# *Is a Better Universal Model of CCQE in Sight?*

INT
Seattle, WA
Dec 2013

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# Outline

- **Why a better model of CCQE is desirable**

- **Shortcomings of some present models**
  **list and evidence**

- **Principle Difficulty – treatment of correlated nucleons**
  **wave function of pair (S,T,r)**
  **current conservation-2 body currents**

- **Example of a “better model”**

- **Approximation to the “better model”**

- **Generalizing the model**

# Why Should WE Care About CCQE?

Experiments investigating neutrino oscillations employ (CCQE) neutrino-nucleus interactions.

For $0.3 < E_\nu < 3.0$ GeV it is a dominant interaction.

**CCQE is assumed to be readily calculable, experimentally identifiable, allowing assignment of the neutrino flavor and energy.**

***Some 50 calculations published since 2005***

Relevant neutrino oscillation parameter, L/E:

**$1.27\ \Delta m_{ij}^2(ev^2)$**

# Many Short Comings in Present State

I find present day CCQE event generators for .2 > $E_\nu$ > 2 GeV inadequate as they employ mean field, impulse approximation models.

Some generators even use 40 year old nuclear physics, produce wrong cross sections, assign incorrect incident neutrino energies, possibly seriously effecting the determination of neutrino oscillation parameters.

Nucleon – Nucleon interactions are absent.

For A ≥ 12 20% of the nucleons are in correlated pairs . These correlated pairs typically have momenta much greater than the mean field momentum.

N-N interactions + current conservation, $\nabla \cdot \vec{j} = \frac{\partial \rho}{\partial t}$ gives rise to two- body nucleonic currents that considerably enhance the transverse response.

Hadronic states in nuclei are <u>not</u> plane waves.

The physics to improve modeling of the neutrino-nucleus CCQE interaction is in hand.

# SOME KNOWN ISSUES

Nucleon-Nucleon interaction required to obtain realistic nucleon momentum distributions:

A< 12 full nuclear wave function –non rel. yields $(\vec{q},\omega)$

A< 40 simplified N-N interaction–non rel. yields $(\vec{q})$ $\omega$ to be inferred. Certain universalities are observed. Can crucial rel. effects be added?

Hadronic currents satisfying CVC and PCAC with N-N interactions, demand 2-body currents effecting both $\vec{q}$ and $\omega$ response , esp. in transverse sector. Quantitative evaluation requires proper 2 body WFs. (not PWs)

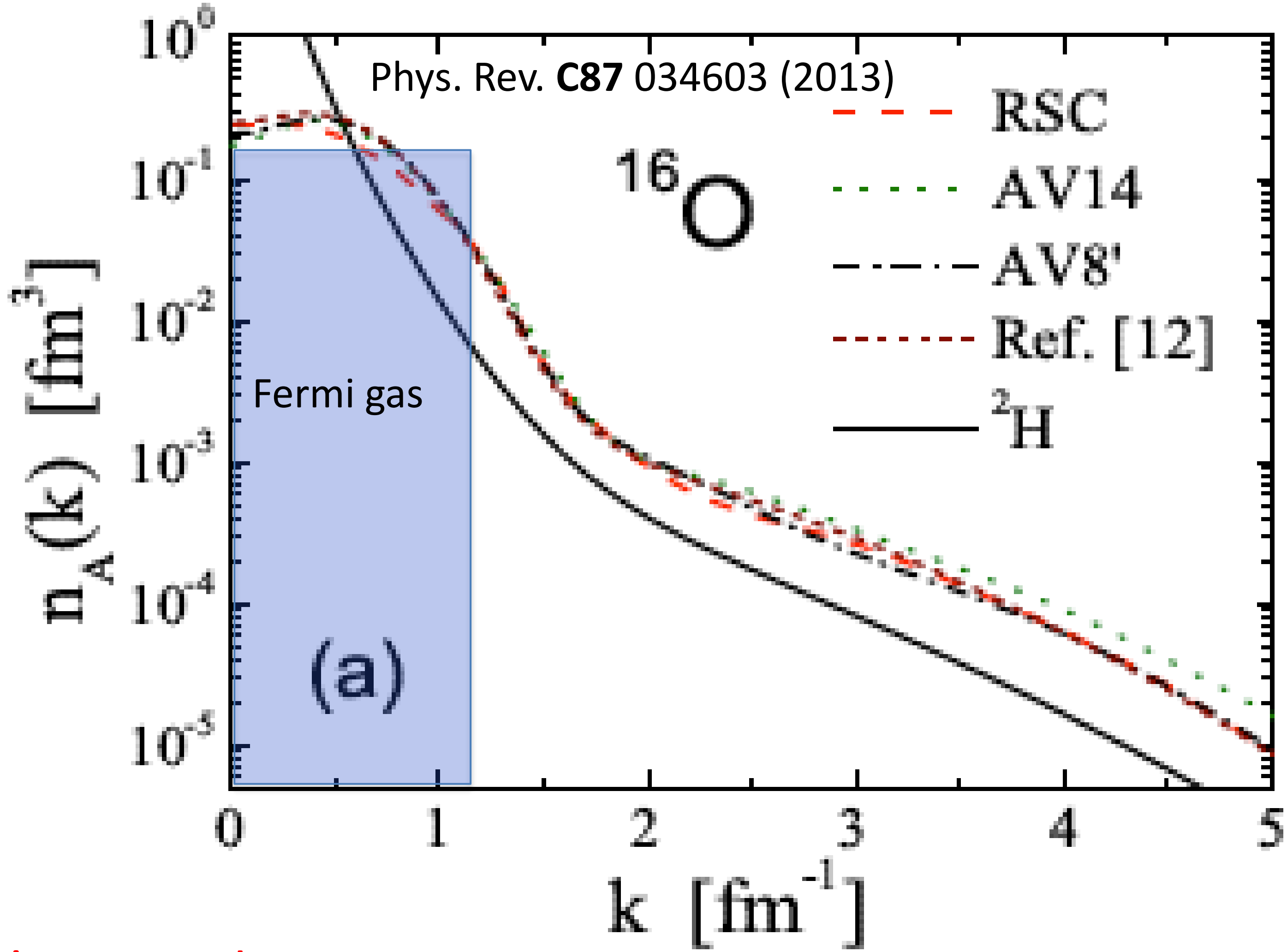
Free nucleon FFs: Not an issue at this point. Should nuclear effects be incorporated into the nucleon FFs? NO

**Reliable Predictions Require Reliable Fluxes!** How can reliable neutrino fluxes be obtained and ascertained? Essential!!

Suppose a good description (~5%) of the neutrino-nucleus CCQE and QE scattering is obtained, How do we deal with FSIs?

# How Bad is It?

For example:The nucleon momentum distribution

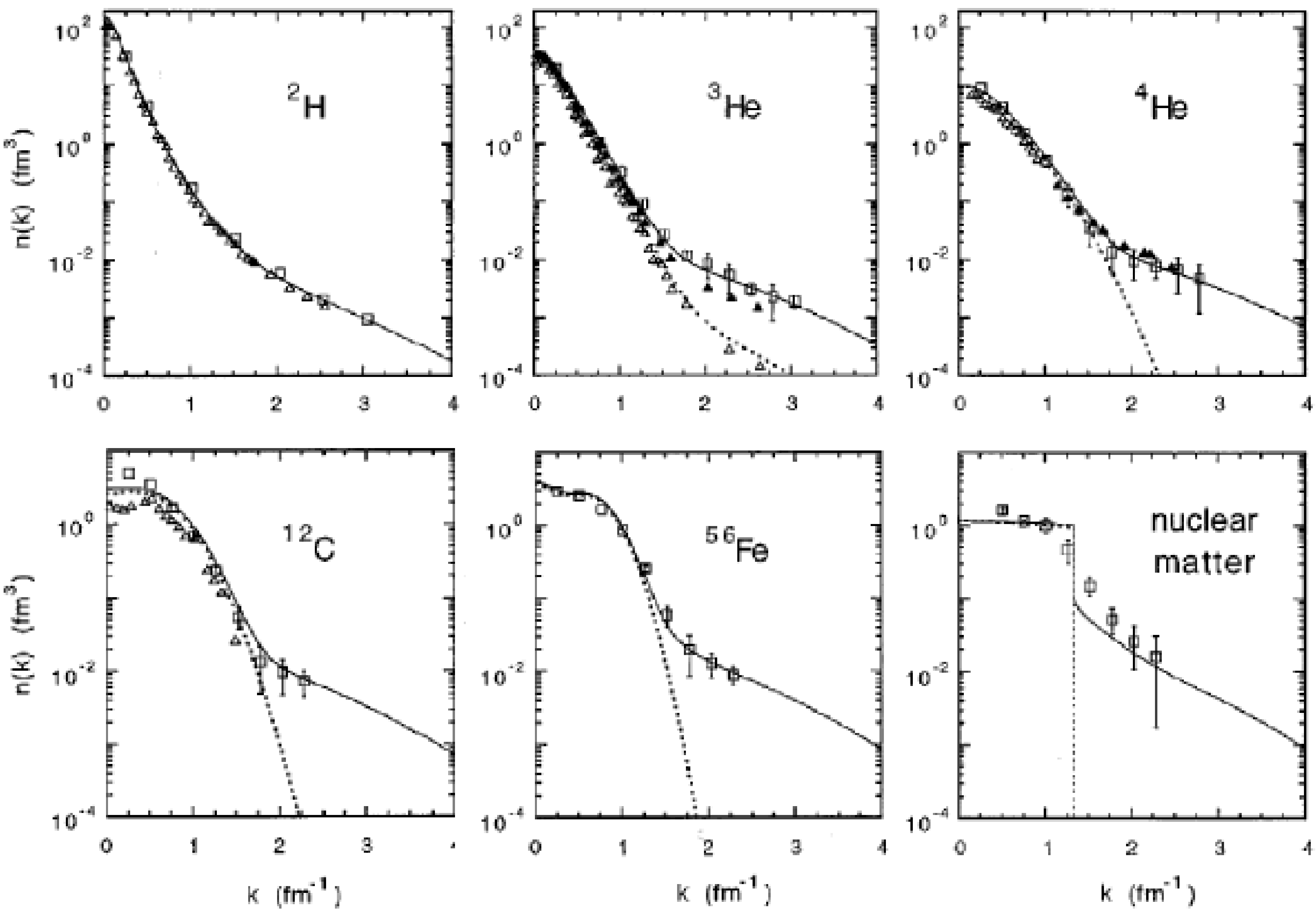



It's worse than it looks

$$P_A(k) = 4\pi \int_0^\infty n_A(k) k^2 \, dk$$

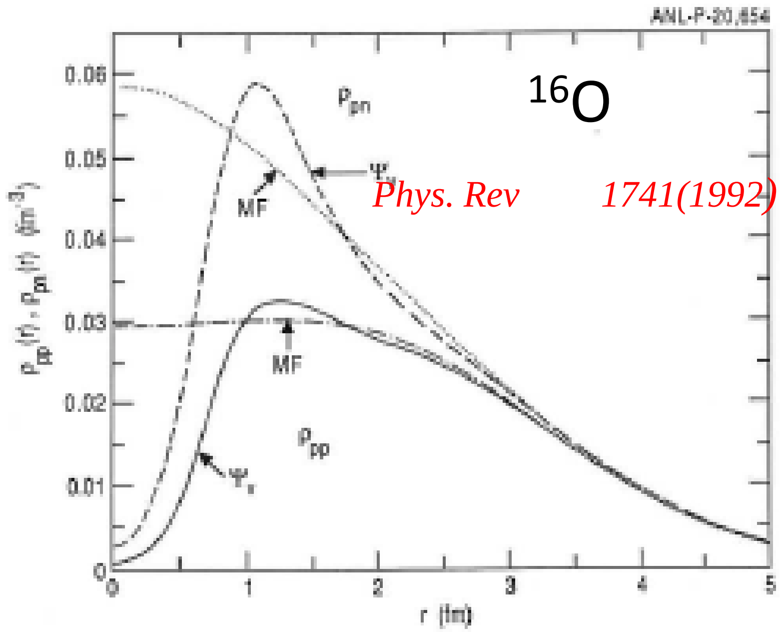
# Is it Really So? **YES!**

Phys. Rev. **C43** 1155

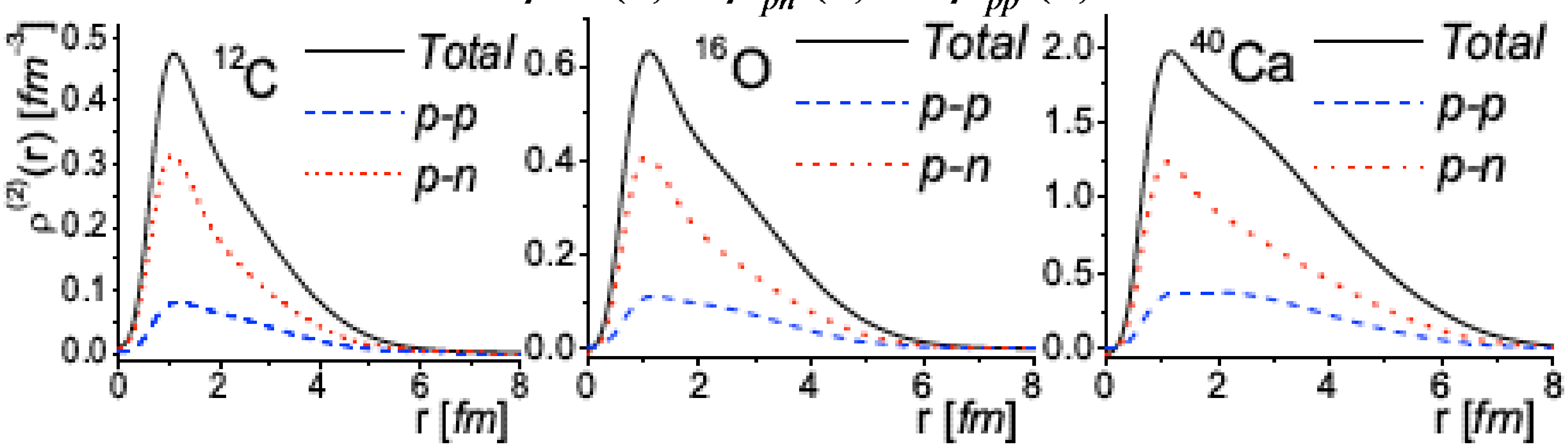


# 2-Body Distributions in Nuclei

arXiv:1306.6235v1 [nucl-th]

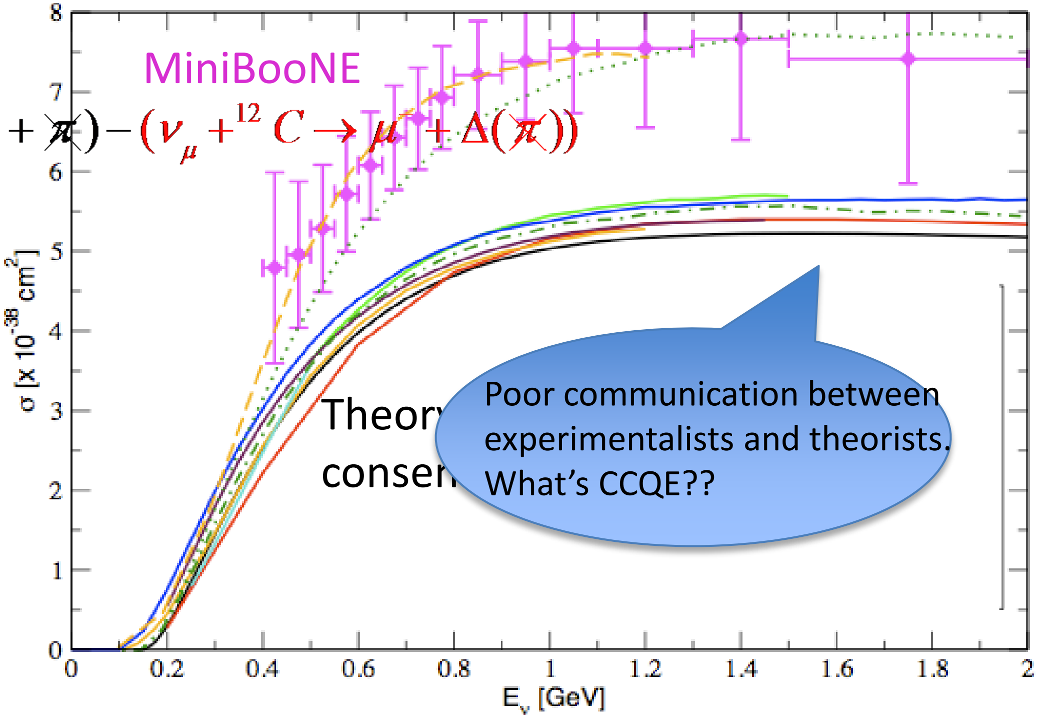



$$\rho^{(2)}(r) = \rho^{(2)}_{pn}(r) + 2\rho^{(2)}_{pp}(r)$$



# What's the deal ... rrelations??

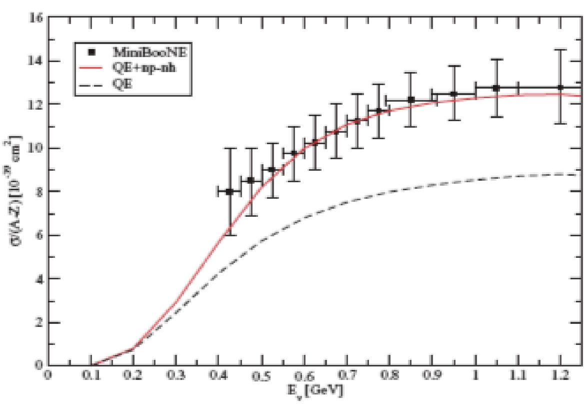
$(\nu_\mu + {}^{12}C \to \mu^- + \not\pi) - (\nu_\mu + {}^{12}C \to \mu^- + \Delta(\not\pi))$




MB fits the observed $Q^2$ distribution and cross-section by increasing $M_A$ to 1.35 GeV

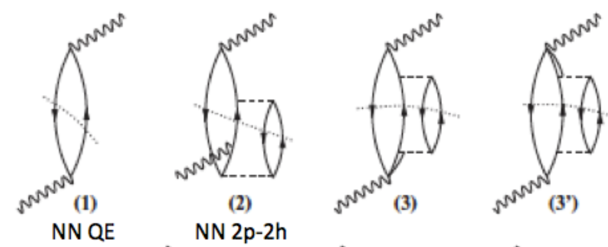
M. Martini, M. Ericson, G. Chanfray, and J. Marteau, PHYS. REV. **C 80**, 065501 (2009)

**They use $M_A$=1.03 GeV, in an RPA formalism**

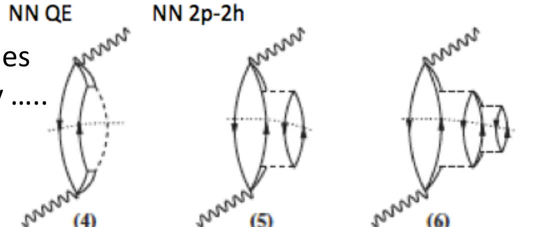



*"We suggest that the proposed increase of the axial mass from the standard value to a larger one to account for the quasielastic data, reflects the presence of a polarization cloud, mostly due to tensor interaction, which surrounds a nucleon in the nuclear medium. It translates into a final state with ejection of two nucleons, which in the present stage of the experiments is indistinguishable from the quasielastic final state."*

Some RPA p-h diagrams from Martini et al




Particle lines crossed by ….. are put on shell

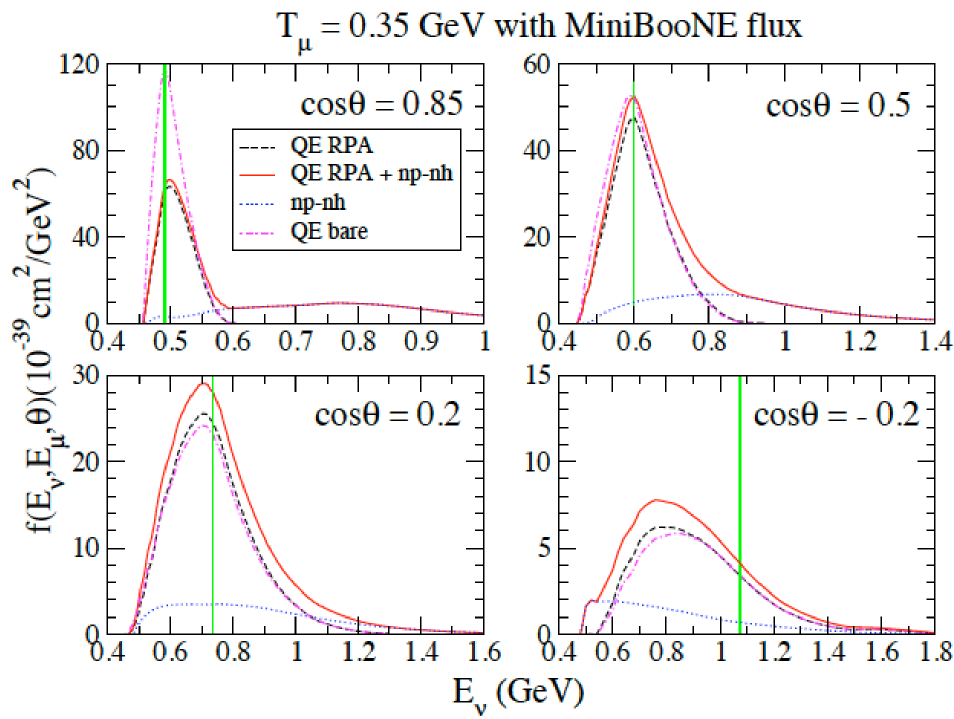


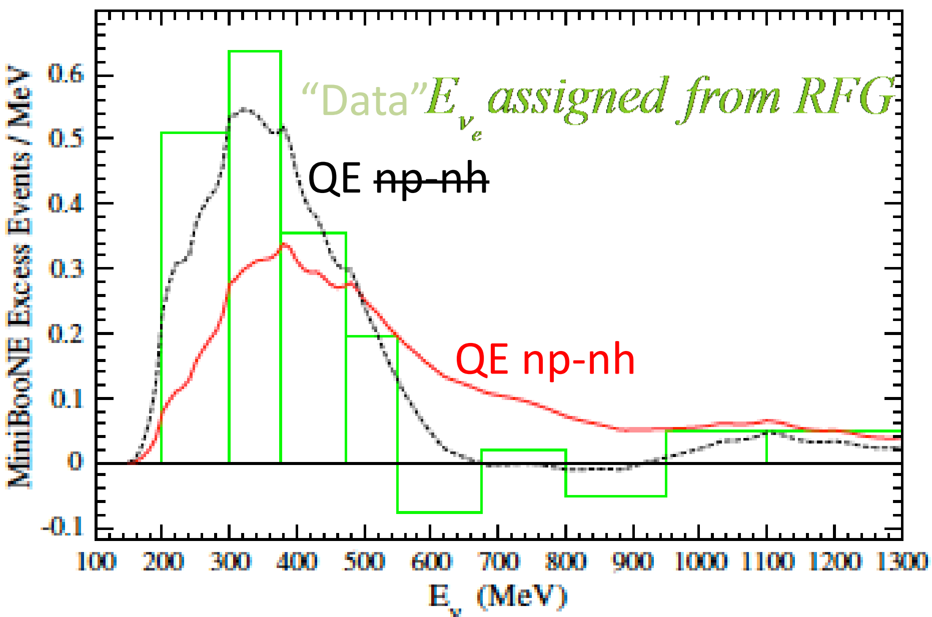
# Enhancement ➡ Uncertainty in Assigned $E_\nu$

**Martini et al**: Phys.Rev. D85, 093012 (2012)

$$
\begin{aligned}
\frac{\partial^2\sigma}{\partial\Omega\,\partial k'} &= \frac{G_F^2\cos^2\theta_c\,(k')^2}{2\pi^2}\cos^2\frac{\theta}{2}\left[G_E^2\left(\frac{q_\mu^2}{q^2}\right)^2 R_\tau^{NN}\right.\\
&+ G_A^2\frac{(M_\Delta-M_N)^2}{2q^2}R^{N\Delta}_{\sigma\tau(L)} + G_A^2\frac{(M_\Delta-M_N)^2}{q^2}R^{\Delta\Delta}_{\sigma\tau(L)}\\
&+ \left(G_M^2\frac{\omega^2}{q^2}+G_A^2\right)\left(-\frac{q_\mu^2}{q^2}+2\tan^2\frac{\theta}{2}\right)\left(R^{NN}_{\sigma\tau(T)}+2R^{N\Delta}_{\sigma\tau(T)}+R^{\Delta\Delta}_{\sigma\tau(T)}\right)\\
&\pm \left.2G_A G_M\frac{k+k'}{M_N}\tan^2\frac{\theta}{2}\left(R^{NN}_{\sigma\tau(T)}+2R^{N\Delta}_{\sigma\tau(T)}+R^{\Delta\Delta}_{\sigma\tau(T)}\right)\right]
\end{aligned}
$$

$$(\nu_e + {}^{12}C \rightarrow e^- + \not\pi) - (\nu_e + {}^{12}C \rightarrow e^- + \Delta(\not\pi))$$



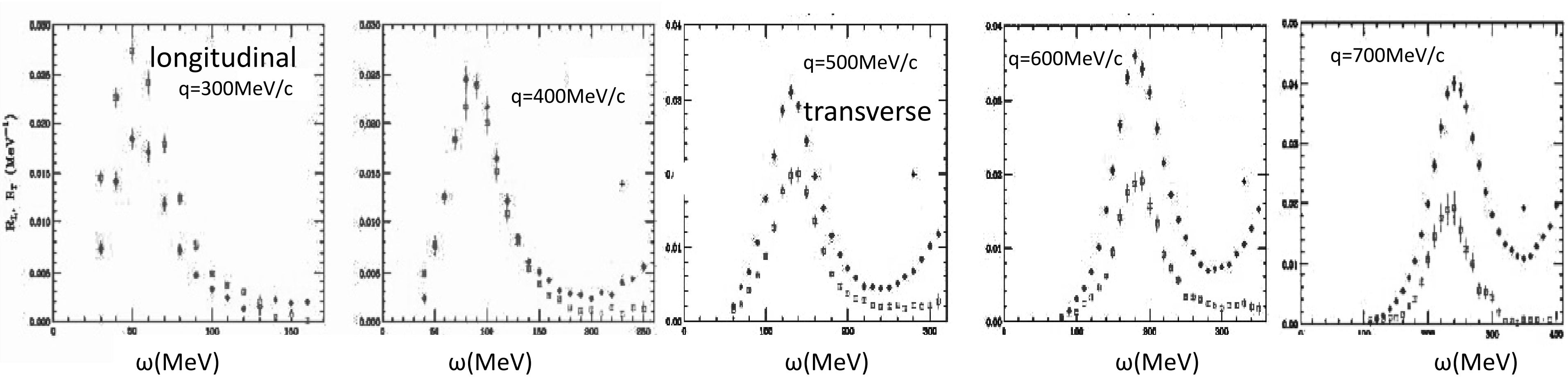

Significant impact on the neutrino energy assignment!

Ulrich"Robust"

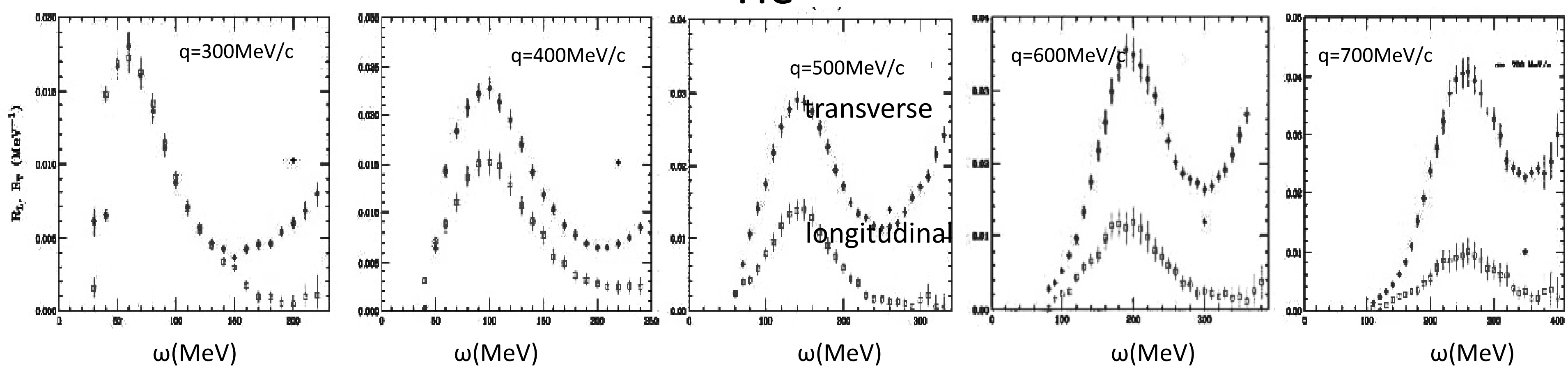
# Carlson et al Phys. Rev. C65 024002 (2002)

## Longitudinal and Transverse Response Functions from $^3He$ and $^4He$ from (e,e') Quasi-elastic Scattering

### $^3He$



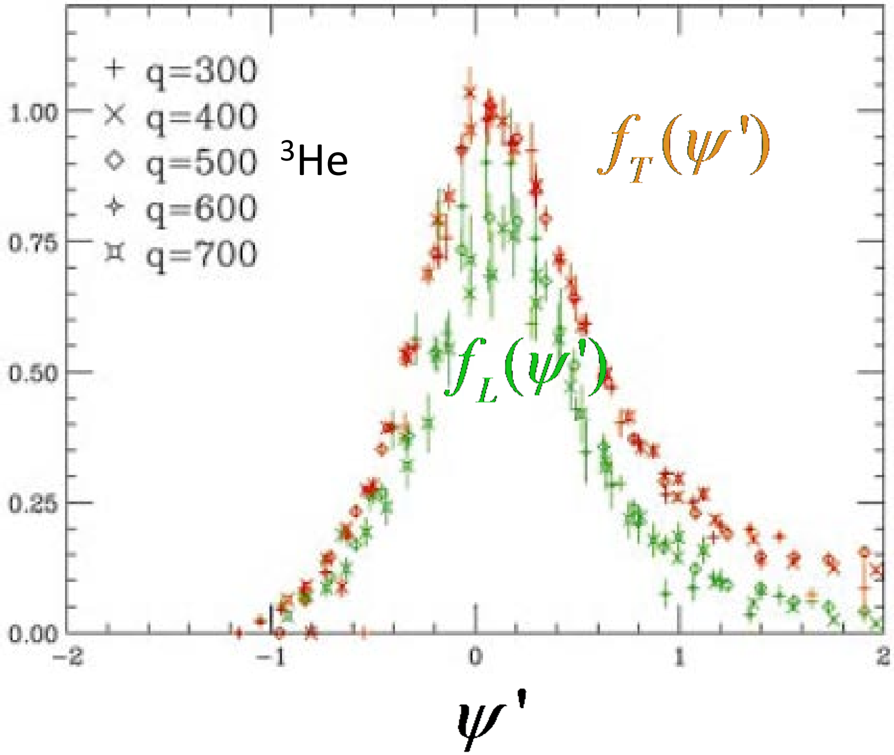

### $^4He$

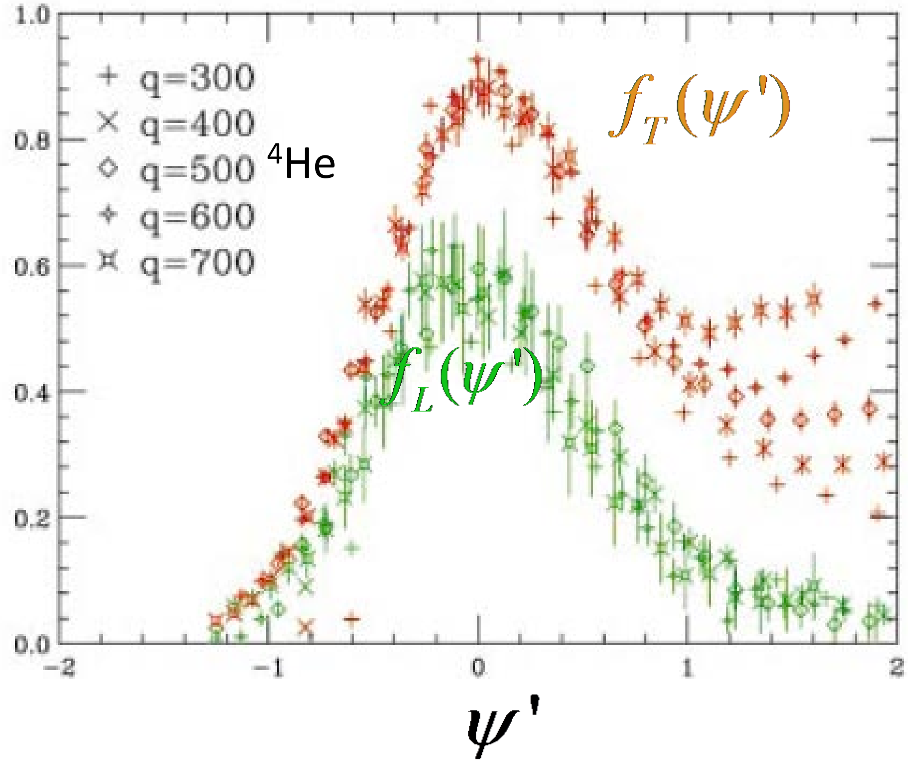


# $^3$He and $^4$He Longitudinal and Transverse Scaled Response Functions

$$\psi' = \frac{y}{k_F} = \frac{\sqrt{\omega^2 + 2m\omega} - q}{k_F}$$

$$f_{L,T}(\psi', q) = k_F \frac{R_{L,T}(q,\omega)}{G_{L,T}(q)} \qquad f_T(\psi', q) = \frac{k_F 2mqR_T(q,\omega)}{Q^2(ZG^2_{M_p}(q) + NG^2_{M_n}(q))}$$




Note : Change in $f_T/f_L$ and shift to higher values of $\psi'$ between $^3$He and $^4$He,

PHYSICAL REVIEW C, VOLUME 65, 024002

# $^4$He Longitudinal and Transverse e,e' QE Response

$$\tilde{E}_{T,L}(q,\tau) = \int_{\omega_{th}}^{\infty} e^{-(\omega - E_0)\tau} R_{T,L}(q,\omega)\, d\omega$$

$$\tilde{E}_T(q,\tau) = \langle 0 | j_T^*(\vec{q}) e^{-(H-E_0)\tau} j_T(\vec{q}) | 0 \rangle - e^{\frac{q^2\tau}{2AM}} \left| \langle 0(\vec{q}) | j_T(\vec{q}) | 0 \rangle \right|^2$$

$$E_{T,L}(q,\tau) = \frac{e^{\frac{q^2\tau}{2m}}}{G_D(Q^2)} \tilde{E}_{T,L}(q,\tau)$$

**Calculation; Uses 2 & 3 body NN forces, includes 2 body current operators.**

# One and Two Body EM Currents and Charges

One-body current and charge:

$$\rho^{(1)}_{i,\mathrm{NR}}(\mathbf{q}) = \epsilon_i e^{i\mathbf{q}\cdot\mathbf{r}_i}, \qquad \mathbf{j}^{(1)}_i(\mathbf{q}) = \frac{1}{2m}\epsilon_i[\mathbf{p}_i, e^{i\mathbf{q}\cdot\mathbf{r}_i}]_+ - \frac{i}{2m}\mu_i \mathbf{q}\times\boldsymbol{\sigma}_i e^{i\mathbf{q}\cdot\mathbf{r}_i},$$

$$\epsilon_i = G_{E,p}(Q^2)\frac{1}{2}(1+\tau_{z,i}) + G_{E,n}(Q^2)\frac{1}{2}(1-\tau_{z,i}),$$

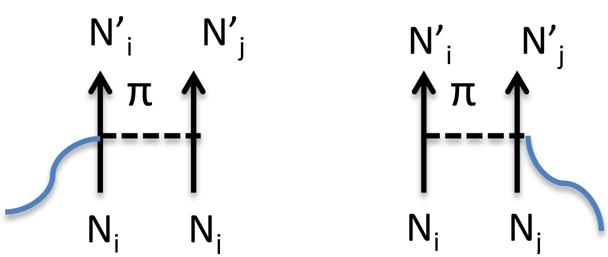
$$\mu_i \equiv G_{M,p}(Q^2)\frac{1}{2}(1+\tau_{z,i}) + G_{M,n}(Q^2)\frac{1}{2}(1-\tau_{z,i}),$$

Continuity eq.:

$$\nabla\cdot\vec{j} + i[H,\rho] = 0 \qquad H = \sum_i T_i + \sum_{i>j} V_{i,j}$$

$$j = \sum_i j_i^{(1)} + \sum_{i>j} j_{i,j}^{(2)} \qquad \nabla\cdot j_i^{(1)} + i[T_i, \rho_i^{(1)}] = 0 \qquad \nabla\cdot j_{i,j}^{(2)} + i[V_{i,j}, \rho_i^{(1)} + \rho_j^{(1)}] = 0$$

Two-body current:

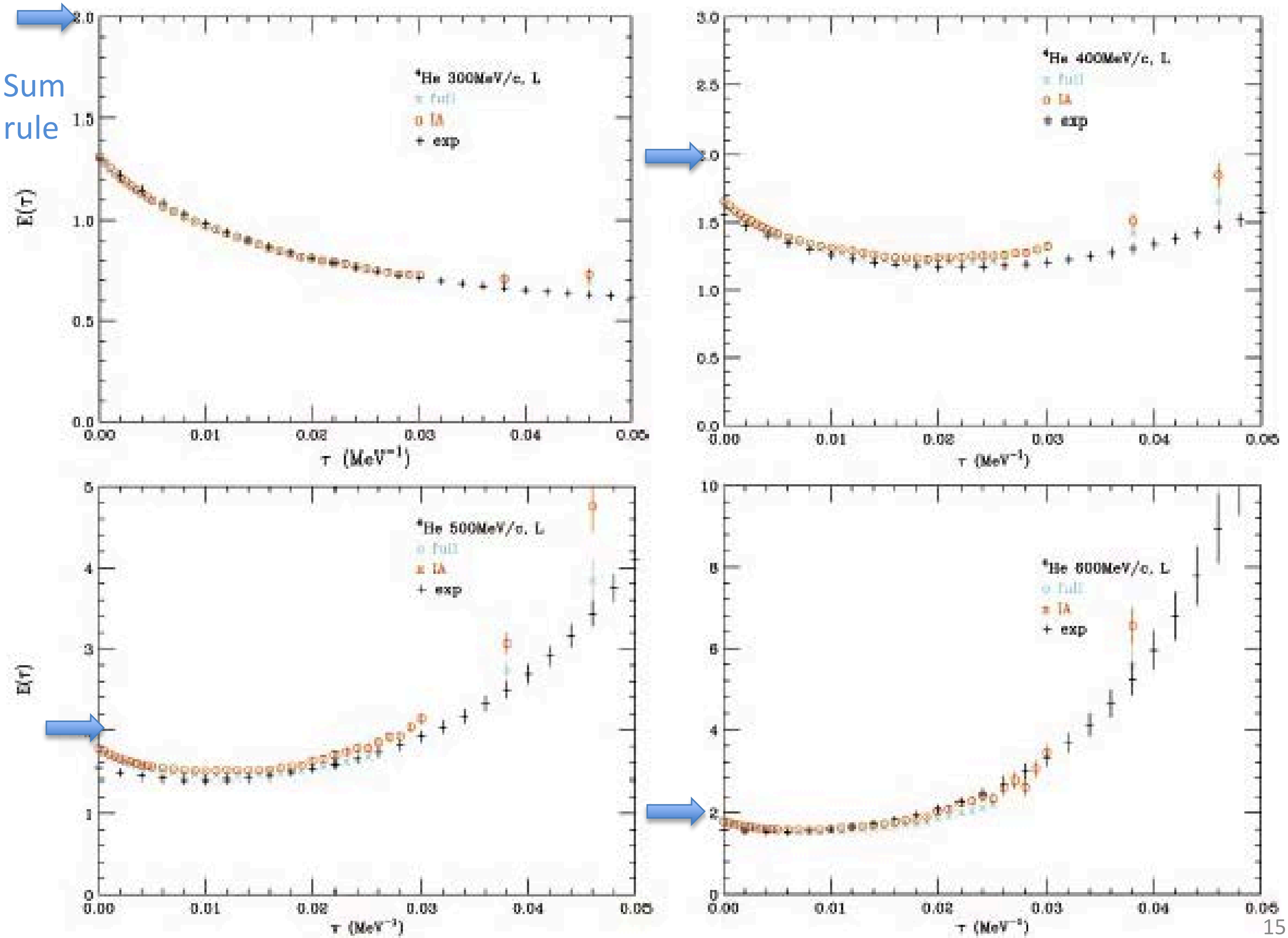



$$\mathbf{j}^{(2)}_{ij}(\mathbf{q};\pi) = G_E^V(Q^2)(\boldsymbol{\tau}_i\times\boldsymbol{\tau}_j)_z[e^{i\mathbf{q}\cdot\mathbf{r}_i} f_{PS}(r)\boldsymbol{\sigma}_i(\boldsymbol{\sigma}_j\cdot\hat{\mathbf{r}}) + e^{i\mathbf{q}\cdot\mathbf{r}_j} f_{PS}(r)\boldsymbol{\sigma}_j(\boldsymbol{\sigma}_i\cdot\hat{\mathbf{r}}) - (\boldsymbol{\sigma}_i\cdot\boldsymbol{\nabla}_i)(\boldsymbol{\sigma}_j\cdot\boldsymbol{\nabla}_j)(\boldsymbol{\nabla}_i - \boldsymbol{\nabla}_j) g_{PS}(\mathbf{q};\mathbf{R},\mathbf{r})],$$

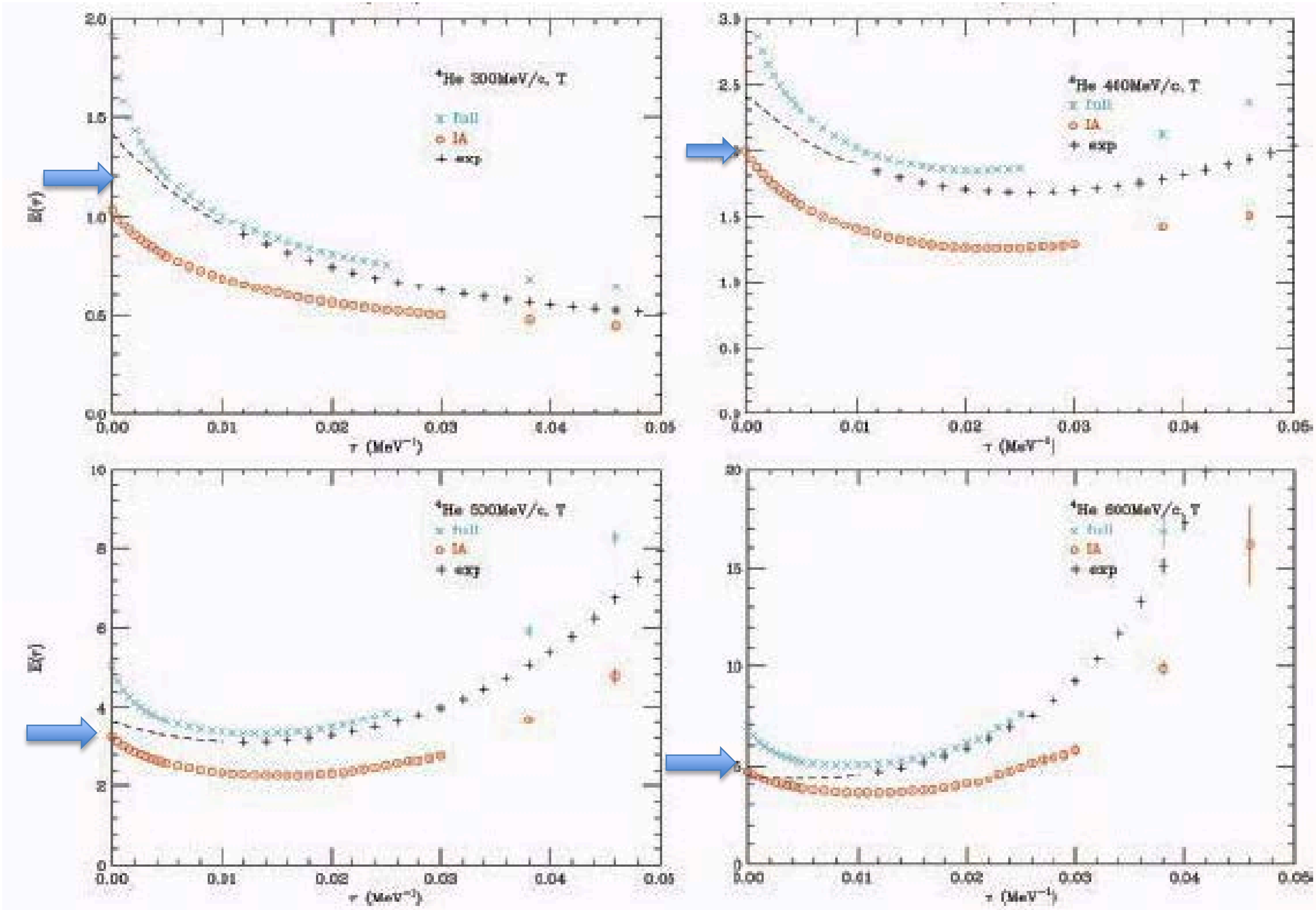
$$f_{PS}(r) = \frac{d}{dr}\int \frac{d\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{r}} v_{PS}(k)$$

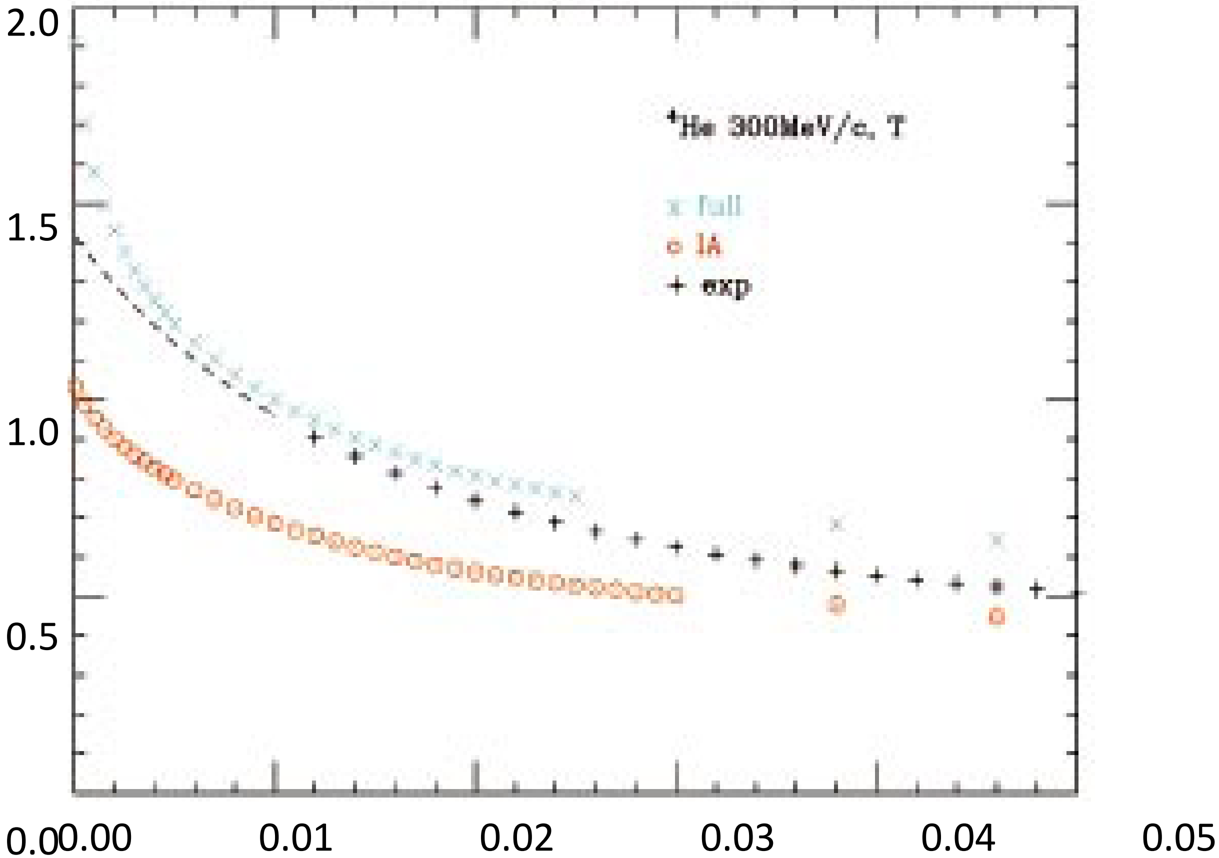
$$v_{PS}(k) \to v_\pi(k) \equiv -\frac{f_\pi^2}{m_\pi^2}\frac{f_\pi^2(k)}{k^2+m_\pi^2}$$

# $^4$He Euclidian **Longitudinal** Response: Calculated versus Data



# $^4$He **Transverse** Response Calculated Versus Data






)

$$\tilde{E}_{T,L}(q,\tau) = \int_{\omega_{th}}^{\infty} e^{-(\omega - E_0)\tau} R_{T,L}(q,\omega)\, d\omega$$

$$\tilde{E}_T(q,\tau) = \left\langle 0 \right| j_T^*(\vec{q}) e^{-(H-E_0)\tau} j_T(\vec{q}) \left| 0 \right\rangle - e^{\frac{q^2\tau}{2AM}} \left| \left\langle 0(\vec{q}) \right| j_T(\vec{q}) \left| 0 \right\rangle \right|^2$$

$$E_{T,L}(q,\tau) = \frac{e^{\frac{q^2\tau}{2m}}}{G_D(Q^2)} \tilde{E}_{T,L}(q,\tau)$$

# Sum Rules

Normalizes to unity the summed contribution from each nucleon to the CS

$$S_{T,L}(q) = C_{T,L} \int_{\omega_{th}}^{\infty} d\omega S_{T,L}(q,\omega) = C_{T,L} \left[ \left\langle 0 \middle| \tilde{O}^{*}_{T,L}(\vec{q}) O_{T,L}(\vec{q}) \middle| 0 \right\rangle - | \left\langle 0 \middle| O_{T,L}(\vec{q}) \middle| 0 \right\rangle |^2 \right]$$

$$S_{T,L}(q) = C_{T,L} E_{T,L}(q, \tau = 0)$$

$$E_{T,L}(q,\tau) = \frac{e^{\frac{q^2 \tau}{2m}}}{G_D(Q^2)} \tilde{E}_{T,L}(q,\tau)$$

$$C_T = \frac{2m^2}{Z\mu_p^2 + N\mu_n^2} \frac{1}{q^2}$$

$$C_L = \frac{1}{Z}$$

PHYSICAL REVIEW C, VOLUME 65, 024002

# $^4$He Longitudinal and Transverse e,e' QE Response

TABLE I. Longitudinal sum rule obtained with one body only and with both one- and two-body charge operators.

| $q$ (MeV/$c$) | $^3$He | | $^4$He | | $^6$Li | |
|---|---|---|---|---|---|---|
| | 1 | 1+2 | 1 | 1+2 | 1 | 1+2 |
| 300 | 0.787 | 0.763 | 0.670 | 0.649 | 0.977 | 0.933 |
| 400 | 0.921 | 0.875 | 0.859 | 0.815 | 0.995 | 0.932 |
| 500 | 0.964 | 0.901 | 0.941 | 0.881 | 0.990 | 0.921 |
| 600 | 0.982 | 0.908 | 0.973 | 0.910 | 0.990 | 0.924 |
| 700 | 0.994 | 0.914 | 0.994 | 0.942 | 0.994 | 0.938 |

TABLE II. Transverse sum rule obtained with one body only and with both one- and two-body current operators.

| $q$ (MeV/$c$) | $^3$He | | $^4$He | | $^6$Li | |
|---|---|---|---|---|---|---|
| | 1 | 1+2 | 1 | 1+2 | 1 | 1+2 |
| 300 | 0.929 | 1.31 | 0.893 | 1.67 | 0.912 | 1.57 |
| 400 | 0.987 | 1.30 | 0.970 | 1.62 | 0.974 | 1.52 |
| 500 | 1.01 | 1.28 | 1.00 | 1.55 | 0.999 | 1.46 |
| 600 | 1.01 | 1.25 | 1.01 | 1.49 | 1.01 | 1.41 |
| 700 | 1.01 | 1.23 | 1.01 | 1.44 | 1.011 | 1.37 |

An important issue

Using Plane Wave Initial and Final States

TABLE VII. Excess-strength contributions $\Delta S_L$ and $\Delta S_T$ to the Fermi-gas sum rules from terms involving two-nucleon currents.

| $q$ (MeV/$c$) | $\Delta S_L$ | $\Delta S_T$ |
|---|---|---|
| 300 | 0.004 | 0.114 |
| 400 | 0.007 | 0.081 |
| 500 | 0.011 | 0.066 |
| 600 | 0.017 | 0.060 |
| 700 | 0.024 | 0.056 |

***Plane waves will not generate adequate enhancement!!***

# Potentially Bad News!!

**Conclusion from** *Phys. Rev. C65 024002 (2002)*

"it is now clear that this enhancement arises from the concerted interplay of tensor interactions and correlations in both ground and scattering states. A successful prediction of the longitudinal and transverse response functions in the quasielastic region demands an accurate description of nuclear dynamics, based on realistic interactions and currents."

# Universality of nucleon-nucleon short-range correlations and nucleon momentum distributions

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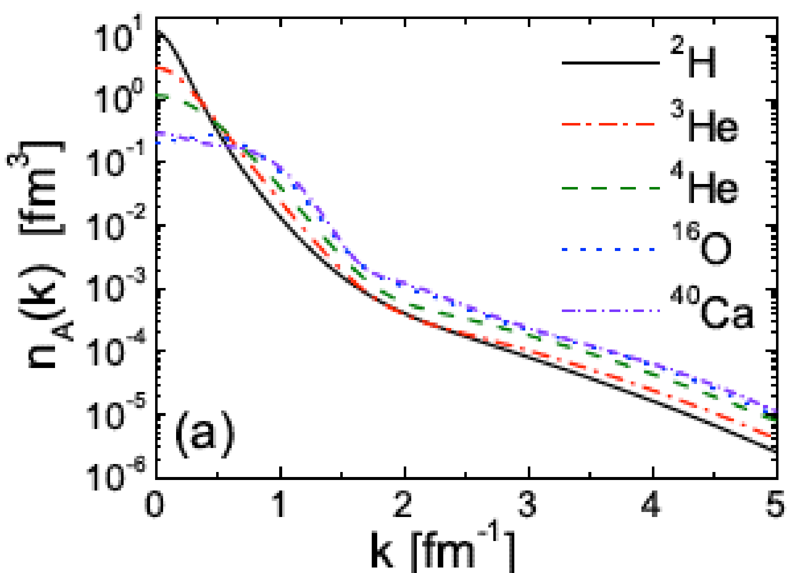
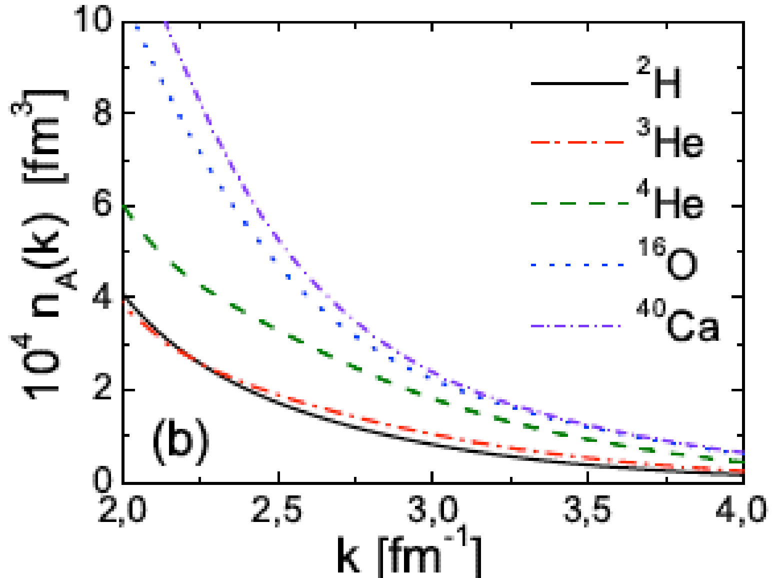
arXiv:1306.6235v1 [nucl-th] 26 Jun 2013

# A Procedure for ν-N CCQE with A ≤ 40

- Start with a momentum distribution generated from realistic N-N interactions. ***C87***
- This produces a mean field momentum distribution (80%) plus correlated pairs (20%).
- Scattering off nucleons in the mean field can be carried out in existing programs.
- The correlated pairs appear to have a universal momentum distribution however calculating their response to interactions is non-trivial.



# Recent Calculation of Nucleon Momentum Distributions with “Realistic” Interactions

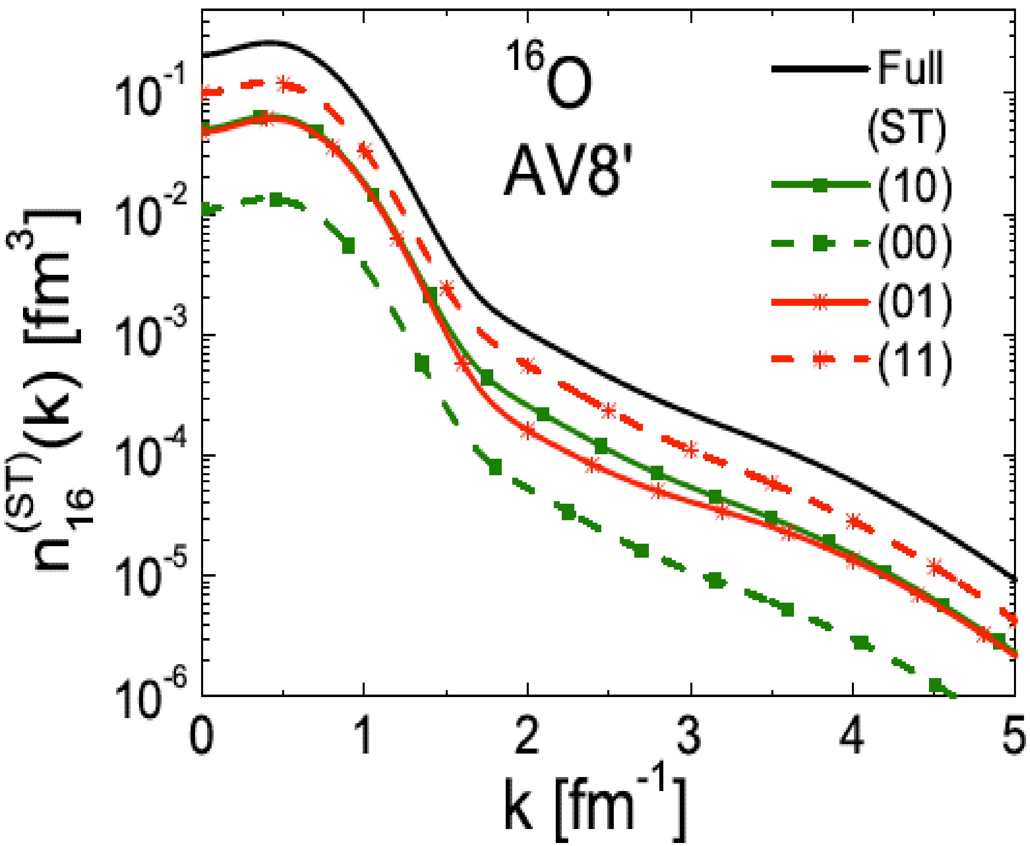
Phys. Rev. **C87** 034603 (2013)

$$\mathcal{P}^{N_1}_{0(1)}(k_1^{\pm}) = 4\pi \int_{k_1^-}^{k_1^+} n^{N_1}_{0(1)}(k_1) k_1^2 \, dk_1$$

0 corresponds to nucleons in mean field
1 corresponds to nucleons in 2-body correlation

| | $^2H$ | $^3$He(n) | $^3$He(p) | | $^4$He | | $^{16}$O | | $^{40}$Ca | |
|---|---|---|---|---|---|---|---|---|---|---|
| $k_1^-$ [fm$^{-1}$] | $\mathcal{P}$ | $\mathcal{P}_1$ | $\mathcal{P}_0$ | $\mathcal{P}_1$ | $\mathcal{P}_0$ | $\mathcal{P}_1$ | $\mathcal{P}_0$ | $\mathcal{P}_1$ | $\mathcal{P}_0$ | $\mathcal{P}_1$ |
| 0.00 | 1.000 | 0.999 | 0.677 | 0.323 | 0.84621 | 0.15285 | 0.79999 | 0.20016 | 0.80 | 0.19321 |
| 0.50 | 0.3078 | 0.568 | 0.277 | 0.201 | 0.53643 | 0.14032 | 0.66972 | 0.19635 | 0.69997 | 0.18301 |
| 1.00 | 0.081 | 0.163 | 0.038 | 0.0723 | 0.10479 | 0.1045 | 0.17588 | 0.14794 | 0.24706 | 0.13771 |
| 1.50 | 0.0366 | 0.067 | 0.0049 | 0.036 | 0.0079 | 0.0791 | 0.00792 | 0.09417 | 0.01022 | 0.10143 |
| 2.00 | 0.0221 | 0.041 | 0.0015 | 0.024 | $6.9512\,10^{-4}$ | 0.06156 | $5.9\,10^{-5}$ | 0.06344 | $3.28\,10^{-4}$ | 0.07124 |

# Correlations are Not as Simple as $^2$H (S=1,T=O)




| Nucleus | | (ST) (10) | (01) | (00) | (11) |
|---|---|---|---|---|---|
| $^2$H | | 1 | - | - | - |
| $^3$He | IPM | 1.50 | 1.50 | - | - |
| | SRC (Present work) | 1.488 | 1.360 | 0.013 | 0.139 |
| | SRC [44] | 1.50 | 1.350 | 0.01 | 0.14 |
| | SRC [23] | 1.489 | 1.361 | 0.011 | 0.139 |
| $^4$He | IPM | 3 | 3 | - | - |
| | IPM(0s states) [46] | 3 | 3 | - | - |
| | SRC (Present work) | 2.99 | 2.57 | 0.01 | 0.43 |
| | SRC [44] | 3.02 | 2.5 | 0.01 | 0.47 |
| | SRC [23] | 2.992 | 2.572 | 0.08 | 0.428 |
| $^{16}$O | IPM | 30 | 30 | 6 | 54 |
| | IPM(0s states) [46] | 20 | 18 | - | - |
| | SRC(Present work) | 29.8 | 27.5 | 6.075 | 56.7 |
| | SRC [44] | 30.05 | 28.4 | 6.05 | 55.5 |
| $^{40}$Ca | IPM | 165 | 165 | 45 | 405 |
| | IPM(0s states) [46] | 90 | 20 | - | - |
| | SRC(Present work) | 165.18 | 159.39 | 45.10 | 410.34 |

At k=2fm$^{-1}$: pn/2pp=1.2 ????

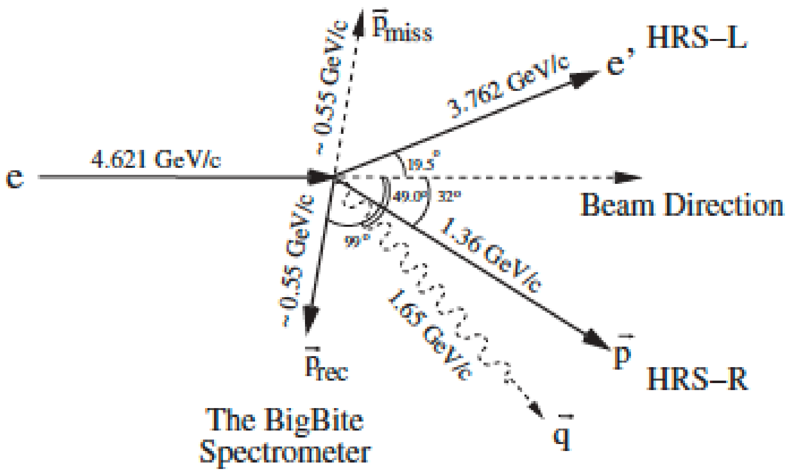
T=1, nn,np,pp

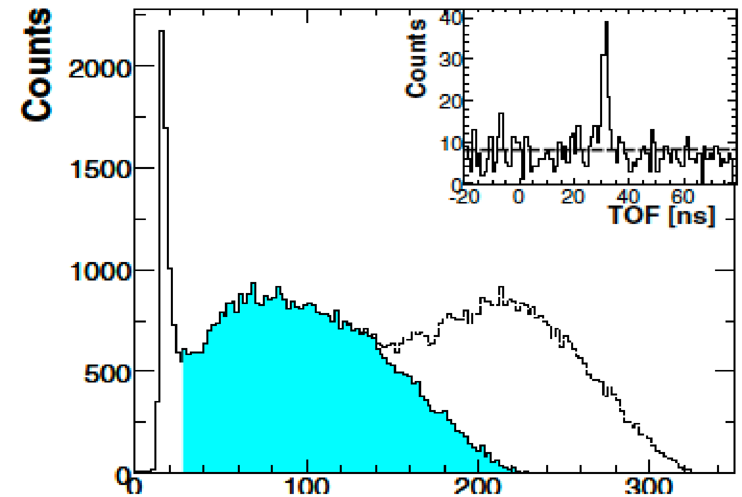
Tensor interaction requires S=1

Pair WF must be AS under exchange

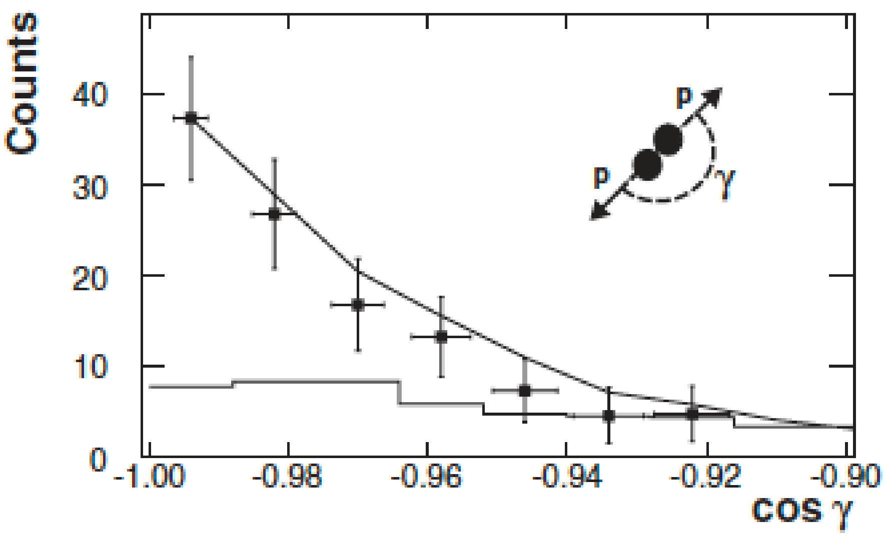
# p-p short Range Correlations via $^{12}$C(e,e'p,p) Reaction

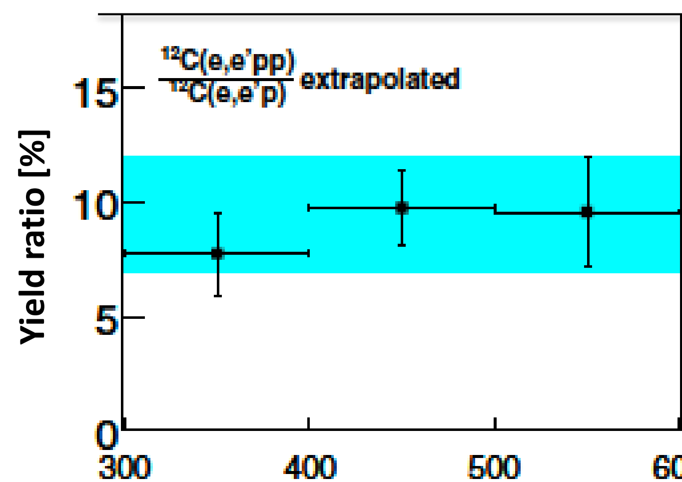
PRL 99,072501 (2007)




$E_{miss}=\omega-T_p-T_{A-1}$ (MeV)




Missing Momentum (MeV/c)

# How do we get a UM for CCQE?

- Use a simplified nucleon-nucleon interaction to establish the nucleon momentum distribution in a nucleus (N,Z).
- The ν-nucleus response in the mean field sector (80%) can be calculated via standard prescriptions with the neutrino flux provided.
- The response involving correlated pairs (20%) requires special treatment. The response of the various two body correlations can be calculated off line by experts. Appropriate 2-body currents and nucleon-nucleon 2-body wave functions must be applied. Hopefully sufficiently general results can be obtained that are added to the mean field result. Values for $\vec{q}$ and ω emerge from the calculation.
- While more complicated than present day neutrino event generators this approach provides a solid base to build upon as further improvements and discoveries are made.

# *Concluding Remarks*

*Better nucleon momentum distributions, consistent 2-body currents and the response of various pair correlation should yield a better description of CCQE and NCE.*

*This approach also provides a more solid foundation to incorporate improvements in theory and data.*

*Better cross sections put much greater emphasis on*

*. Role for $^{2}H$? Phys. Rev. C 86, 035503 (2012) and more to come.*

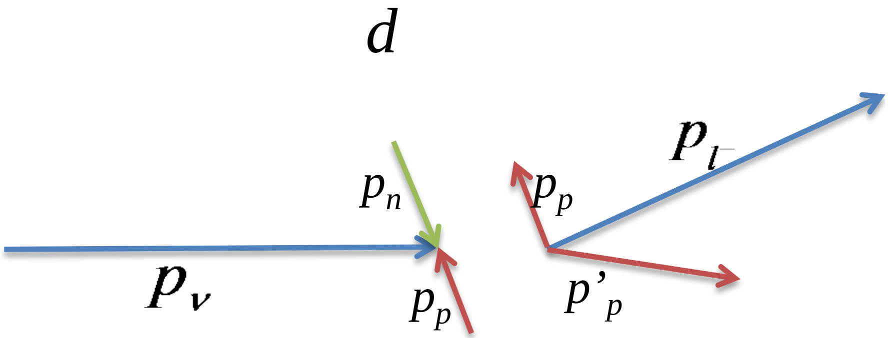
*This approach should also effect calculations of delta resonance production.*

*Note hadronic final state interactions have not been addressed in this presentation*

*Realization of the full capability of LAr detectors will require dealing with FSI, a difficult task. FSI must start with the right conditions a the weak vertex.*

# Absolute Normalization of the ν Flux

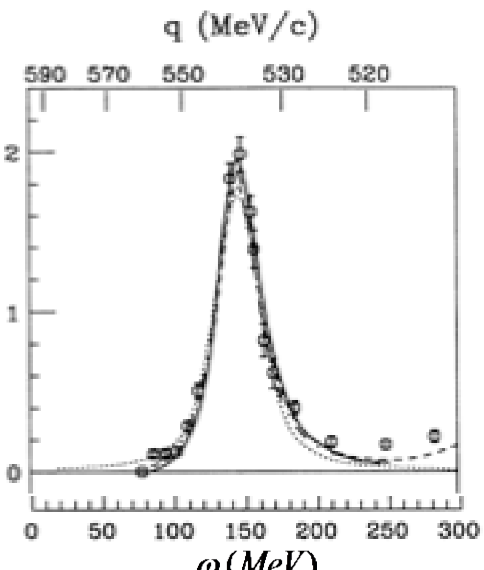
$$\nu_l + d \rightarrow l^- + 2p$$

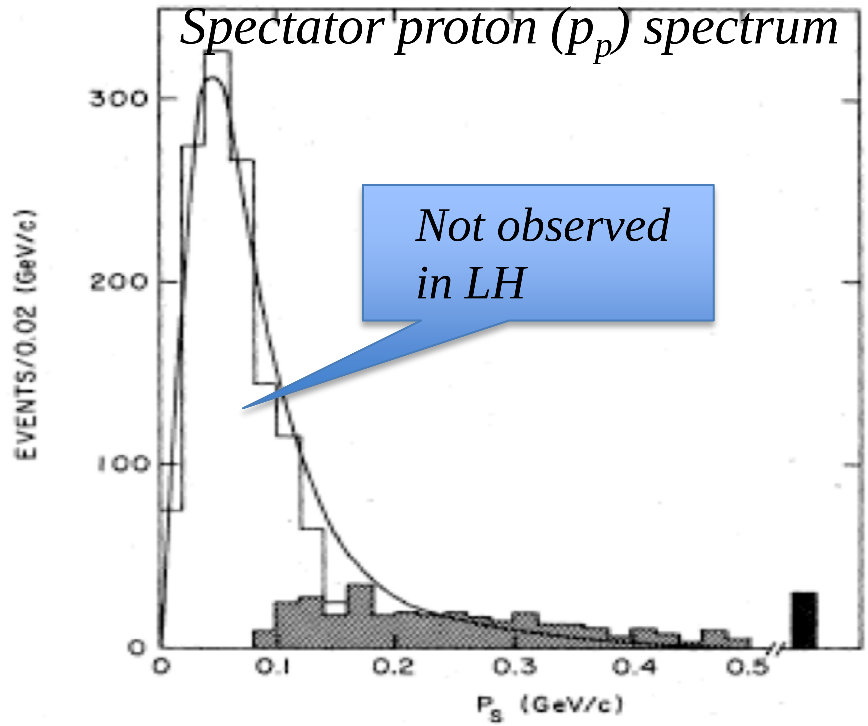



$$p_\nu - p_{l^-} = (\omega, \vec{q}) = p'_p - p_n = p'_p + p_p$$

*e+d inclusive scattering*

596.8 MeV, 60.0°



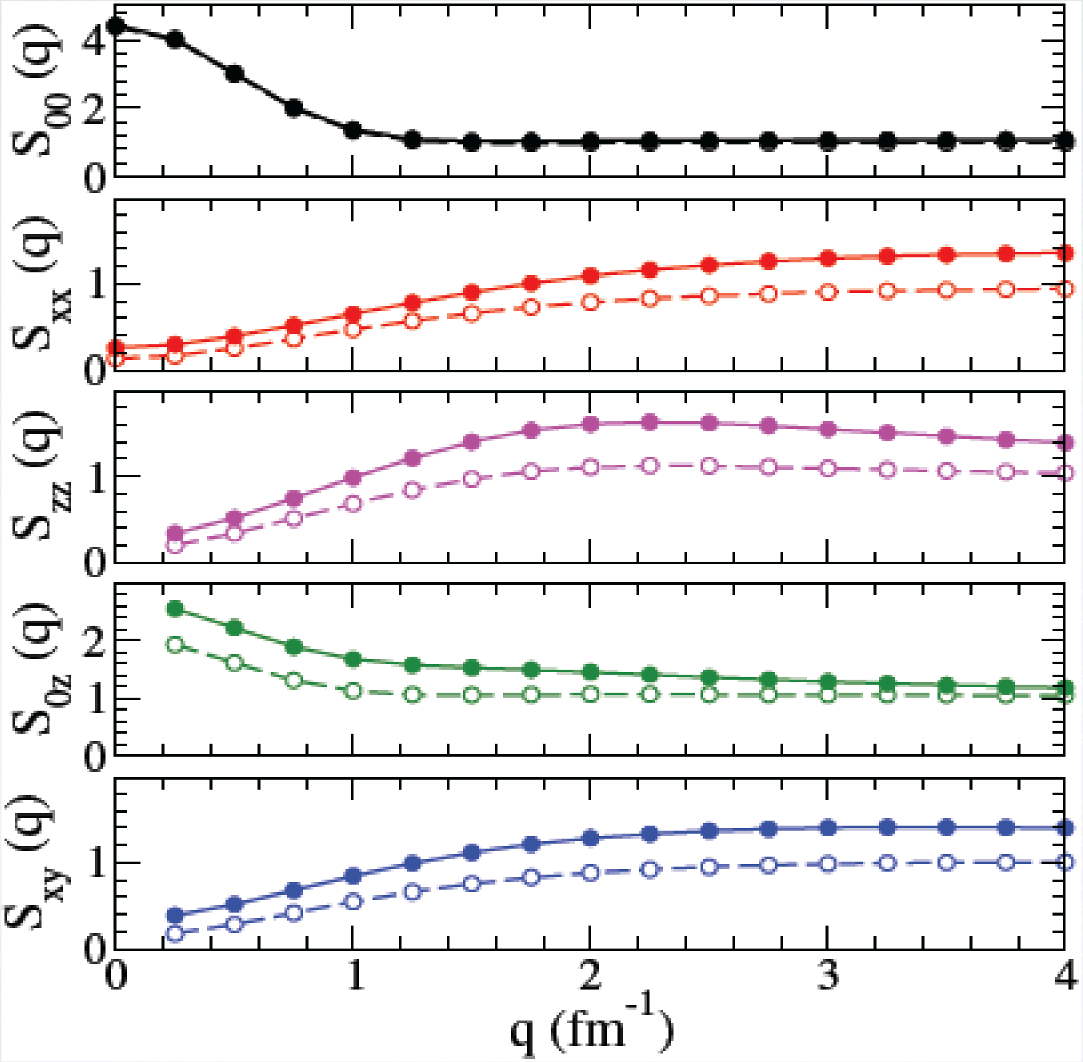

**With ω and $E_\mu$ known, $E_\nu$ is determined!!**
**With q and ω known, $y=(\omega+2m\omega)^{1/2} - q <$ O can be selected if necessary.**

# Neutrino Scattering:

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\epsilon'\mathrm{d}\Omega}\right)_{\nu/\bar{\nu}} = \frac{G_F^2}{2\pi^2} k'\epsilon' \cos^2\frac{\theta}{2}\left[R_{00} + \frac{\omega^2}{q^2} R_{zz} - \frac{\omega}{q} R_{0z} + \left(\tan^2\frac{\theta}{2} + \frac{Q^2}{2\,q^2}\right) R_{xx} \mp \tan\frac{\theta}{2}\sqrt{\tan^2\frac{\theta}{2} + \frac{Q^2}{q^2}}\, R_{xy}\right]$$

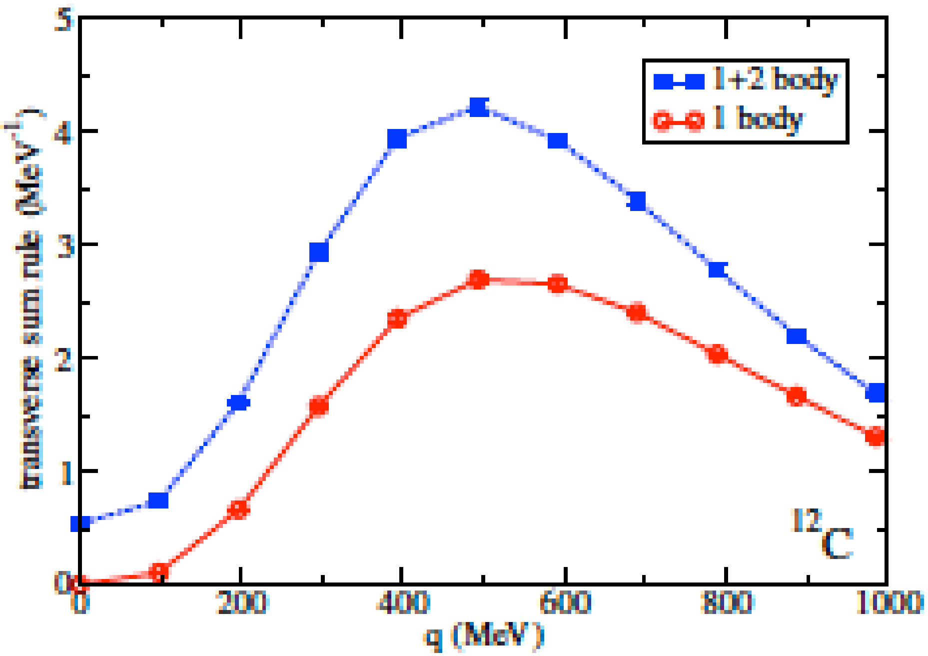
$$\begin{aligned}
R_{00}(q,\omega) &= \overline{\sum_i}\sum_f \delta(\omega + m_A - E_f)\,|\langle f|\, j^0(\mathbf{q},\omega)\,|i\rangle|^2 , \\
R_{zz}(q,\omega) &= \overline{\sum_i}\sum_f \delta(\omega + m_A - E_f)\,|\langle f|\, j^z(\mathbf{q},\omega)\,|i\rangle|^2 , \\
R_{0z}(q,\omega) &= \overline{\sum_i}\sum_f \delta(\omega + m_A - E_f)\Big[\langle f|\, j^0(\mathbf{q},\omega)\,|i\rangle \\
&\quad \times \langle f|\, j^z(\mathbf{q},\omega)\,|i\rangle^* + \mathrm{c.c.}\Big] , \\
R_{xx}(q,\omega) &= \overline{\sum_i}\sum_f \delta(\omega + m_A - E_f)\Big[\,|\langle f|\, j^x(\mathbf{q},\omega)\,|i\rangle|^2 \\
&\quad + |\langle f|\, j^y(\mathbf{q},\omega)\,|i\rangle|^2\,\Big] , \\
R_{xy}(q,\omega) &= \overline{\sum_i}\sum_f \delta(\omega + m_A - E_f)\Big[\langle f|\, j^x(\mathbf{q},\omega)\,|i\rangle \\
&\quad \times \langle f|\, j^y(\mathbf{q},\omega)\,|i\rangle^* - \mathrm{c.c.}\Big] ,
\end{aligned}$$

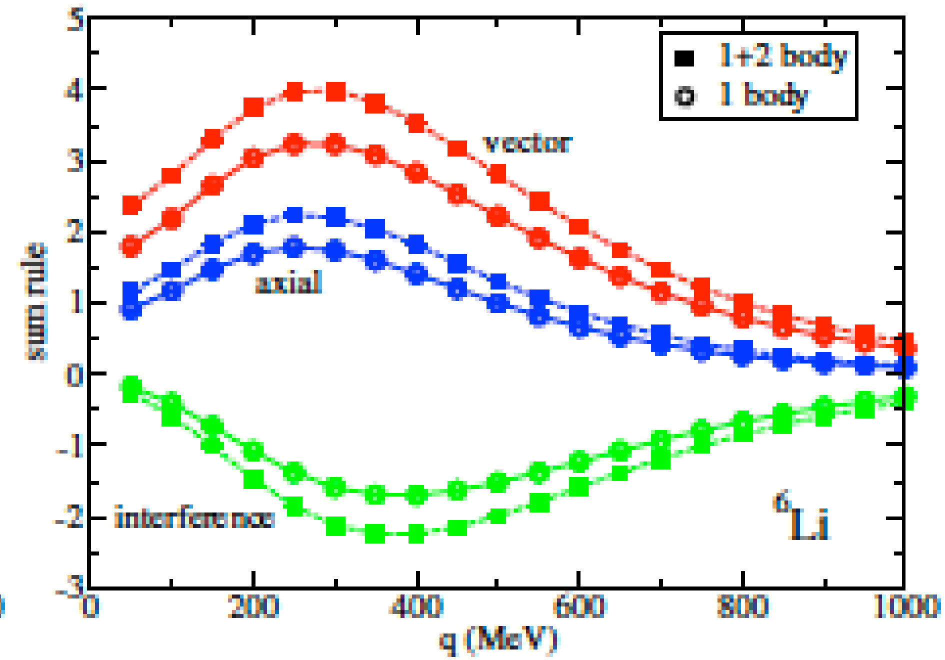
# Neutrino/Anti-neutrino Scattering
# 5 response functions
# Neutral current sum rules for 12C



# Some Effects Beyond $^4$He

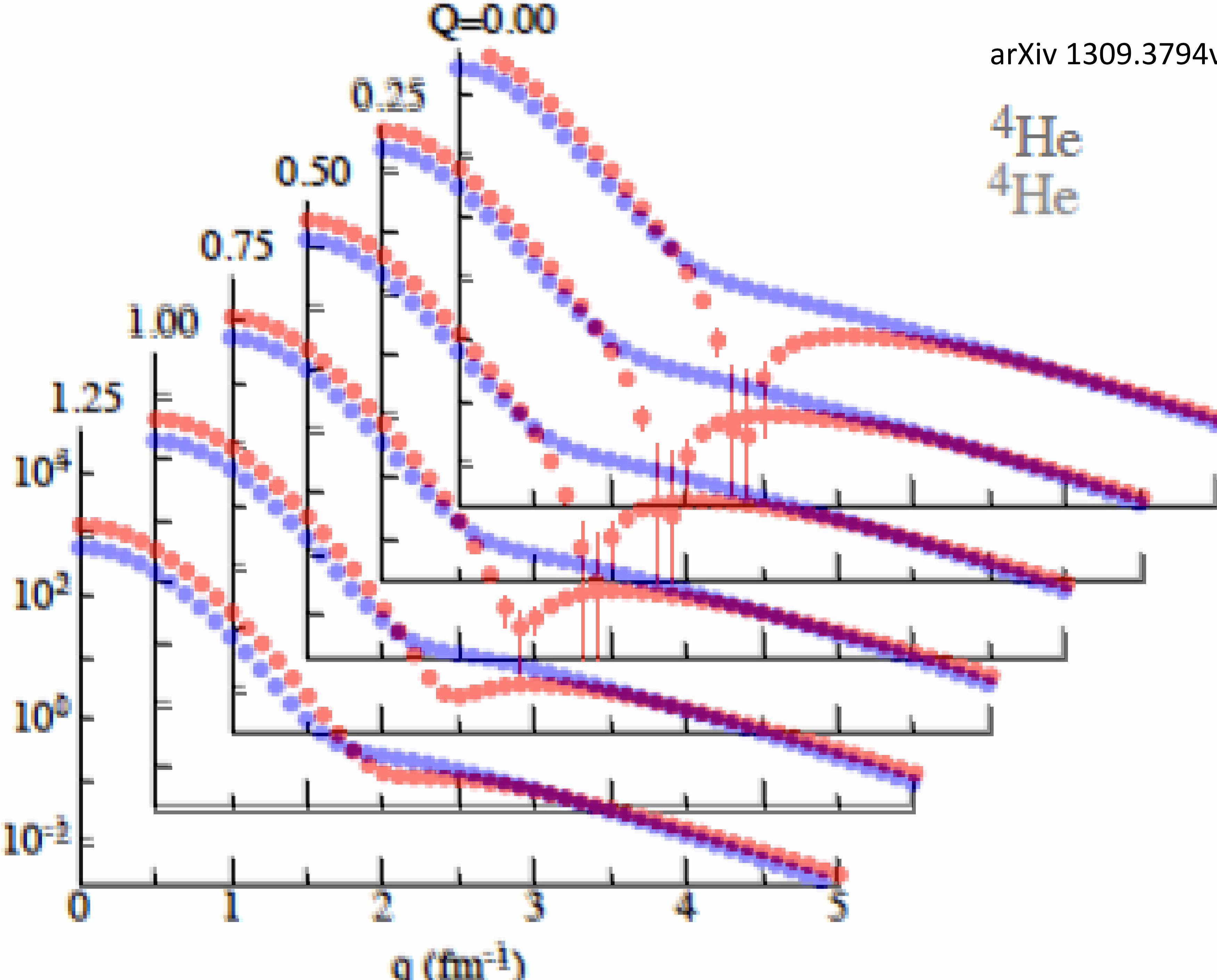
recent results at Los Alamos:
Gandolfi, Carlson,




$$(\tau = 0)$$

arXiv 1309.3794v1

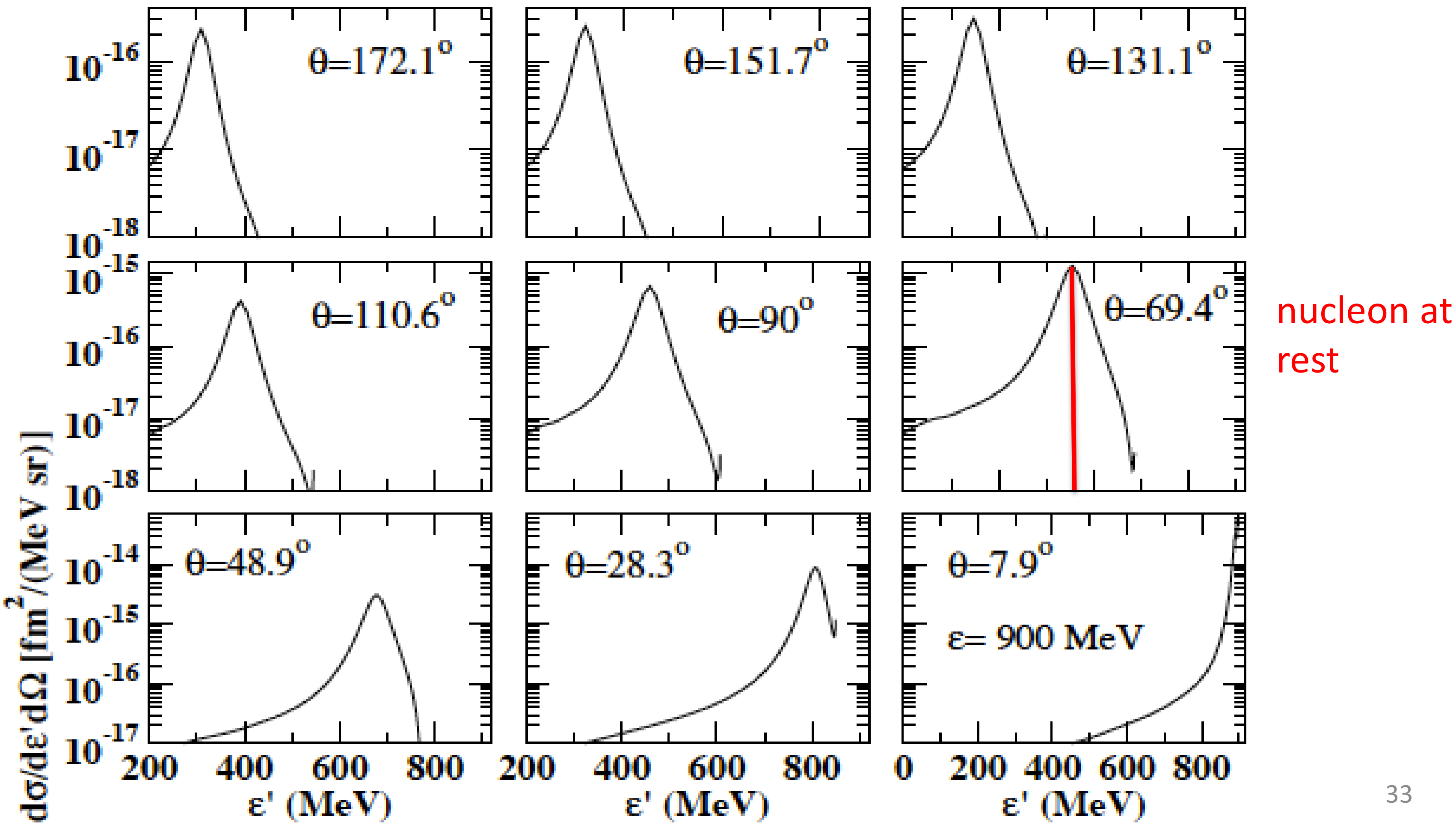


# ν-$^2$H Scattering (Theory)

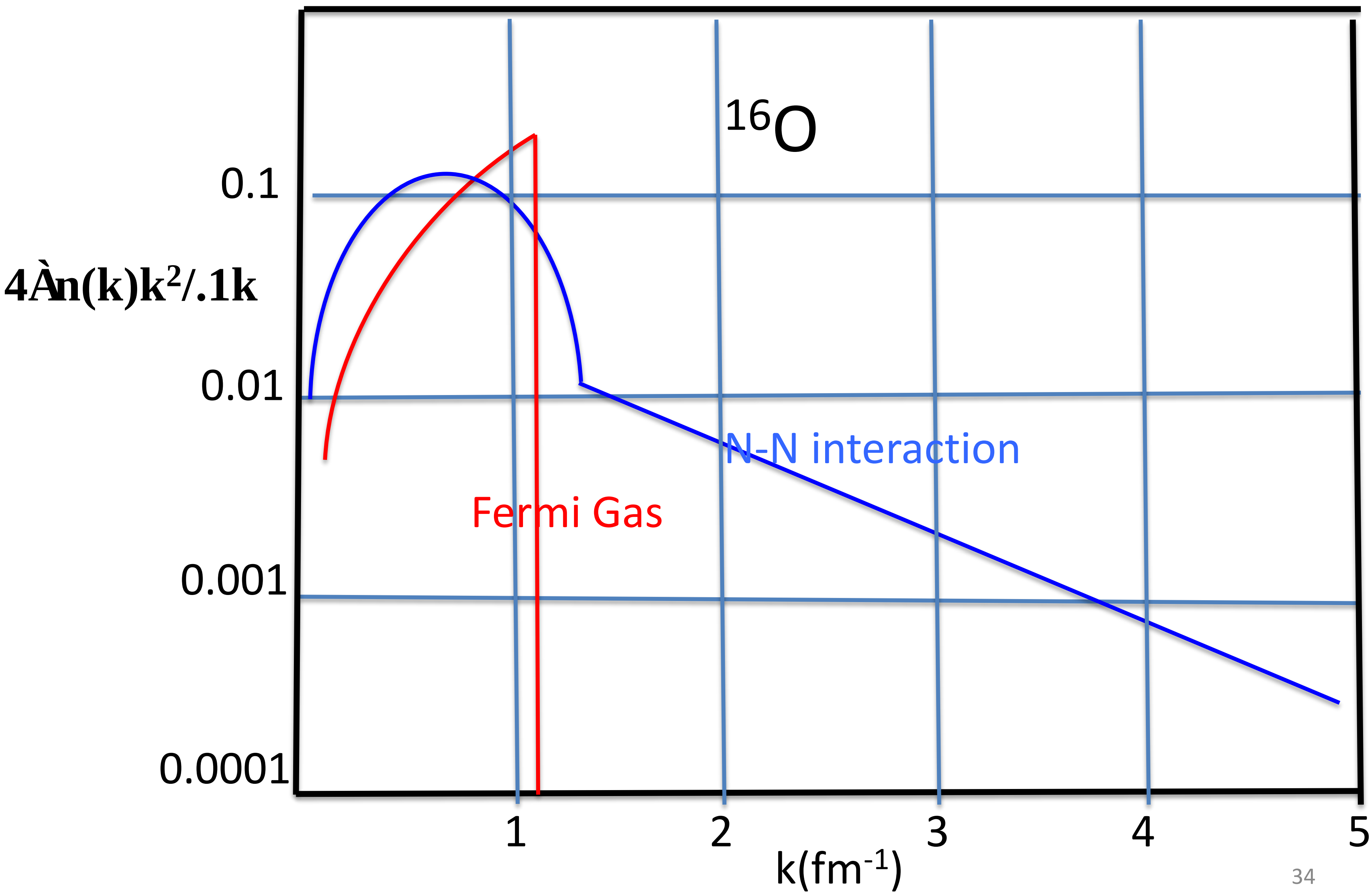
Phys. Rev. C 86, 035503 (2012)

$$\left(\frac{d\sigma}{d\epsilon' d\Omega}\right)_{\nu/\overline{\nu}} = \frac{G^2}{2\pi^2} k'\epsilon' F(Z,k') \cos^2\frac{\theta}{2}\left[R_{00} + \frac{\omega^2}{q^2}R_{zz} - \frac{\omega}{q}R_{0z} + \left(\tan^2\frac{\theta}{2} + \frac{Q^2}{2q^2}\right)R_{xx+yy} \mp \tan\frac{\theta}{2}\sqrt{\tan^2\frac{\theta}{2} + \frac{Q^2}{q^2}}\,R_{xy}\right]$$

**Calculated Lepton Energies for 900 MeV incident Neutrinos**



# Nucleon Momentum Distribution



$$\rho(\boldsymbol{r}_1,\boldsymbol{r}_2)=\sum_{ST}\rho_{(ST)}^{N_1N_2}(\mathbf{r}_1,\mathbf{r}_2)=\rho(\mathbf{r}_{rel},\mathbf{R}_{c.m.}),$$

$$\boldsymbol{r}_{rel}=\boldsymbol{r}_1-\boldsymbol{r}_2\equiv\boldsymbol{r} \qquad \boldsymbol{R}_{c.m.}=\frac{\boldsymbol{r}_1+\boldsymbol{r}_2}{2}\equiv\boldsymbol{R},$$

$$\int\rho(\boldsymbol{r}_1,\boldsymbol{r}_2)d\,\mathbf{r}_1\,d\,\mathbf{r}_2=\sum_{ST}\int\rho_{(ST)}^{N_1N_2}(\mathbf{r}_1,\mathbf{r}_2)d\,\mathbf{r}_1\,d\,\mathbf{r}_2=\sum_{ST}N_{(ST)}^{N_1N_2}=\frac{A(A-1)}{2}$$

$$\rho_{rel}(\boldsymbol{r})=\int\rho(\boldsymbol{r},\boldsymbol{R})\,d\boldsymbol{R} \quad \rho_{c.m.}(\boldsymbol{R})=\int\rho(\boldsymbol{r},\boldsymbol{R})d\,\boldsymbol{r}.$$