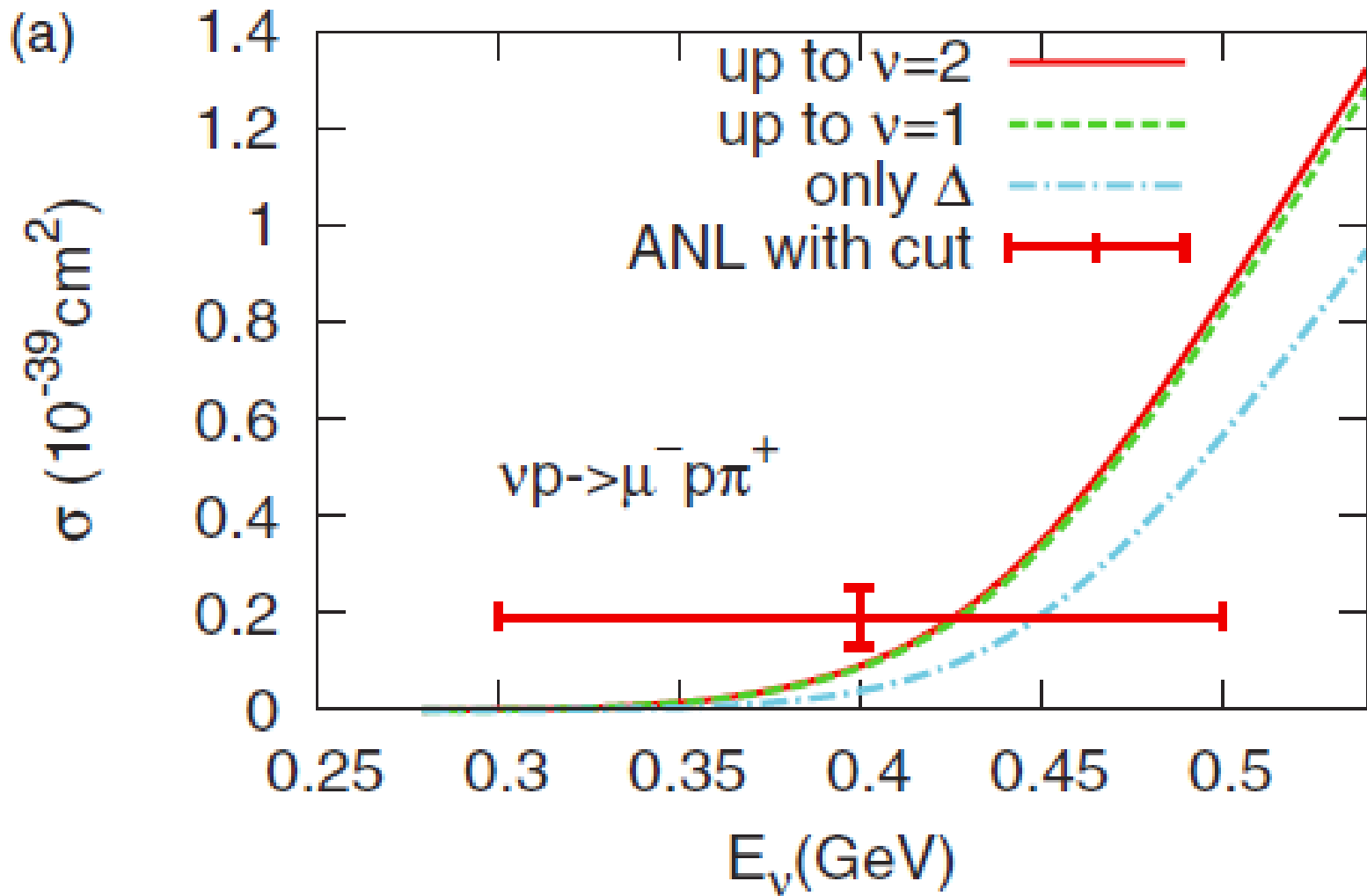




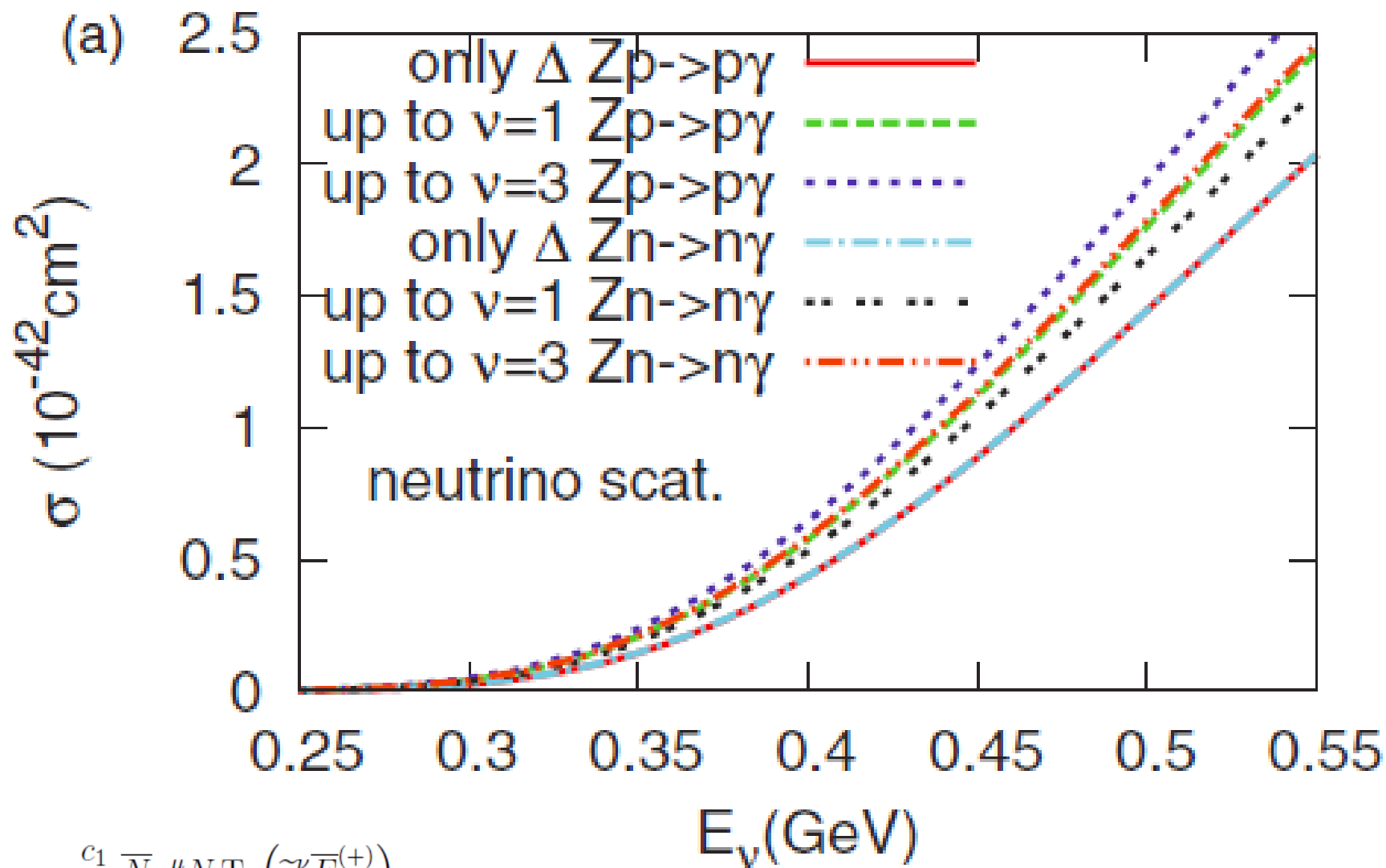
K. Graczyk, D. Kietczewska, P. Przewłocki, and J. Sobczyk, *PRD* **80**, 093001 (2009).  
 E. Hern´andez, J. Nieves, and M. Valverde, *PRD* **76**, 033005 (2007).  
 G. M. Radecky et al., *PRD* **25**, 1161 (1982); T. Kitagaki et al., *PRD* **34**, 2554 (1986).







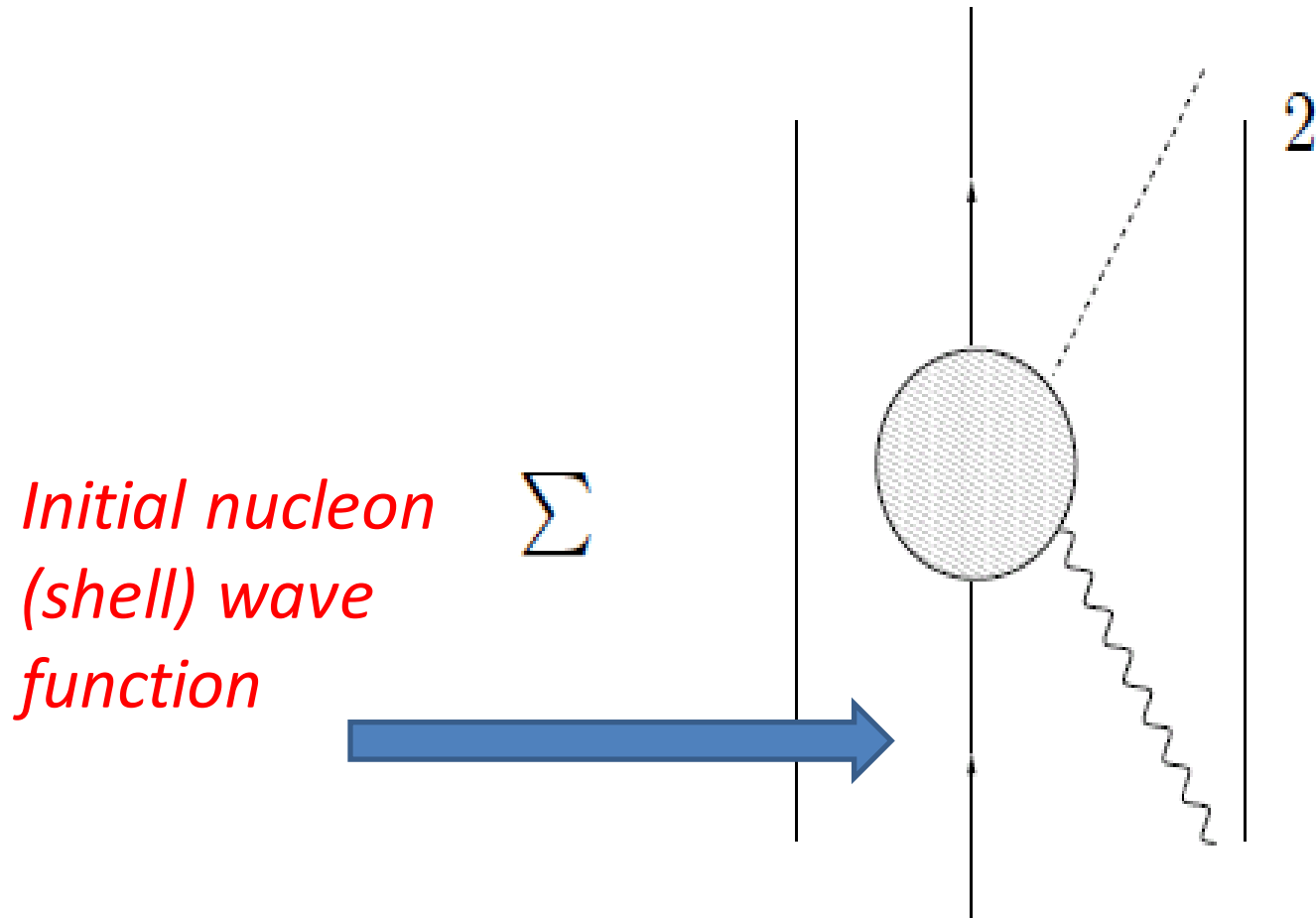
*Power counting of the calculation*



$$\frac{c_1}{M^2} \bar{N} \gamma^\mu N \text{Tr} \left( \tilde{a}^\nu \bar{F}_{\mu\nu}^{(+)} \right),$$

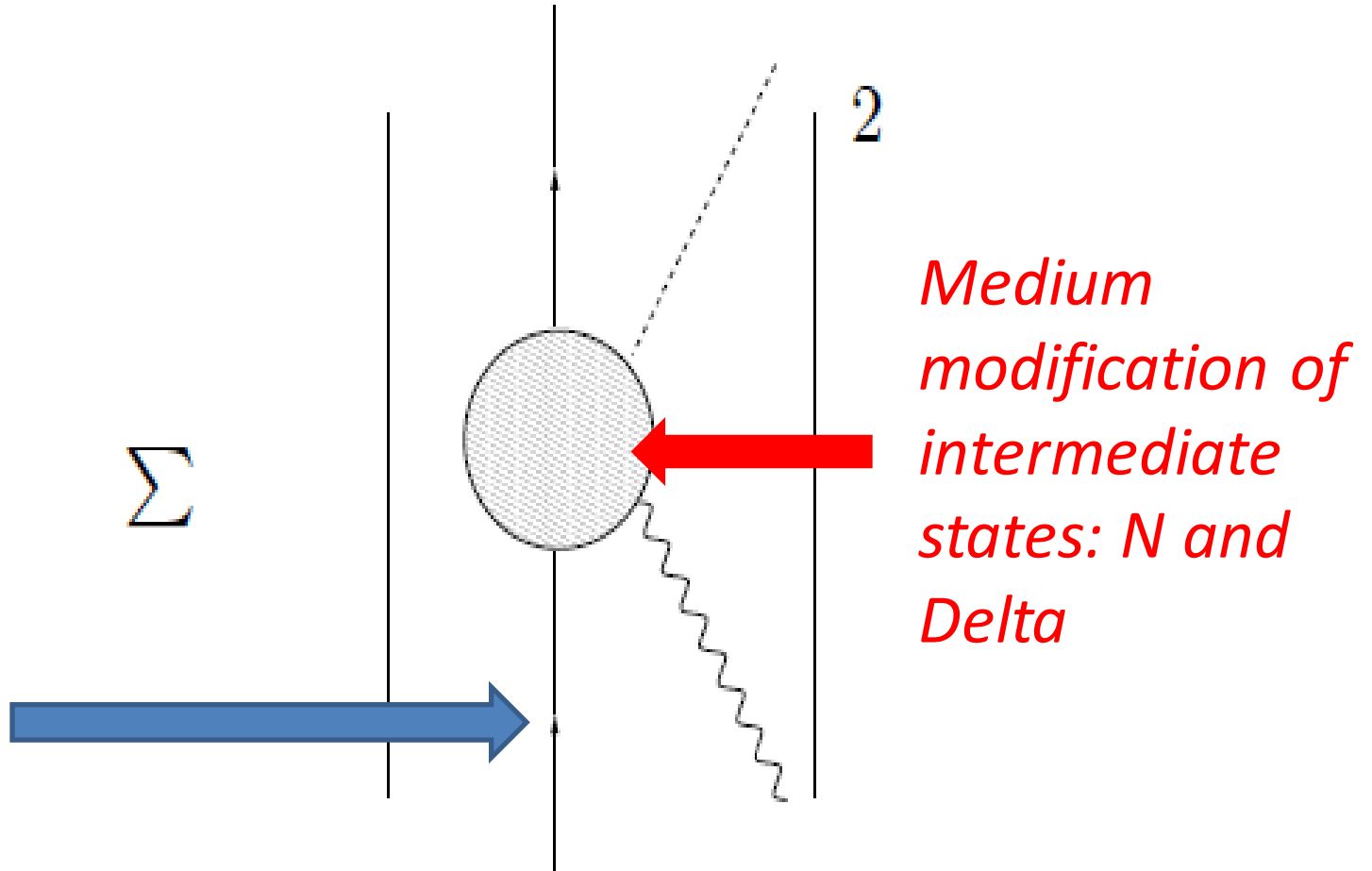
$$\frac{e_1}{M^2} \bar{N} \gamma^\mu \tilde{a}^\nu N \bar{f}_{s\mu\nu}.$$

# Incoherent productions





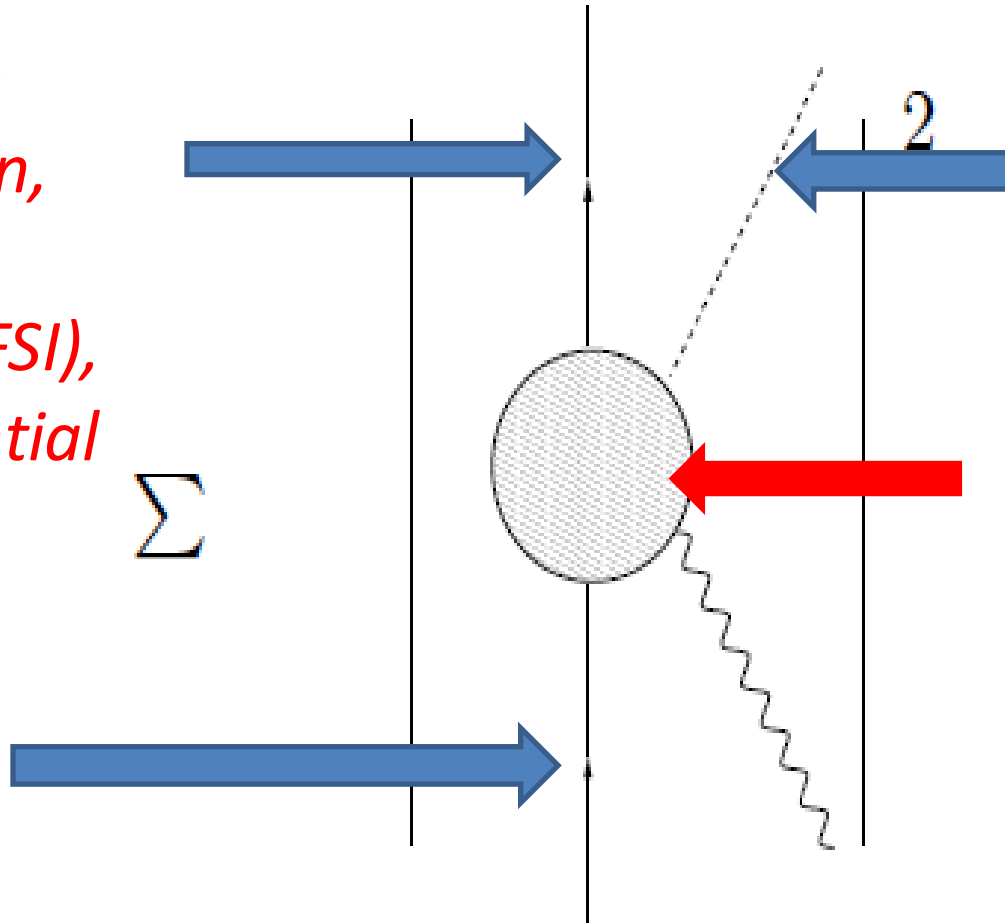
# Incoherent productions



# Incoherent productions

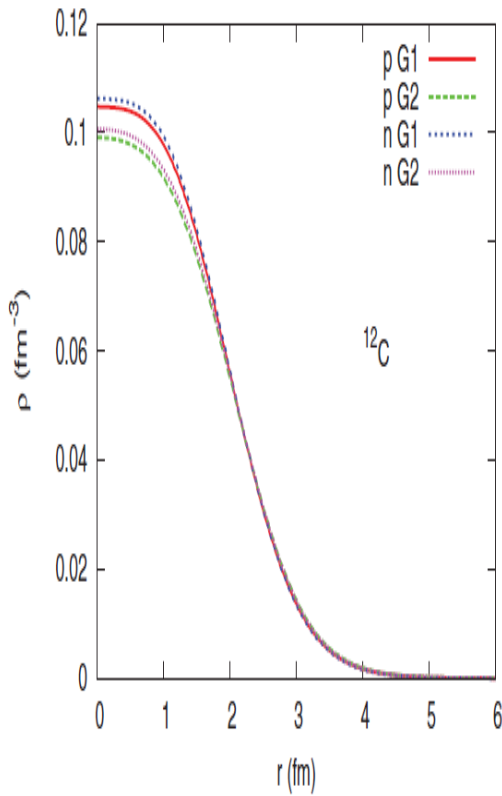
*Final nucleon  
wave function,  
final state  
interaction (FSI),  
optical potential*

$\Sigma$

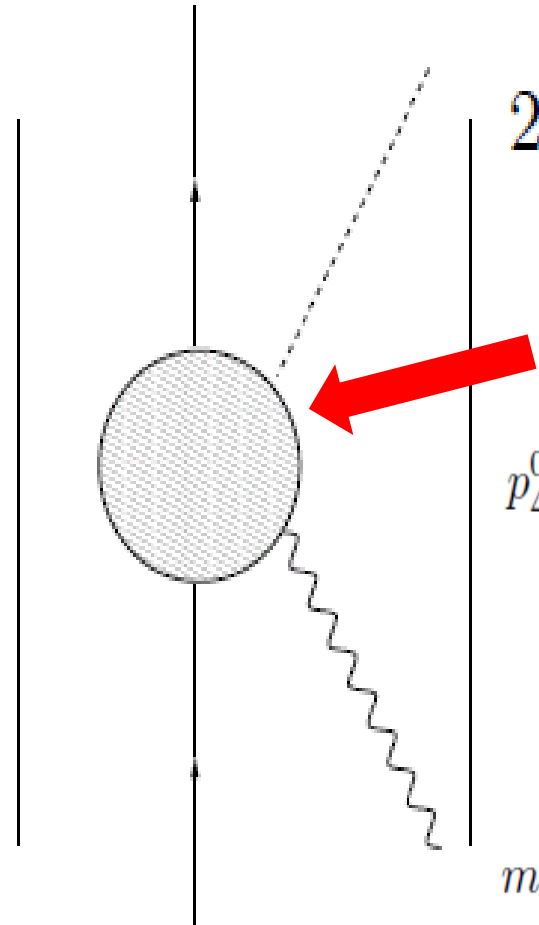


*Pion FSI,  
optical potential;  
Photon has  
no FSI*

# Local Fermi Gas



$$\sum_{\vec{r}} \sum_{\vec{p}} k_F(\vec{r})$$



*Delta:*

$$\begin{aligned} p_{\Delta}^0 &= h_v \langle V^0 \rangle + \sqrt{m^{*2} + \vec{p}_{\Delta}^2} \\ &\equiv h_v \langle V^0 \rangle + p_{\Delta}^{*0} \\ &= h_v \langle V^0 \rangle + \sqrt{m^{*2} + \vec{p}_{\Delta}^{*2}}, \\ m^* &\equiv m - h_s \langle \phi \rangle. \end{aligned}$$

*Nucleon:*

$$E(\vec{p}) = \sqrt{\vec{p}^2 + M^{*2}} + g_v \langle V^0 \rangle$$

$$M^* = M - g_s \langle \phi \rangle$$

# Delta in the nuclear medium

- Self energy: real part  $\rightarrow$  spin-orbital coupling in nucleus

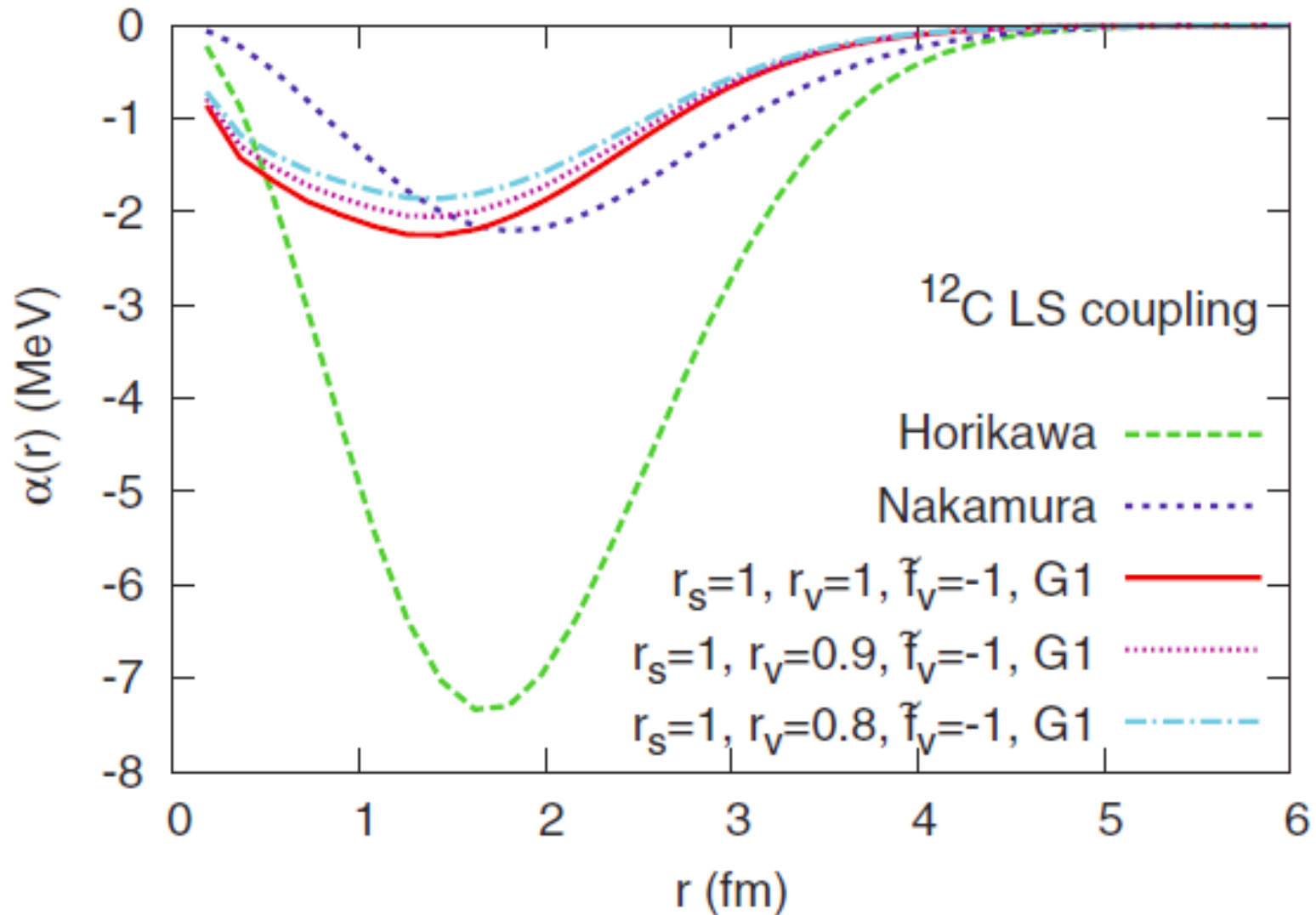
$$\mathcal{L}_{\Delta;\pi,\rho,V,\phi} = \frac{-i\overline{\Delta}^a}{2} \left\{ \sigma^{\mu\nu}, \left( i \tilde{\not{\partial}} - h_\rho \not{\rho} - h_v \not{V} - m + h_s \phi \right) \right\}_a^b \Delta_{bv}$$

$$h_\Delta = \frac{1}{3} \left[ \frac{1}{2\overline{m}^2} \frac{d}{r dr} (h_s \langle \phi \rangle + h_v \langle V^0 \rangle) - \frac{\tilde{f}_v}{m\overline{m}} \frac{d}{r dr} (h_v \langle V^0 \rangle) \right] \vec{S} \cdot \vec{L}$$

$$\equiv \alpha(r) \vec{S} \cdot \vec{L} .$$

$$r_s = h_s / g_s$$

$$r_v = h_v / g_v$$



Y. Horikawa, M. Thies, and F. Lenz, *Nucl.Phys.A* **345**, 386 (1980).

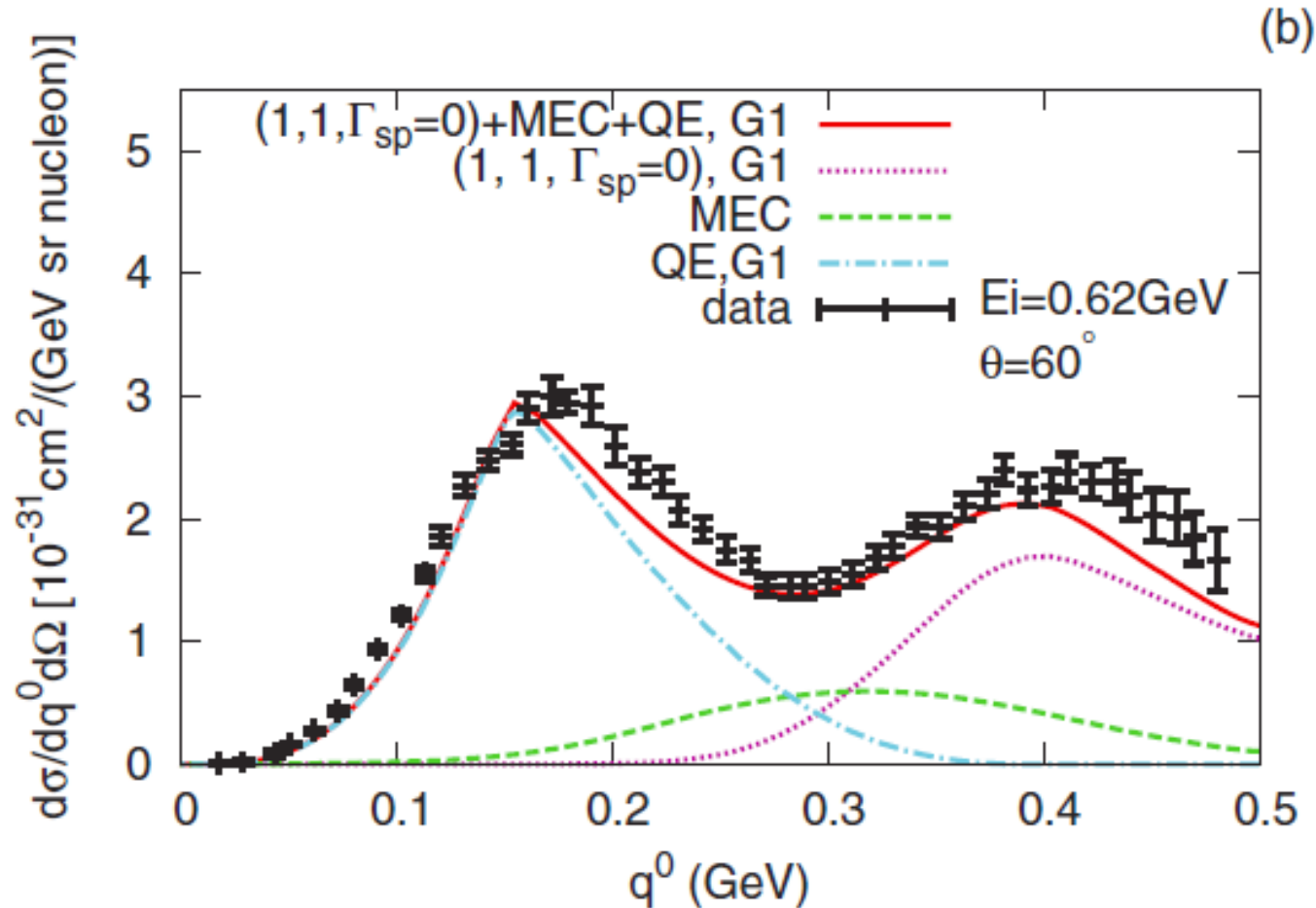
S. X. Nakamura, T. Sato, T.-S. H. Lee, B. Szczerbinska, and K. Kubodera, *Phys.Rev.C* **81**, 035502 (2010).

# Delta in the nuclear medium

- Self energy: real part  $\rightarrow$  spin-orbital coupling in nucleus
- Self energy: imaginary part; collision broadening

$$\Gamma_{\Delta} = \Gamma_{\pi} + \Gamma_{\text{sp}} , \quad [V_0 \approx 80 \text{ MeV}]$$
$$\Gamma_{\text{sp}} = V_0 \times \frac{\rho(r)}{\rho(0)}$$

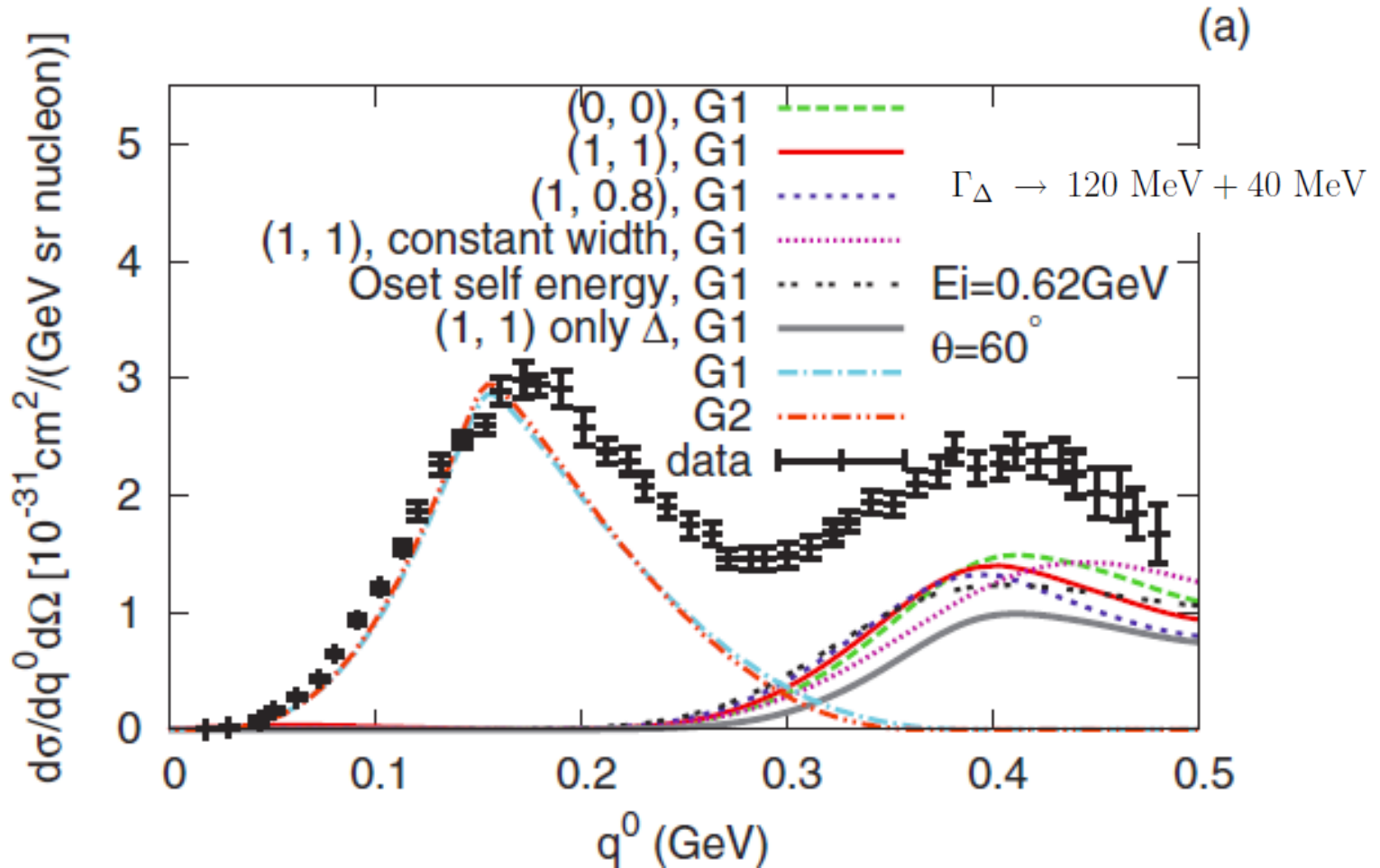
- Check: incoherent electro-production of pion from C12.



*T. W. Donnelly (private communication).*

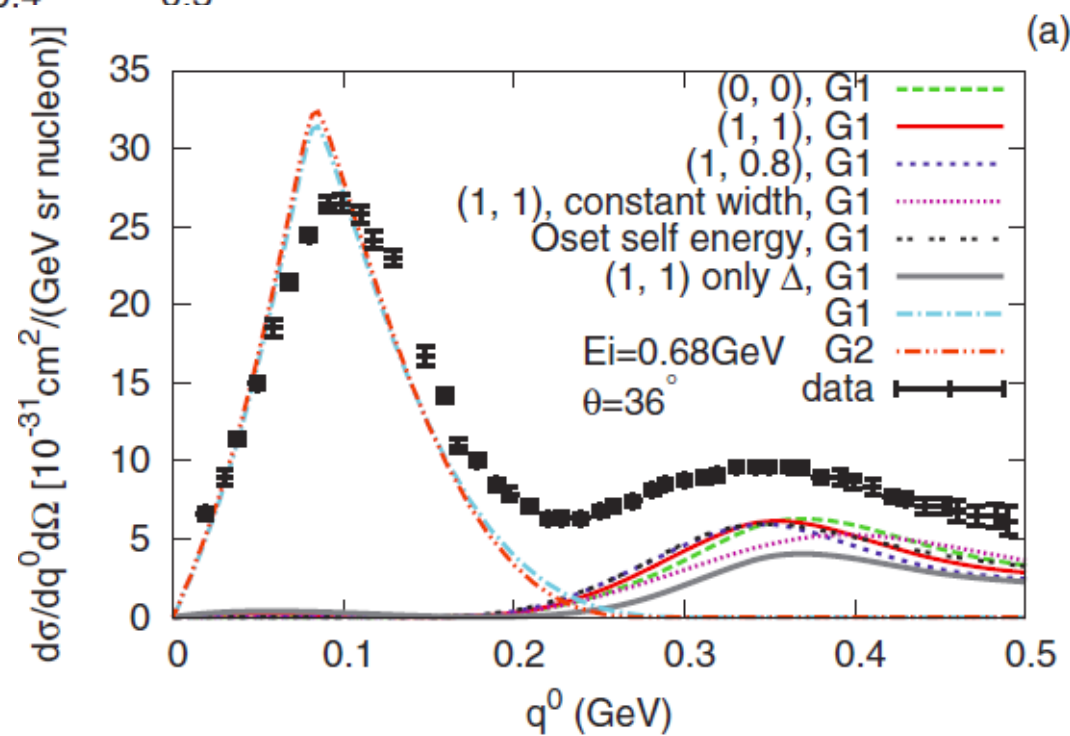
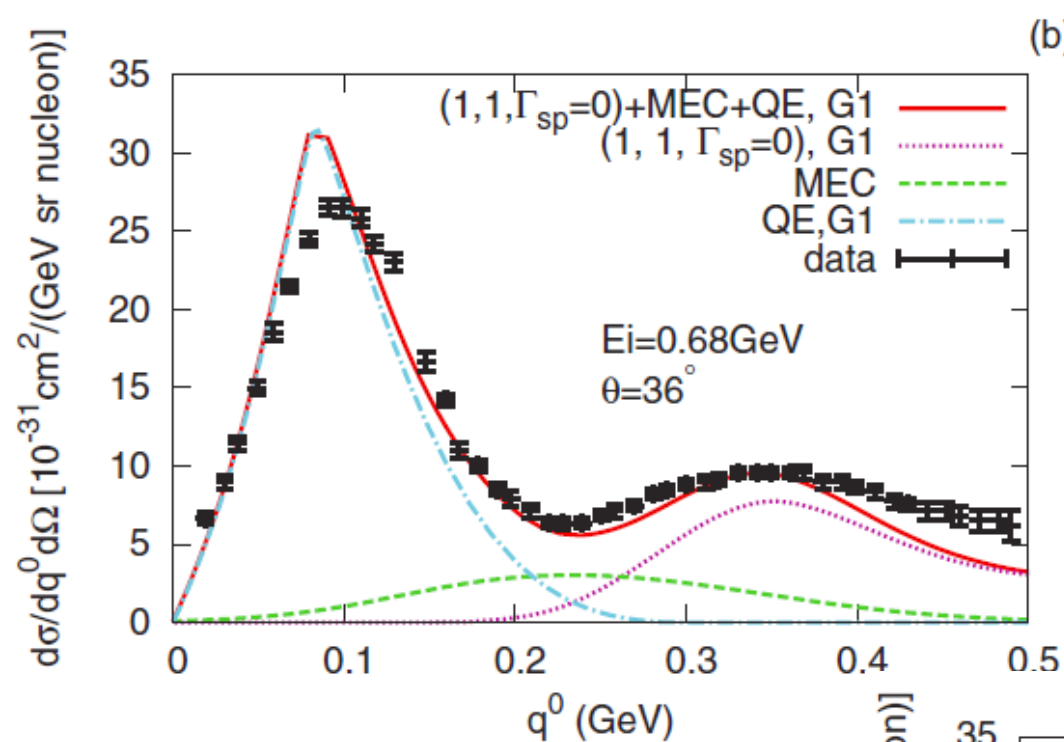
*P. Barreau et al., Nucl.Phys.A **402**, 515 (1983).*

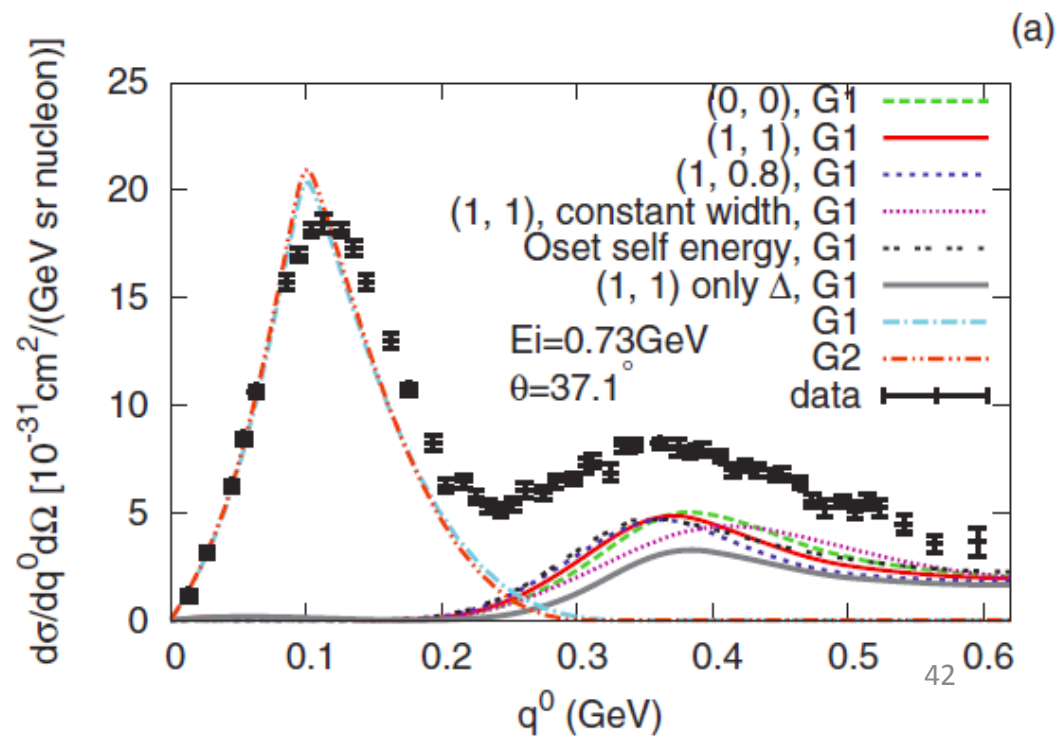
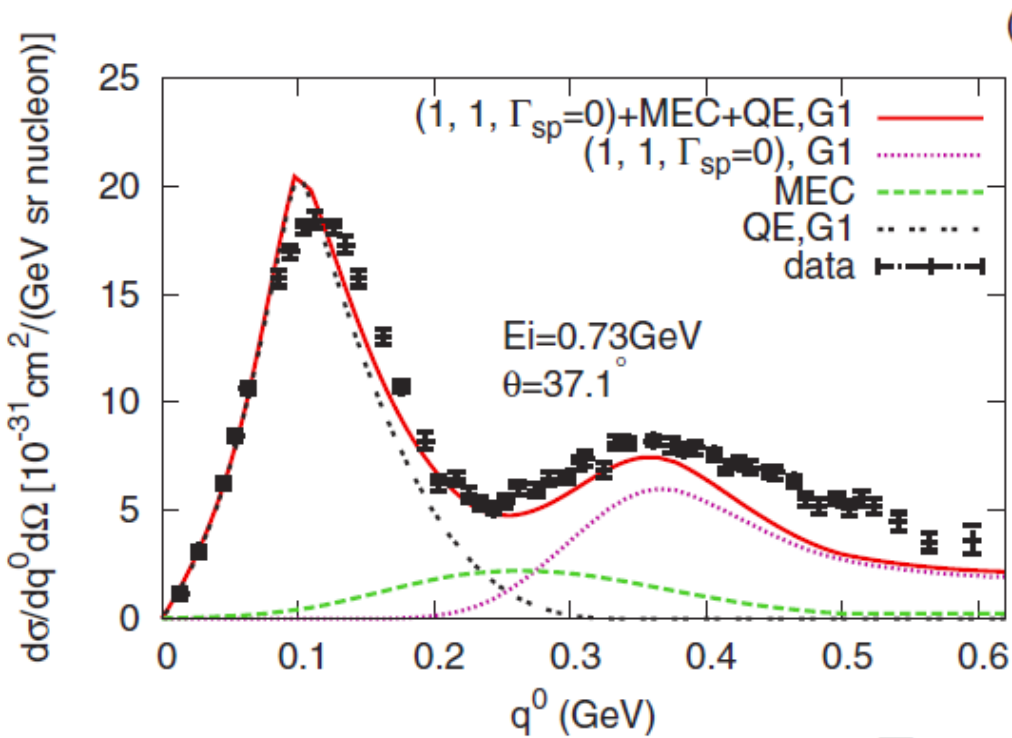
- Check: incoherent electro-production of pion from C12.



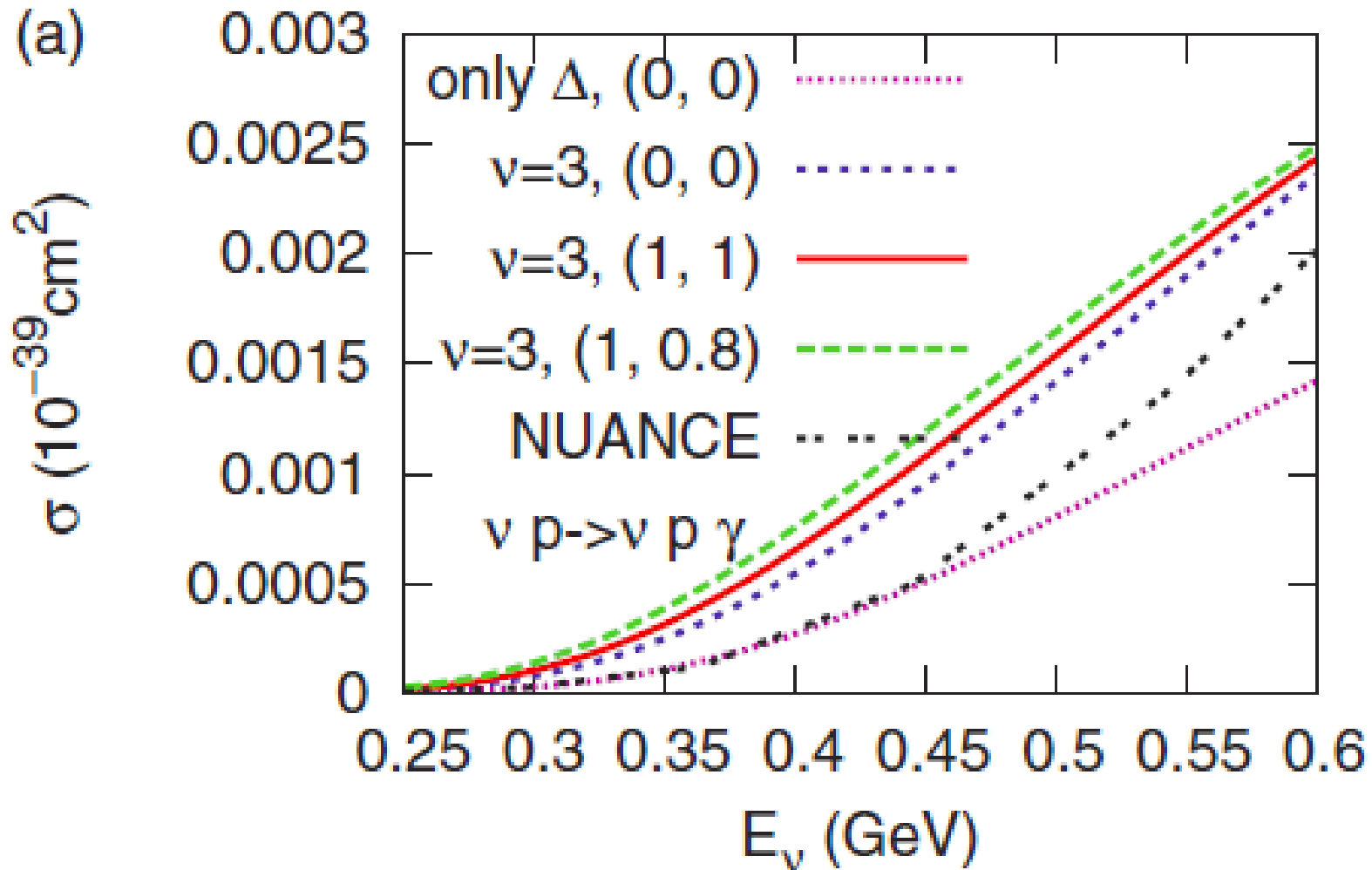
*P. Barreau et al., Nucl.Phys.A* **402**, 515 (1983).



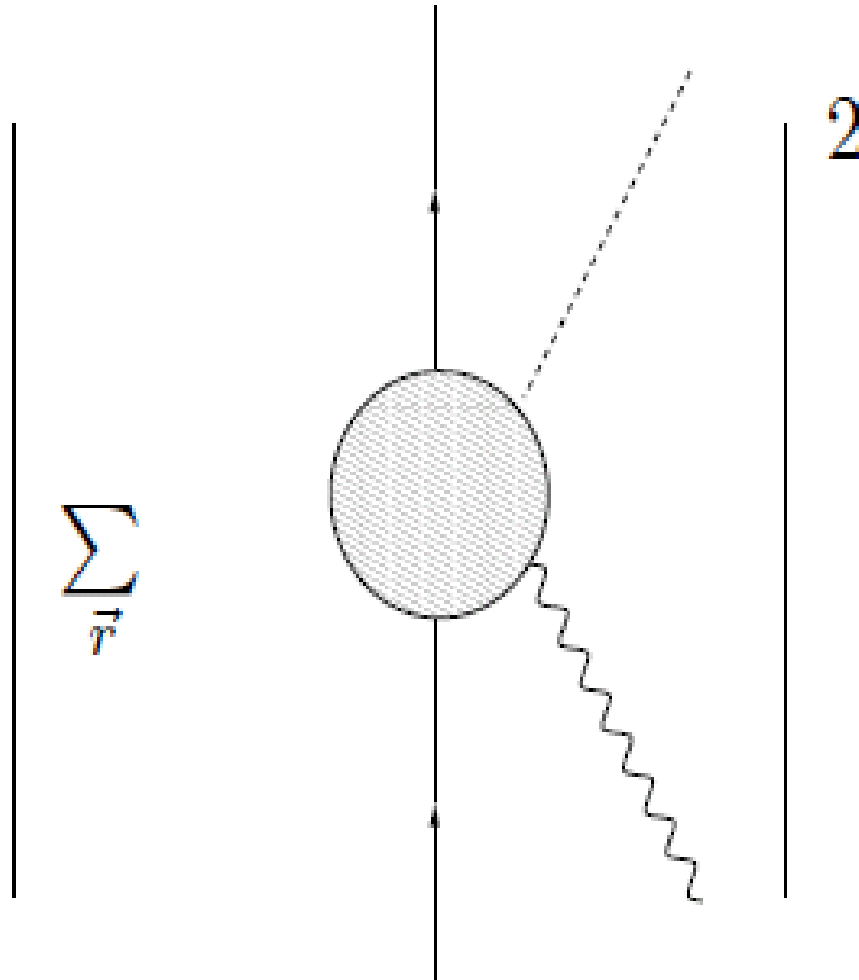




# NC photon



# Coherent production of pion



# Coherent production of pion

- “Optimal” approximation (factorization): ?

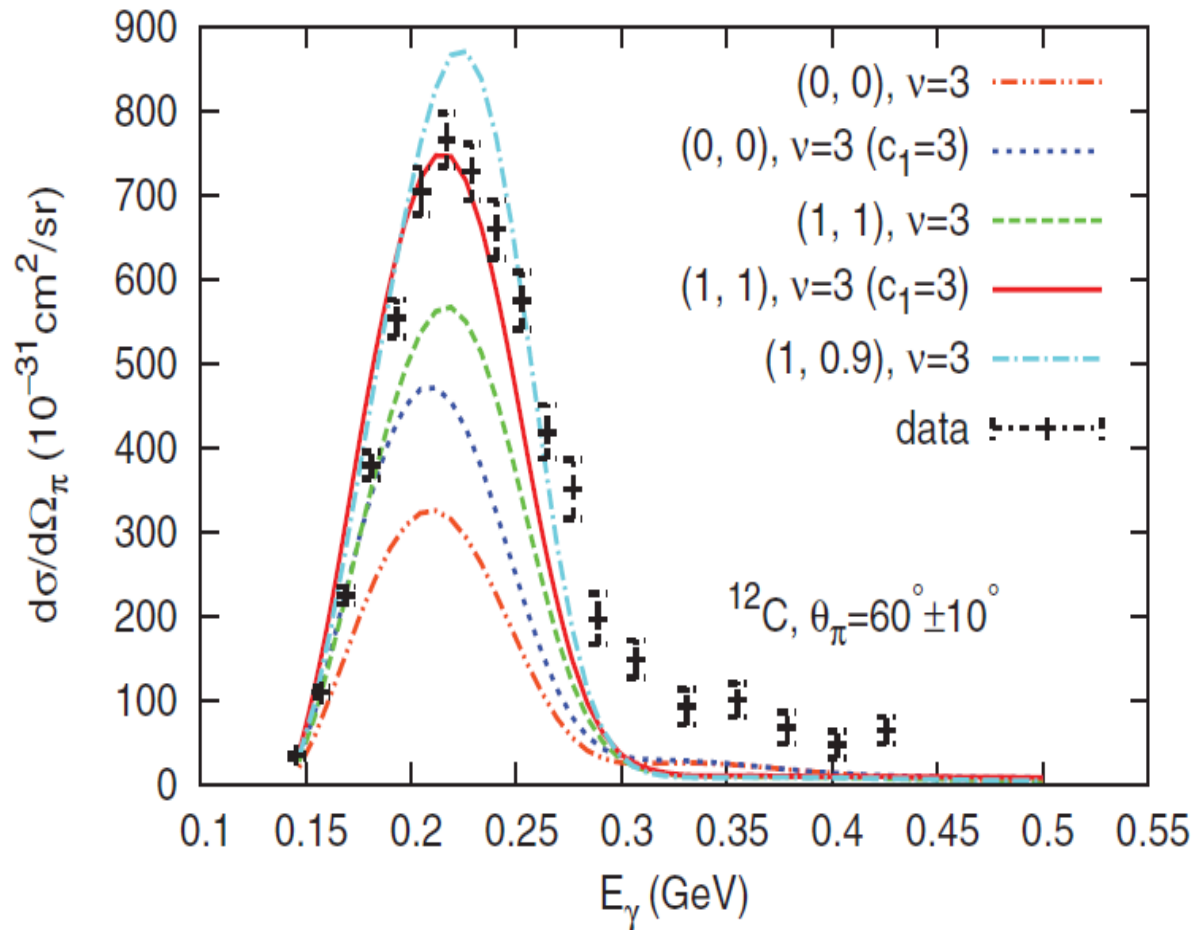
$$\frac{1}{m_A} \langle A, \pi(\vec{k}_\pi) | J_{had}^\mu | A \rangle$$

*X.Z. and B. Serot, PRC 86,  
035504 (2012)  
arXiv:1208.1553)*

$$\approx \begin{cases} \int_A d\vec{r} e^{i(\vec{q}-\vec{k}_\pi)\cdot\vec{r}} \langle J_{had}^\mu(\vec{q}, \vec{k}_\pi, \vec{r}) \rangle & \text{PW,} \\ \int_A d\vec{r} e^{i(\vec{q}-\vec{k}_\pi)\cdot\vec{r}} e^{-i \int_z^\infty \frac{\Pi(\rho, l)}{2|\vec{k}_\pi|} dl} \langle J_{had}^\mu(\vec{q}, \vec{k}_\pi, \vec{r}) \rangle & \text{DW.} \end{cases}$$

$$\begin{aligned} \langle J_{had}^\mu(\vec{q}, \vec{k}_\pi, \vec{r}) \rangle &\approx \rho_n(\vec{r}) \frac{1}{2} \sum_{s_z} \frac{1}{p_{ni}^{*0}} \langle n, s_z, \frac{\vec{q}-\vec{k}_\pi}{2} | J_{had}^\mu(\vec{q}, \vec{k}_\pi) | n, s_z, \frac{\vec{k}_\pi-\vec{q}}{2} \rangle \\ &+ \rho_p(\vec{r}) \frac{1}{2} \sum_{s_z} \frac{1}{p_{pi}^{*0}} \langle p, s_z, \frac{\vec{q}-\vec{k}_\pi}{2} | J_{had}^\mu(\vec{q}, \vec{k}_\pi) | p, s_z, \frac{\vec{k}_\pi-\vec{q}}{2} \rangle . \end{aligned}$$

- photo-production of pions from C12.



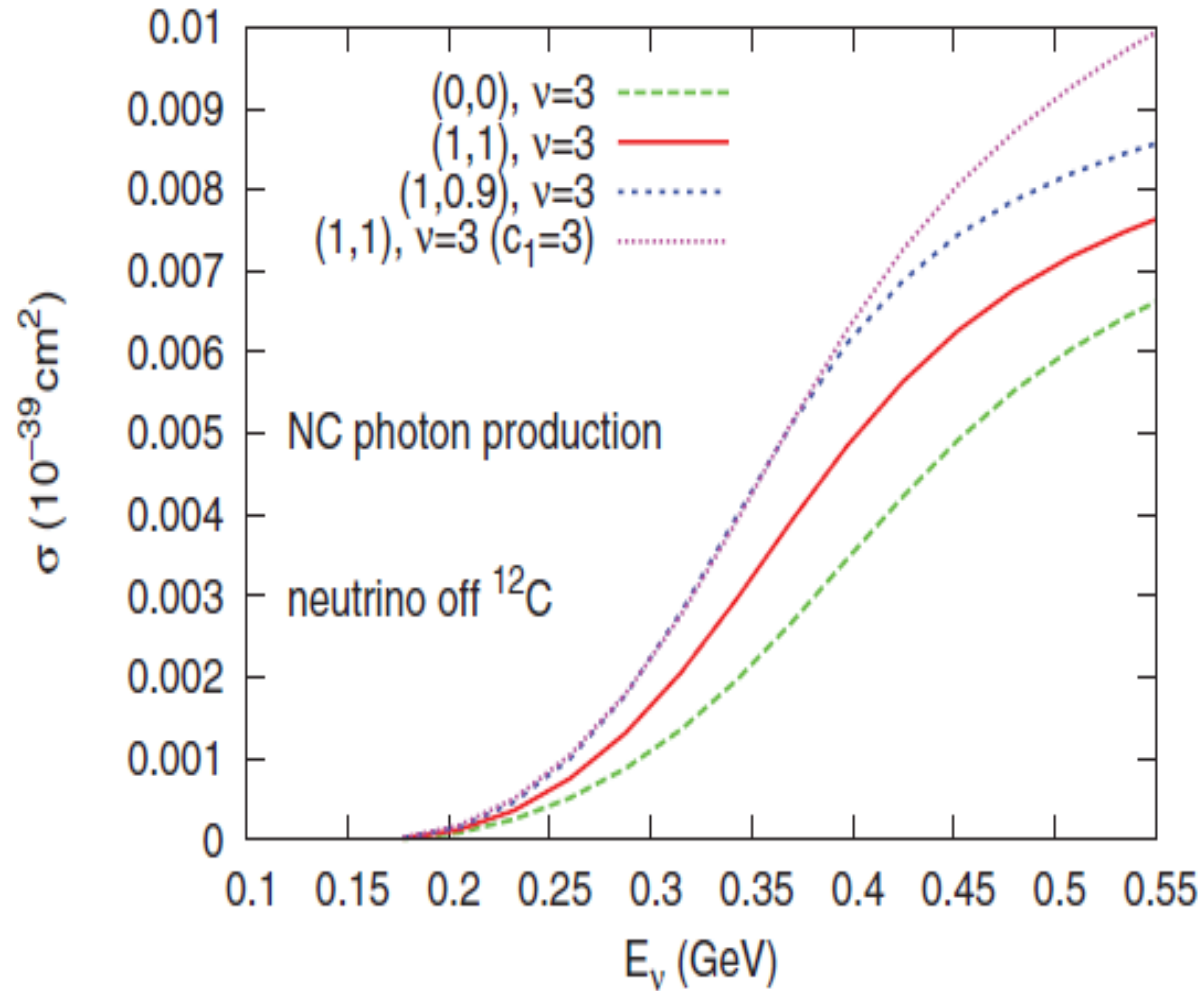
*Also related to pion  
electro(photo)-production*

$$\frac{c_1}{M^2} \bar{N} \gamma^{\mu} N \text{Tr}(\tilde{a}^{\nu} \bar{F}_{\mu\nu}^{(+)}), \quad \frac{e_1}{M^2} \bar{N} \gamma^{\mu} \tilde{a}^{\nu} N \bar{f}_{s\mu\nu}$$

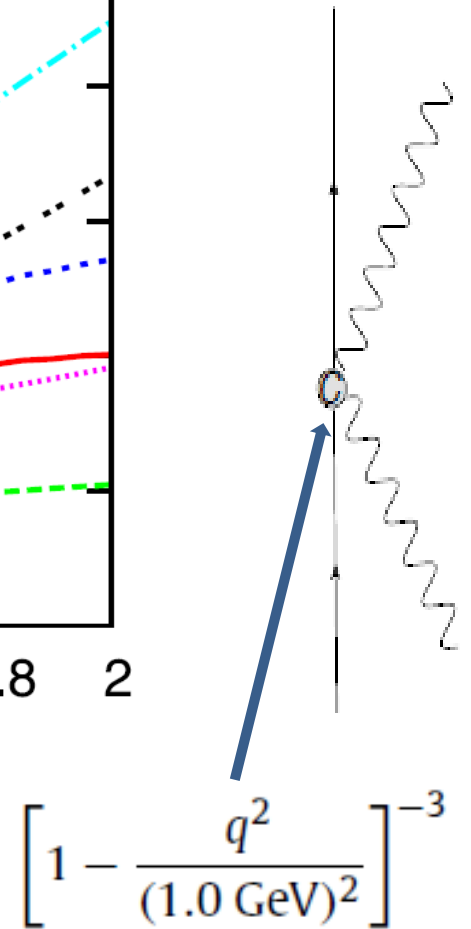
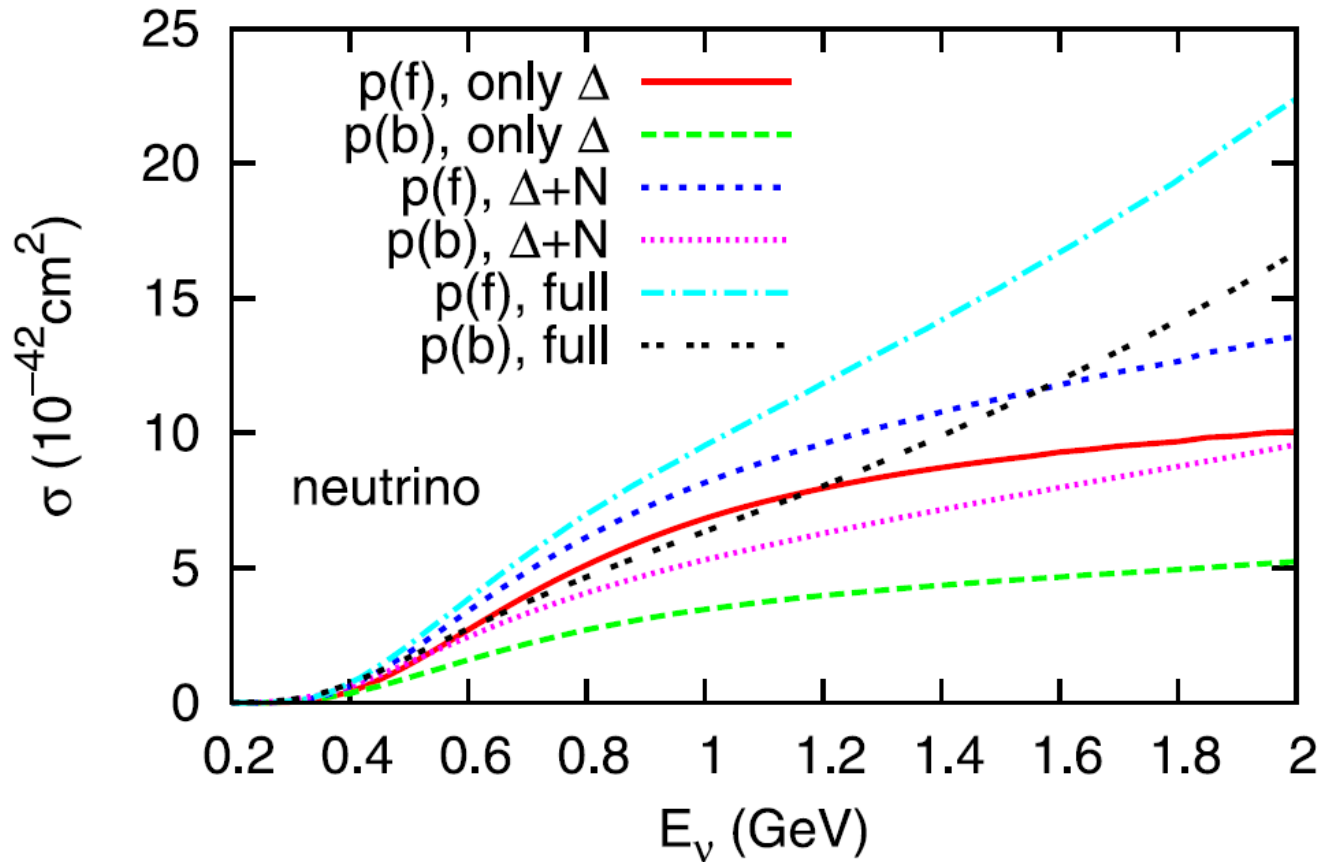
*M. Schmitz, Ph.D. thesis, Johannes Gutenberg Universität Mainz, 1996.*

*W. Peters, H. Lenske, and U. Mosel, NPA **640**, 89 (1998).*

# Coherent NC photon



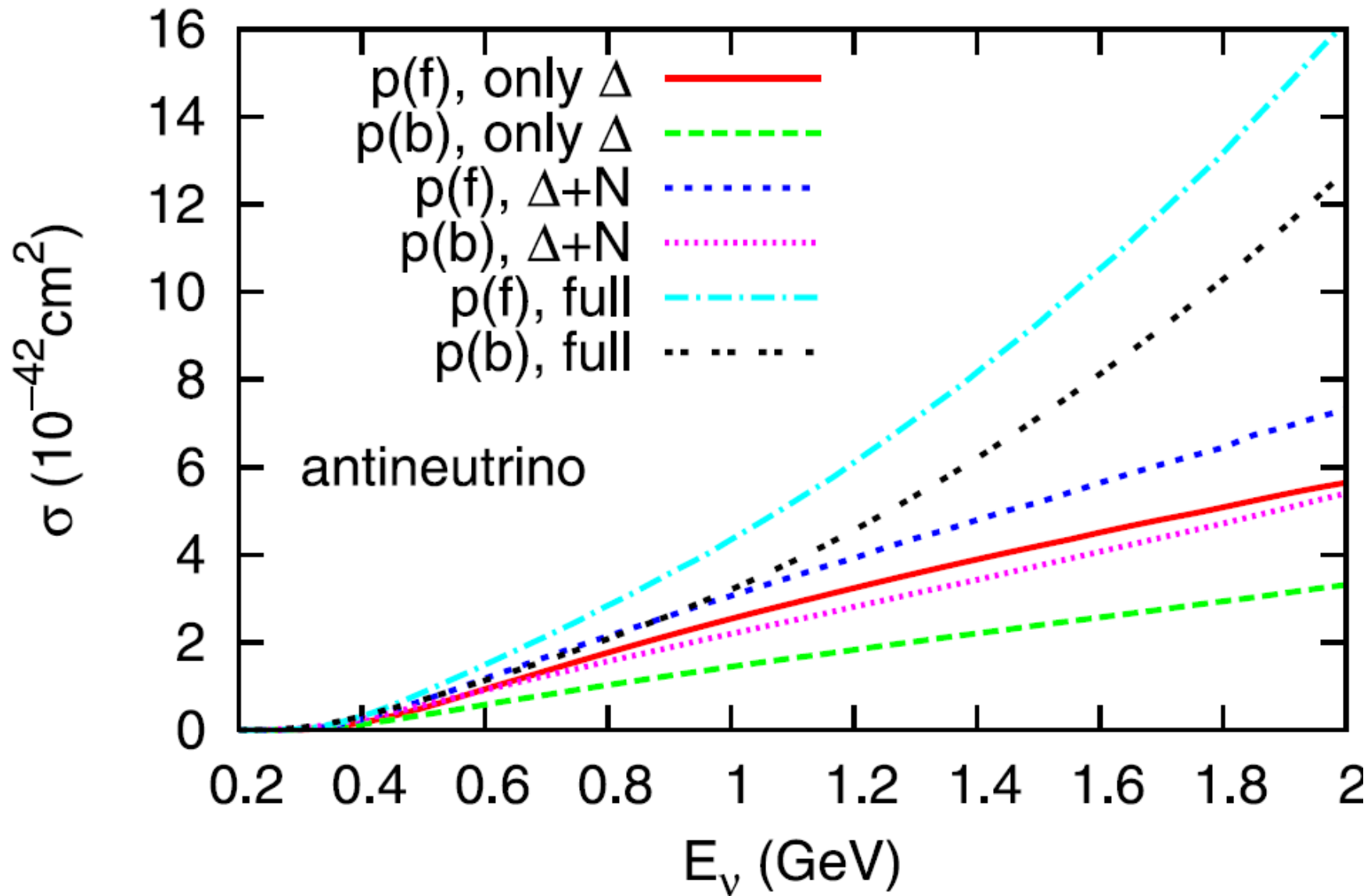
# MiniBooNE NC photon

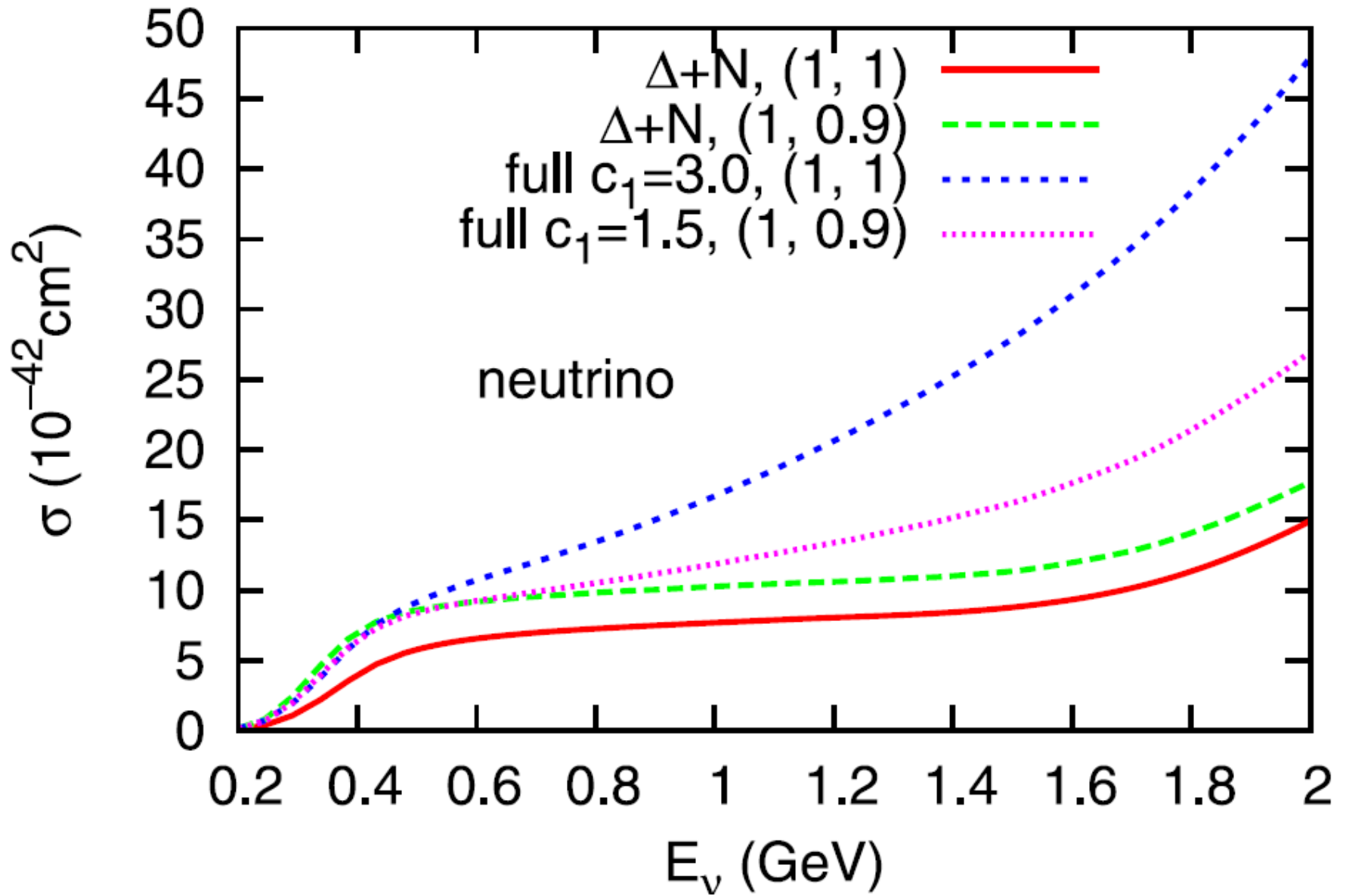


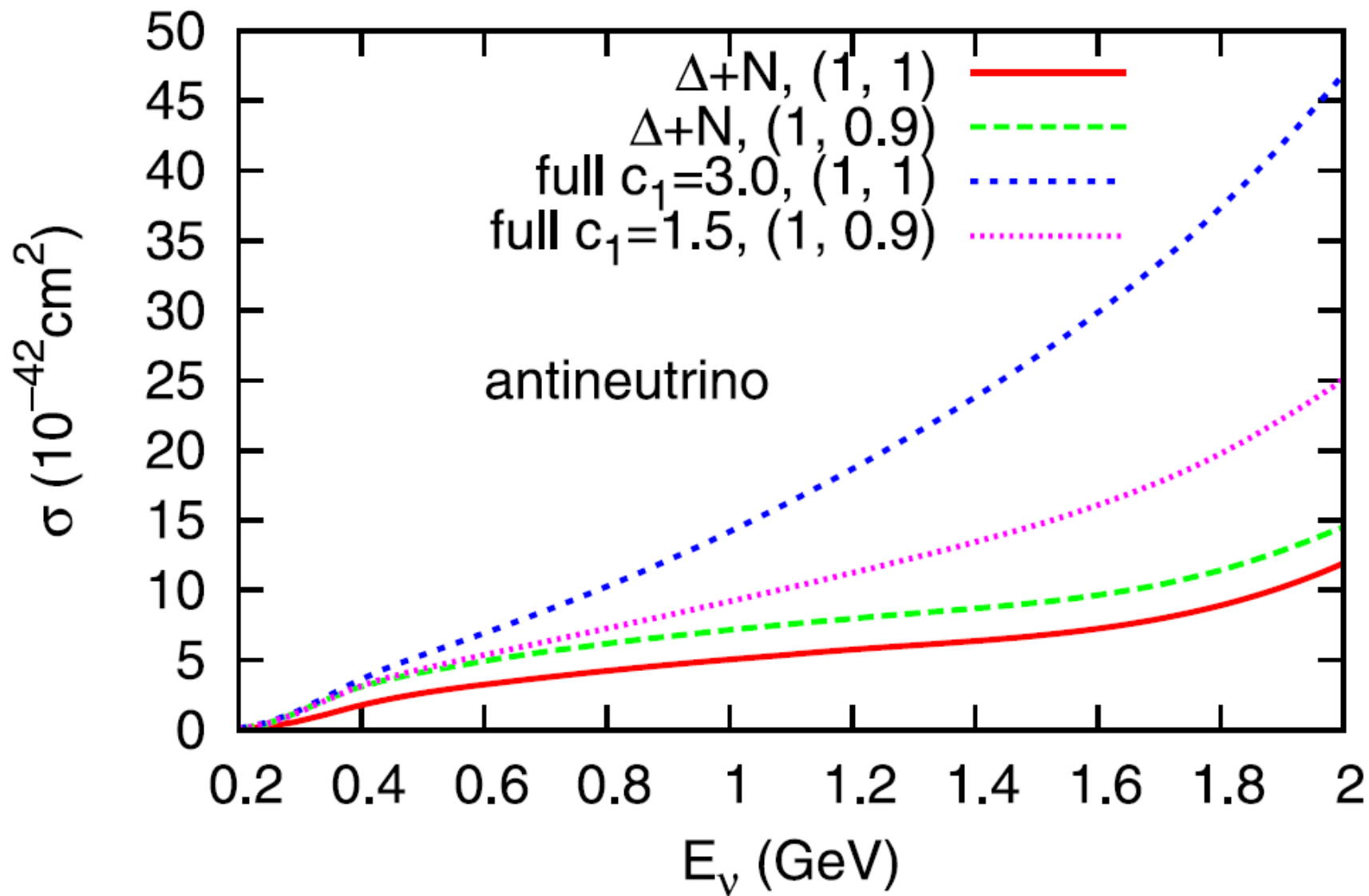
K. Graczyk, D. Kietczewska, P. Przewłocki, and J. Sobczyk, *Phys.Rev.D* **80**, 093001 (2009).

X.Z. and B. Serot, *Phys.Lett.B* 719, 409 (2013) (arXiv: 1210.3210)









# MiniBooNE NC photon events

$E_{QE}$ (GeV)	[0.2, 0.3]	[0.3, 0.475]	[0.475, 1.25]
coh	1.5 (2.9)	6.0 (9.2)	2.1 (8.0)
inc	12.0 (14.1)	25.5 (31.1)	12.6 (23.2)
H	4.1 (4.4)	10.6 (11.6)	4.6 (6.3)
Total	17.6 (21.4)	42.1 (51.9)	19.3 (37.5)
MiniBN	19.5	47.3	19.4
Excess	$42.6 \pm 25.3$	$82.2 \pm 23.3$	$21.5 \pm 34.9$

$E_{QE}$ (GeV)	[0.2, 0.3]	[0.3, 0.475]	[0.475, 1.25]
coh	1.0 (2.2)	3.1 (5.5)	0.87 (5.4)
inc	4.5 (5.3)	10.0 (12.2)	4.0 (10.2)
H	1.3 (1.6)	3.6 (4.3)	1.1 (2.4)
Total	6.8 (9.1)	16.7 (22.0)	6.0 (18.0)
MiniBN	8.8	16.9	6.8
Excess	$34.6 \pm 13.6$	$23.5 \pm 13.4$	$20.2 \pm 22.8$

Xection  
needs to be  
doubled at  
least.

# MiniBooNE NC photon events

$E_\gamma$ (GeV)	coh	inc	H	Total	MiniBN	Excess
[0.1 , 0.2]	0.72 (1.5)	14.0 (15.0)	4.4 (4.6)	19.1 (21.1)	10.6	52.5
[0.2 , 0.3]	3.2 (5.5)	22.7 (25.2)	7.8 (8.5)	33.7 (39.2)	32.5	61.2
[0.3 , 0.4]	3.7 (5.4)	12.7 (15.0)	5.0 (5.6)	21.4 (26.0)	24.7	58.4
[0.4 , 0.5]	1.0 (1.7)	5.4 (7.3)	2.1 (2.4)	8.5 (11.4)	12.7	-9.7
[0.5 , 0.6]	0.32 (1.0)	2.3 (3.9)	0.75 (1.0)	3.4 (5.9)	4.4	10.5

$E_\gamma$ (GeV)	coh	inc	H	Total	MiniBN	Excess
[0.1 , 0.2]	0.55 (1.2)	4.9 (5.5)	1.4 (1.6)	6.9 (8.3)	4.3	18.8
[0.2 , 0.3]	2.0 (3.8)	8.7 (10.3)	2.9 (3.3)	13.6 (17.4)	14.3	22.6
[0.3 , 0.4]	1.8 (3.0)	4.0 (5.4)	1.5 (1.8)	7.3 (10.2)	9.1	11.5
[0.4 , 0.5]	0.36 (1.0)	1.3 (2.6)	0.43 (0.66)	2.1 (4.3)	3.6	18.7
[0.5 , 0.6]	0.10 (0.72)	0.51 (1.7)	0.14 (0.36)	0.75 (2.8)	1.1	8.4

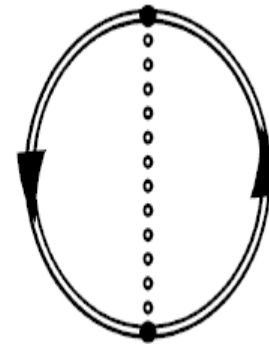
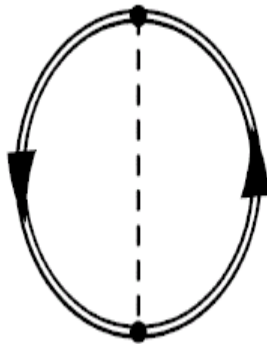
# Summary & questions

- QHD → bound state, electroweak currents, pion dynamics, baryon spectrum modification
- Reaction kernel, medium modification, and approximation schemes
- NC photon event at MiniBooNE
- Axial transition form factors?
- The contact term: couplings and form factors?
- LFG and optimal factorization?
- Experimental measurement?
- Would photon production help constrain FSIs?

# Back up

# Where Are the Pions?

- For nuclear equation of state (EOS), 1- and 2-loop calculations (including pion) are done by Y. Hu, J. McIntire, and B. Serot (NPA 794:187, 2007); Infrared Regularization.





# Spin-3/2 Particle in EFT

- Redundant degrees of freedom in Rarita-Schwinger representation ( $\psi^\mu$ ) do NOT show up.

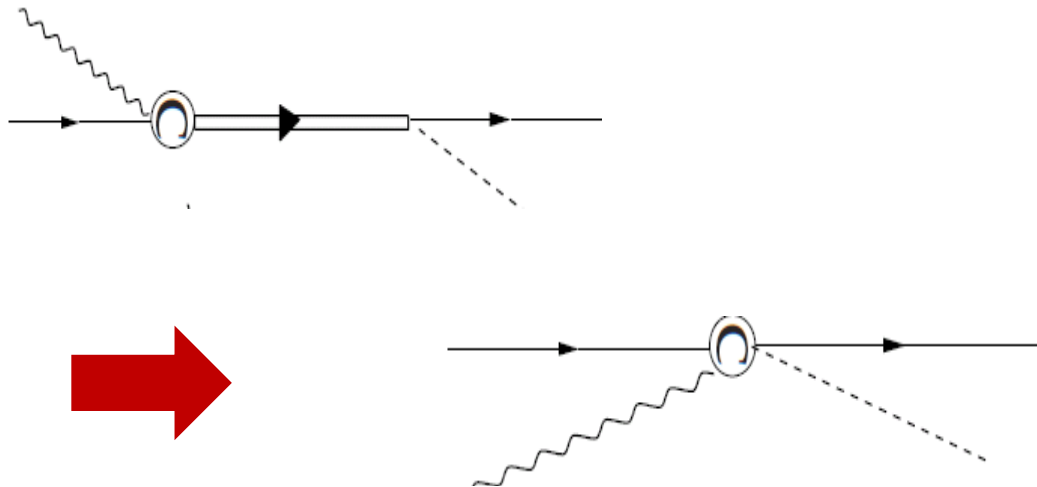
$$\begin{aligned}
 S_F &= (S_F^{0(\frac{3}{2})} + S_F^{0(\frac{3}{2}\perp)}) + (S_F^{0(\frac{3}{2})} + S_F^{0(\frac{3}{2}\perp)})(\Sigma^{(\frac{3}{2})} + \Sigma^{(\frac{3}{2}\perp)})(S_F^{0(\frac{3}{2})} + S_F^{0(\frac{3}{2}\perp)}) + \dots \\
 &= S_F^{0(\frac{3}{2})} + S_F^{0(\frac{3}{2})}\Sigma^{(\frac{3}{2})}S_F^{0(\frac{3}{2})} + \dots \\
 &\quad + S_F^{0(\frac{3}{2}\perp)} + S_F^{0(\frac{3}{2}\perp)}\Sigma^{(\frac{3}{2}\perp)}S_F^{0(\frac{3}{2}\perp)} + \dots
 \end{aligned}$$

*This can be generalized to other spins*

Related work: V. Pascalutsa, PRD 58: 096002, 1998; V. P and D. Phillips, PRC 67: 055202, 2003; H. Krebs, E. Epelbaum, and U. Meissner, PRC 80: 028201, 2009; PLB 683: 222, 2010

# Spin-3/2 Particle in EFT

- Redundant degrees of freedom in Rarita-Schwinger representation ( $\psi^\mu$ ) do NOT show up.
- Off-shell couplings  $\gamma_\mu \psi^\mu$ ,  $\partial_\mu \psi^\mu$ ,  $\bar{\psi}^\mu \gamma_\mu$ , and  $\partial_\mu \bar{\psi}^\mu$



A quick look:

$$SU(2)_L \otimes SU(2)_R \otimes U(1)_B$$

- Chiral symmetry in QCD:

$$\mathcal{L} = \mathcal{L}_{\text{QCD}} + \bar{q} \gamma_\mu (v^\mu + B v_{(s)}^\mu + \gamma_5 a^\mu) q - \bar{q} (s - i \gamma_5 p) q$$

$$q_{LA} \rightarrow \exp \left[ -i \frac{\theta(x)}{3} \right] \left( \exp \left[ -i \theta_{Li}(x) \frac{\tau^i}{2} \right] \right)_A^B q_{LB} \equiv \exp \left[ -i \frac{\theta(x)}{3} \right] (L)_A^B q_{LB},$$

$$q_R \rightarrow \exp \left[ -i \frac{\theta(x)}{3} \right] \exp \left[ -i \theta_{Ri}(x) \frac{\tau^i}{2} \right] q_R \equiv \exp \left[ -i \frac{\theta(x)}{3} \right] R q_R,$$

# A quick look:

- Chiral symmetry in QCD:
- Its nonlinear realization at low

$$U \equiv \exp \left[ 2i \frac{\pi_i(x)}{f_\pi} t^i \right] \rightarrow LUR^\dagger,$$

$$\xi \equiv \sqrt{U} = \exp \left[ i \frac{\pi_i}{f_\pi} t^i \right] \rightarrow L\xi h^\dagger = h \xi R^\dagger,$$

$$\tilde{v}_\mu \equiv \frac{-i}{2} [\xi^\dagger (\partial_\mu - il_\mu) \xi + \xi (\partial_\mu - ir_\mu) \xi^\dagger] \equiv \tilde{v}_{i\mu} t^i \rightarrow h \tilde{v}_\mu h^\dagger - ih \partial_\mu h^\dagger,$$

$$\tilde{a}_\mu \equiv \frac{-i}{2} [\xi^\dagger (\partial_\mu - il_\mu) \xi - \xi (\partial_\mu - ir_\mu) \xi^\dagger] \equiv \tilde{a}_{i\mu} t^i \rightarrow h \tilde{a}_\mu h^\dagger,$$

# A quick look:

- Chiral symmetry in QCD:
- Its nonlinear realization at low

$$(\tilde{\partial}_\mu \psi)_\alpha \equiv (\partial_\mu + i \tilde{v}_\mu - i v_{(s)\mu} B)_\alpha^\beta \psi_\beta \rightarrow \exp[-i\theta(x)B] h_\alpha^\beta (\tilde{\partial}_\mu \psi)_\beta ,$$

$$\tilde{v}_{\mu\nu} \equiv -i[\tilde{a}_\mu, \tilde{a}_\nu] \rightarrow h \tilde{v}_{\mu\nu} h^\dagger ,$$

$$F_{\mu\nu}^{(+)} \equiv \tilde{\zeta}^\dagger f_{L\mu\nu} \tilde{\zeta} + \tilde{\zeta} f_{R\mu\nu} \tilde{\zeta}^\dagger \rightarrow h F_{\mu\nu}^{(+)} h^\dagger ,$$

$$F_{\mu\nu}^{(-)} \equiv \tilde{\zeta}^\dagger f_{L\mu\nu} \tilde{\zeta} - \tilde{\zeta} f_{R\mu\nu} \tilde{\zeta}^\dagger \rightarrow h F_{\mu\nu}^{(-)} h^\dagger ,$$

# A quick look:

- ...
- The lagrangian, baryon section:

$$\mathcal{L}_{N(\hat{v} \leq 3)} = \bar{N} (i \gamma^\mu [\tilde{\partial}_\mu + i g_\rho \rho_\mu + i g_v V_\mu] + g_A \gamma^\mu \gamma^5 \tilde{a}_\mu - M + g_s \phi) N$$

$$\mathcal{L}_\Delta = \frac{-i}{2} \bar{\Delta}_\mu^a \{ \sigma^{\mu\nu}, (i \tilde{\not{\partial}} - h_\rho \not{\rho} - h_v \not{V} - m + h_s \phi) \}_a^b \Delta_{b\nu}$$

$$\mathcal{L}_{\Delta, N, \pi} = h_A \bar{\Delta}^{a\mu} T_a^{\dagger iA} \tilde{a}_{i\mu} N_A + \text{c.c.},$$

# A quick look:

- ...
- The lagrangian, baryon section.
- The lagrangian, meson

$$\begin{aligned}\mathcal{L}_{\text{meson}(\hat{v} \leq 4)} = & \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{4} f_\pi^2 \text{Tr}[\tilde{\partial}_\mu U (\tilde{\partial}^\mu U)^\dagger] \\ & + \frac{1}{4} f_\pi^2 m_\pi^2 \text{Tr}(U + U^\dagger - 2) \\ & - \frac{1}{2} \text{Tr}(\rho_{\mu\nu} \rho^{\mu\nu}) - \frac{1}{4} V^{\mu\nu} V_{\mu\nu} \\ & + \frac{1}{2g_\gamma} \left( \text{Tr}(F^{(+)\mu\nu} \rho_{\mu\nu}) + \frac{1}{3} f_s^{\mu\nu} V_{\mu\nu} \right).\end{aligned}$$

*Vector  
meson  
dominanc  
e (VMD)*

# A quick look:

- Chiral symmetry in QCD
- The lagrangian, baryon section.
- The lagrangian, meson

$$\begin{aligned}\mathcal{L}_{\text{meson}(\hat{v} \leq 4)} &= \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{4} f_\pi^2 \text{Tr}[\tilde{\partial}_\mu U (\tilde{\partial}^\mu U)^\dagger] \\ &+ \frac{1}{4} f_\pi^2 m_\pi^2 \text{Tr}(U + U^\dagger - 2) \\ &- \frac{1}{2} \text{Tr}(\rho_{\mu\nu} \rho^{\mu\nu}) - \frac{1}{4} V^{\mu\nu} V_{\mu\nu} \\ &+ \frac{1}{2g_\gamma} \left( \text{Tr}(F^{(+)\mu\nu} \rho_{\mu\nu}) + \frac{1}{3} f_s^{\mu\nu} V_{\mu\nu} \right).\end{aligned}$$

*Vector  
meson  
dominanc  
e (VMD)*



# A quick look:

- ...

- **Electroweak (EW)**

$$l_\mu = -e \frac{\tau^0}{2} A_\mu + \frac{g}{\cos \theta_w} \sin^2 \theta_w \frac{\tau^0}{2} Z_\mu$$

$$- \frac{g}{\cos \theta_w} \frac{\tau^0}{2} Z_\mu - g V_{ud} \left( W_\mu^{+1} \frac{\tau_{+1}}{2} + W_\mu^{-1} \frac{\tau_{-1}}{2} \right) ,$$

$$r_\mu = -e \frac{\tau^0}{2} A_\mu + \frac{g}{\cos \theta_w} \sin^2 \theta_w \frac{\tau^0}{2} Z_\mu ,$$

$$v_{(s)\mu} = -e \frac{1}{2} A_\mu + \frac{g}{\cos \theta_w} \sin^2 \theta_w \frac{1}{2} Z_\mu .$$

# A quick look:

- ...
- Electroweak (EW) interactions:

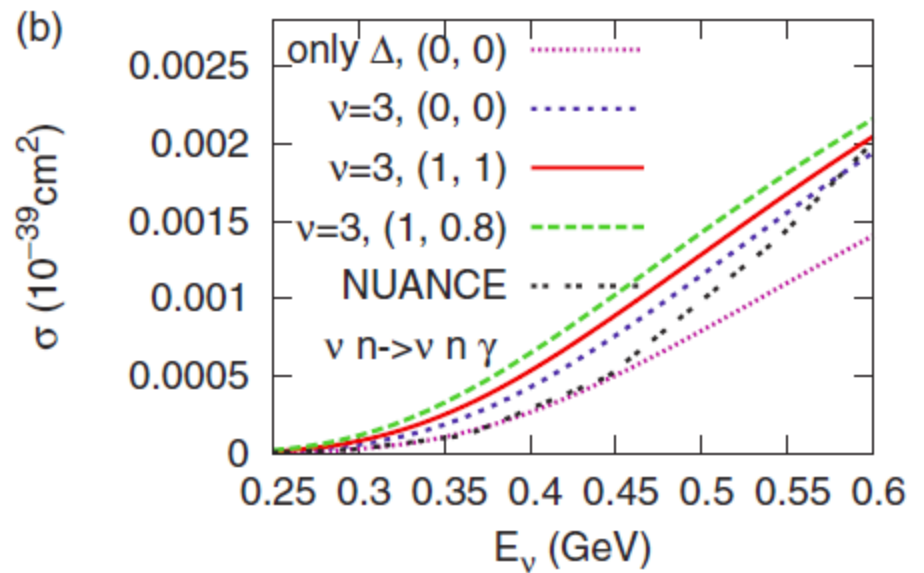
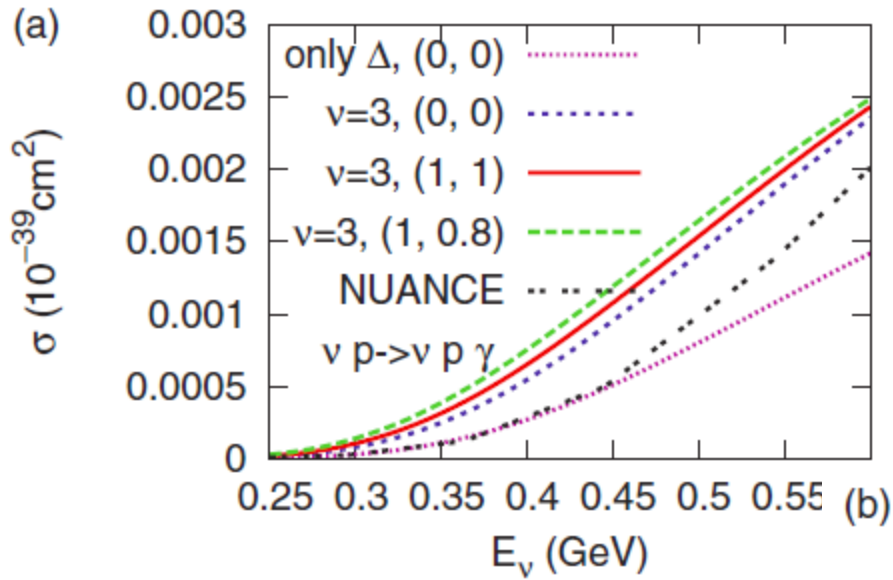
$$\begin{aligned} \mathcal{L}_{\Delta,N,\text{bg}} = & \frac{ic_{1\Delta}}{M} \bar{\Delta}_\mu^a \gamma_\nu \gamma^5 T_a^{\dagger iA} F_i^{(+)\mu\nu} N_A + \frac{ic_{3\Delta}}{M^2} \bar{\Delta}_\mu^a i\gamma^5 T_a^{\dagger iA} (\partial_\nu F^{(+)\mu\nu})_i N_A + \frac{c_{6\Delta}}{M^2} \bar{\Delta}_\lambda^a \sigma_{\mu\nu} T_a^{\dagger iA} (\partial^\lambda \bar{F}^{(+)\mu\nu})_i N_A \\ & - \frac{d_{2\Delta}}{M^2} \bar{\Delta}_\mu^a T_a^{\dagger iA} (\partial_\nu F^{(-)\mu\nu})_i N_A - \frac{id_{4\Delta}}{M} \bar{\Delta}_\mu^a \gamma_\nu T_a^{\dagger iA} F_i^{(-)\mu\nu} N_A - \frac{id_{7\Delta}}{M^2} \bar{\Delta}_\lambda^a \sigma_{\mu\nu} T_a^{\dagger iA} (\partial^\lambda F^{(-)\mu\nu})_i N_A + \text{c.c.}, \end{aligned}$$

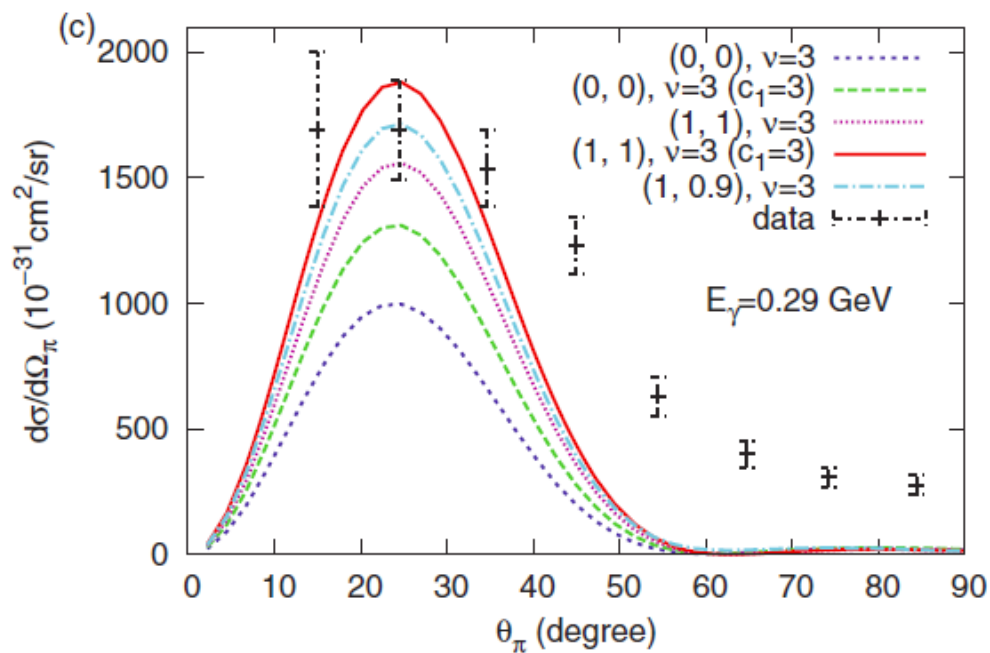
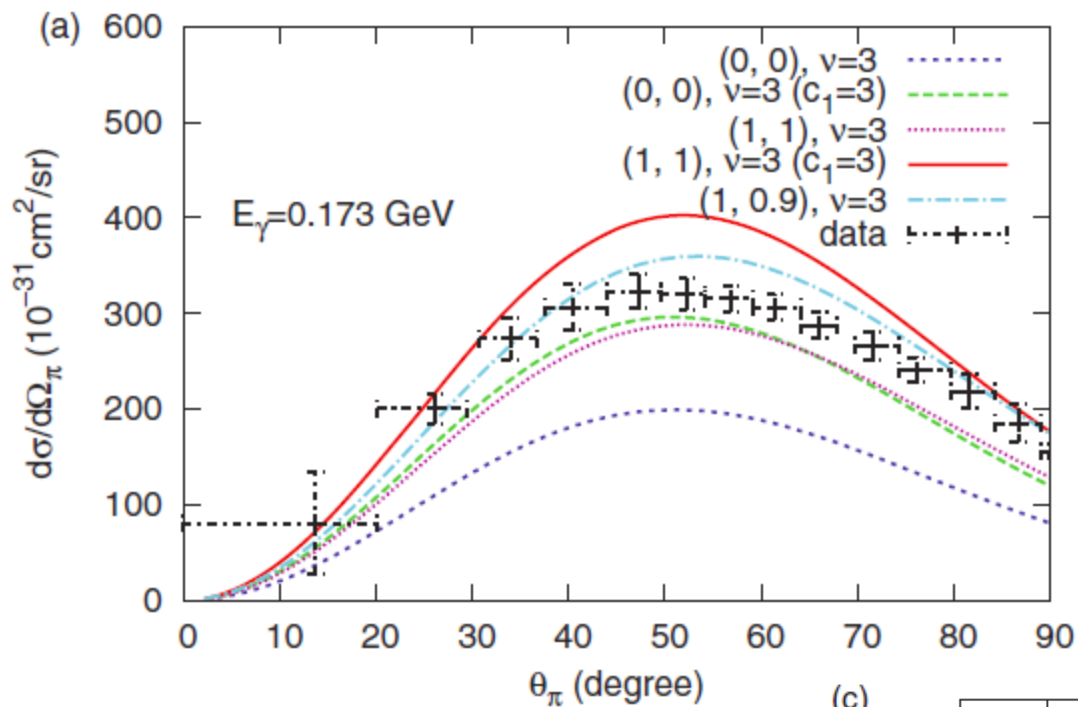
$$\mathcal{L}_{\Delta,N,\rho} = \frac{ic_{1\Delta\rho}}{M} \bar{\Delta}_\mu^a \gamma_\nu \gamma^5 T_a^{\dagger iA} \rho_i^{\mu\nu} N_A + \frac{ic_{3\Delta\rho}}{M^2} \bar{\Delta}_\mu^a i\gamma^5 T_a^{\dagger iA} (\partial_\nu \rho^{\mu\nu})_i N_A + \frac{c_{6\Delta\rho}}{M^2} \bar{\Delta}_\lambda^a \sigma_{\mu\nu} T_a^{\dagger iA} (\partial^\lambda \bar{\rho}^{\mu\nu})_i N_A + \text{c.c.}.$$

# A quick look (recap)

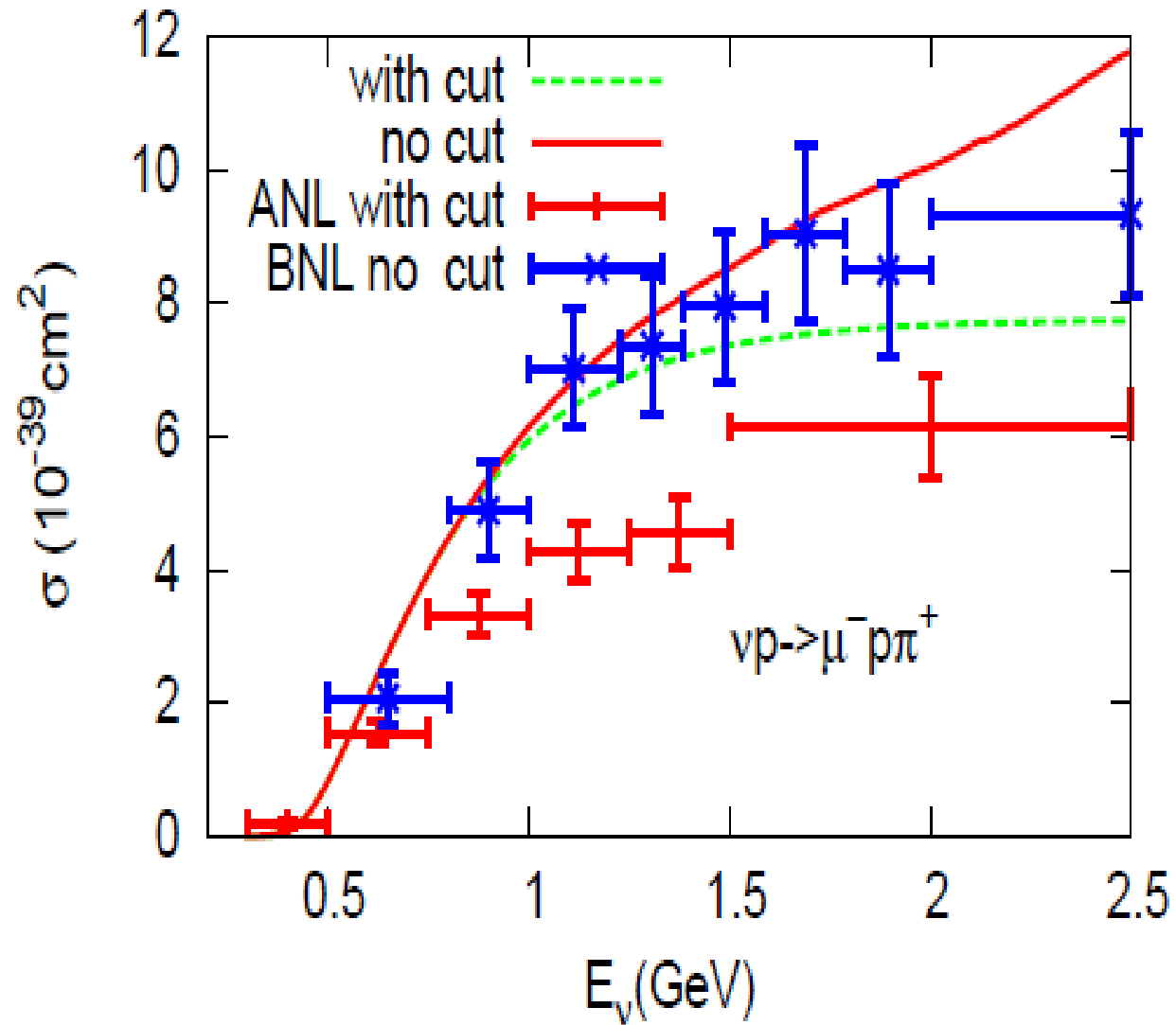
- Chiral symmetry
- The lagrangian
- Electroweak interactions (CVC and PCAC)

# NC photon

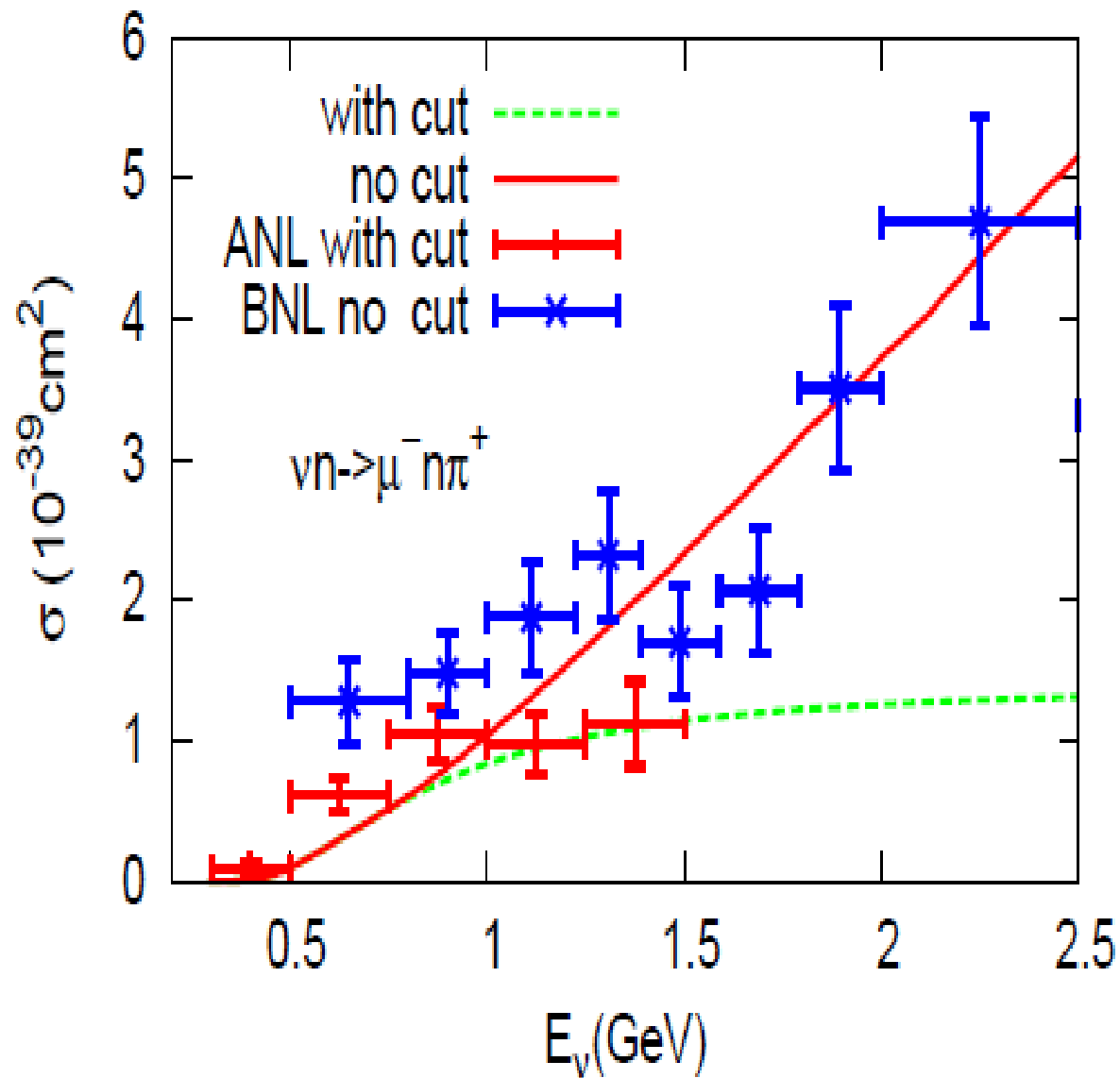




# Benchmarks



# Benchmarks



# Benchmarks

