Row equivalence 31/03

SUMMARY

Let M, M' be two matrices.

We say that M is ***row equivalent*** to M' if M, M' have the same size and M' can be obtained from M by one or more ERO's. We indicate this by writing $M \sim M'$.

It is obvious that, for any matrices M, M', M'', we have

- $M \sim M$,
- $M \sim M', \; M' \sim M'' \;\Rightarrow\; M \sim M''$.

But because each type of ERO can be reversed, we also have

- $M \sim M' \;\Rightarrow\; M' \sim M$.

These three properties mean that $\sim$ is an ***equivalence relation.***

Now suppose that $M = (A \,|\, B)$ and $M' = (A' \,|\, B')$ represent linear systems with B, B' column vectors.

Theorem. $M \sim M'$ iff the two systems $AX = B$ and $A'X = B'$ have the same solutions.

If $M \sim M'$ then $r(M) = r(M')$ and $r(A) = r(A')$ because row reduction preserves the rank of matrices. By (RC1), solutions only exist if all these ranks are all equal.