Subspaces of $\mathbb{R}^n$ 04/04

Summary

$\mathbb{R}^n$ is a 'vector space'. Its elements are vectors (thought of as rows or columns, each with n coordinates) that can be multiplied by scalars, and added or subtracted.

A ***subspace*** is a subset V of $\mathbb{R}^n$ that is closed under

(S1) addition of its elements,

(S2) multiplication by scalars.

Any such V must contain $\mathbf{0}$.

Suppose that $\mathbf{u}_1, \mathbf{u}_2$ belong to a subspace V of $\mathbb{R}^n$. Then

$$a_1\mathbf{u}_1 + a_2\mathbf{u}_2 \in V \qquad \text{for all} \quad a_1, a_2 \in \mathbb{R}.$$

The set $\mathscr{L}\{\mathbf{u}_1, \mathbf{u}_2\}$ of all such LC's is already a subspace.

When $n = 2$ and $\mathbf{u}_1, \mathbf{u}_2$ are LI then $\mathscr{L}\{\mathbf{u}_1, \mathbf{u}_2\} = \mathbb{R}^2$. If $\mathbf{u}_2$ is a multiple of $\mathbf{u}_1$ then $\mathscr{L}\{\mathbf{u}_1, \mathbf{u}_2\} = \mathscr{L}\{\mathbf{u}_1\}$ represents a line passing through the origin $\mathbf{0}$ (or the zero subspace $\{\mathbf{0}\}$).

More generally, the set of linear combinations

$$\mathscr{L}\{\mathbf{u}_1, \dots, \mathbf{u}_k\} = \{a_1\mathbf{u}_1 + \cdots + a_k\mathbf{u}_k : a_i \in \mathbb{R}\}$$

of k vectors is the smallest subspace containing them.