MATHEMATICS OF OPERATIONS RESEARCH
Vol. 00, No. 0, Xxxxx 0000, pp. 000–000
ISSN 0364-765X | EISSN 1526-5471 | 00 | 0000 | 0001

INFORMS
DOI 10.1287/xxxx.0000.0000

Correctional note: "Convex Duality in Stochastic Optimization and Mathematical Finance"

Teemu Pennanen
King's College London, teemu.pennanen@kcl.ac.uk, http://www.mth.kcl.ac.uk/~teemu

---

In my paper [2] (Math. of Oper. Res., 36(2):340–362, 2011), Theorem 2.2 is not valid as stated. It omits certain integrability conditions that are needed in general. A corrected version is given below. The additional conditions are satisfied in most of the applications given in [2]. For the remaining ones, sufficient conditions are given below. The topological results in Section 5 remain unaffected.

We denote

$$\mathcal{N}^\perp := \{v \in L^1(\Omega,\mathcal{F},\mathbb{R}^n) \,|\, E(x\cdot v) = 0 \;\forall x \in \mathcal{N}^\infty\}$$

and $\operatorname{dom}_1 Ef := \{x \in \mathcal{N} \,|\, \exists u \in L^p :\; Ef(x,u) < \infty\}$. As usual, the normal integrand conjugate to f is defined by

$$f^*(v,y,\omega) := \sup_{x,u}\{x\cdot v + u\cdot y - f(x,u,\omega)\}.$$

The corrected form of Theorem 2.2 is as follows. The proof is taken from that of Theorem 2 in Biagini, Pennanen and Perkkiö [1].

THEOREM 1. *The function $\omega \mapsto l(x(\omega),y(\omega),\omega)$ is measurable for any $x \in \mathcal{N}$ and $y \in L^q$ so the integral functional $I_l(x,y) = I_l(x(\omega),y(\omega),\omega)$ is well-defined on $\mathcal{N}\times L^q$. We have*

$$g(y) = \inf_{x\in\mathcal{N}^\infty} I_l(x,y)$$

as long as $\operatorname{dom} I_l(\cdot,y)\cap\mathcal{N}^\infty \subseteq \operatorname{dom}_1 Ef \cap \mathcal{N}^\infty$ and there exists $v \in \mathcal{N}^\perp$ such that $\inf_{x\in\mathcal{N}^\infty} I_l(x,y) = -Ef^*(v,y)$.

Proof. The measurability is proved as in [2]. Let $\tilde g(y) := \inf_{x\in\mathcal{N}^\infty} L(x,y)$. By the interchange rule ([4, Theorem 14.60]), $L(x,y) = I_l(x,y)$ for $x \in \operatorname{dom}_1 Ef$ and thus,

$$\tilde g(y) = \inf_{x\in\mathcal{N}^\infty\cap\operatorname{dom}_1 Ef} L(x,y) = \inf_{x\in\mathcal{N}^\infty\cap\operatorname{dom}_1 Ef} I_l(x,y) \ge \inf_{x\in\mathcal{N}^\infty} I_l(x,y).$$

The reverse inequality follows from the condition on domains. It thus suffices to prove $g = \tilde g$. Clearly, $\tilde g(y) \ge \inf_{x\in\mathcal{N}} L(x,y) = g(y)$. On the other hand, by Fenchel inequality,

$$f(x,u) + f^*(v,y) \ge x\cdot v + u\cdot y.$$

Thus, if $(x,u) \in \operatorname{dom} Ef$ and $v \in \mathcal{N}^\perp$ with $Ef^*(v,y) < \infty$, we have $E[x\cdot v] < \infty$. By [3, Lemma 1] this implies $E(x\cdot v) = 0$ so

$$g(y) = \inf_{x\in\mathcal{N},u\in L^p} E[f(x,u) - u\cdot y] \ge \sup_{v\in\mathcal{N}^\perp} -Ef^*(v,y) = \tilde g(y),$$

where the last equality follows from the last assumption and the fact that

$$\tilde{g}(y) = \inf_{x\in\mathcal{N}^\infty, u\in L^p} E[f(x,u) - u\cdot y] \geq \sup_{v\in\mathcal{N}^\perp} -Ef^*(v,y)$$

by the Fenchel inequality. □

The conditions of the above statement are satisfied in the examples of [2] as follows.

Examples 3.1 and 3.2: The conditions are satisfied under the assumption that $f_j(x,\cdot)\in L^p$ for all $x\in\mathbb{R}^n$, which was made in Example 3.2; see [1] for details.

Examples 3.3 and 3.4 are special cases of Example 3.2.

Example 3.5: It suffices to assume that $\operatorname{dom} I_h \neq \emptyset$ on L^p. Indeed, one then has $\mathcal{N}^\infty \subseteq \operatorname{dom}_1 I_f$ so the domain condition holds. Since

$$l(x,y,\omega) = \sum_{t=0}^T x_t\cdot y_t - h^*(y,\omega) \quad\text{and}\quad f^*(v,y,\omega) = \begin{cases} h^*(y,\omega) & \text{if } v = y, \\ +\infty & \text{if } v\neq y \end{cases}$$

we also have $\inf_{x\in\mathcal{N}^\infty} I_l(x,y) = -I_{f^*}(v,y)$ (both sides being equal to $-\infty$ unless $y\in\mathcal{N}^\perp$).

Example 3.6: Both conditions are satisfied provided for every $x\in\mathcal{N}^\infty$ with $x_t\in\operatorname{cl}\operatorname{dom}_1 L_t$ almost surely for all t, there exists $\bar{x}\in\mathcal{N}^\infty$ with $\bar{x}_t\in\operatorname{rint}\operatorname{dom}_1 L_t$ for all t and $\alpha\bar{x}+(1-\alpha)x\in\operatorname{dom}_1 I_f$ for all $\alpha\in(0,1)$; see [1, Example 5] for proof. Here, "rint" stands for relative interior.

Examples 4.2 and 4.4: Both are instances of Example 3.6 with $\operatorname{dom}_1 L_t = D_t$. In Example 4.2, the above condition posed for Example 3.6 is clearly satisfied with any $\bar{x}\in\mathcal{N}^\infty$ with $\bar{x}_t\in\operatorname{rint} D_t$ almost surely. In Example 4.4, it holds similarly if e.g. the utility functions are normalized so that $U_t(0,\omega)=0$.

Example 4.3: Here the domain condition holds as an equality with both $\operatorname{dom} I_l(\cdot,y)\cap\mathcal{N}^\infty$ and $\operatorname{dom}_1 I_f\cap\mathcal{N}^\infty$ equal to $\{x\in\mathcal{N}^\infty \mid x_t\in D_t\}$. As shown in the example,

$$\inf_{x\in\mathcal{N}^\infty} I_l(x,y) = \begin{cases} -E\left[\sum_{t=0}^{T-1}\sigma_{D_t(\omega)}(E_t\Delta y_{t+1}) + \sum_{t=0}^T \sigma_{C_t(\omega)}(y_t)\right] & \text{if } Ep\cdot y = 1, \\ -\infty & \text{otherwise.} \end{cases}$$

Moreover,

$$\begin{aligned} f^*(v,y,\omega) &= -\inf_x \sum_{t=0}^T [l_t(x_t,y,\omega) - x_t\cdot v_t] \\ &= \begin{cases} \sum_{t=0}^{T-1}\sigma_{D_t(\omega)}(v_t+\Delta y_{t+1}) + \sum_{t=0}^T\sigma_{C_t(\omega)}(y_t) & \text{if } 1-p\cdot y = v_0^0 \\ +\infty & \text{otherwise,} \end{cases} \end{aligned}$$

where v_0^0 denotes the component of v_0 corresponding to α. So $\inf_{x\in\mathcal{N}^\infty} I_l(x,y) = -I_{f^*}(v,y)$ where $v\in\mathcal{N}^\perp$ is given by $v_t := E_t\Delta y_{t+1} - \Delta y_{t+1}$ and $v_0^0 = 1-p\cdot y$.

Acknowledgements. The author is indebted to Ari-Pekka Perkkiö for finding the errors as well as the corrections.

References

[1] S. Biagini, T. Pennanen, and A.-P.. Perkkiö. Duality and optimality conditions in stochastic optimization and mathematical finance. *manuscript*, 2015.

[2] T. Pennanen. Convex duality in stochastic optimization and mathematical finance. *Mathematics of Operations Research*, 36(2):340–362, 2011.

[3] A.-P. Perkkiö. Stochastic programs without duality gaps for objectives without a lower bound. *manuscript*, 2014.

[4] R. T. Rockafellar and R. J.-B. Wets. *Variational analysis*, volume 317 of *Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences]*. Springer-Verlag, Berlin, 1998.