Row reduction and rank 28/03

Summary

The second matrix was obtained by row reducing the first:

$$A = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 3 & 4 & 5 & 6 \\ 6 & 7 & 8 & 9 \\ 9 & 9 & 9 & 9 \end{pmatrix}, \qquad A'' = \begin{pmatrix} \boxed{1} & 0 & -1 & -2 \\ 0 & \boxed{1} & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Since A'' is step-reduced and has 2 markers, A has rank 2.

In this example, *any* two rows of A are linearly independent (neither is a multiple of the other). But each zero row of A'' corresponds to a linear relation between three rows of A:

$$R_3 - 2R_2 + R_1 = \mathbf{0}, \qquad R_4 - 3R_2 + 3R_1 = \mathbf{0}.$$

These relations DO NOT hold for the rows of A''.

Each unmarked column of A'' can be expressed as a linear combination of the marked columns:

$$C_3 = -C_1 + 2C_2, \qquad C_4 = -2C_1 + 3C_2.$$

These relations DO hold for the columns of A (or any matrix obtained from A by ERO's).

In general:

- **The columns of a matrix that become marked after reduction will be LI to start with.**
- **Row operations preserve linear relations between columns, but not those between rows.**