Surfaces of revolution 21/06

Start with a curve

$$\boldsymbol{\gamma}(t) = (f(t),\, 0,\, g(t)), \qquad a \leqslant t \leqslant b,$$

in the xz**-plane and rotate it around the** z**-axis by an angle** u**. The resulting surface is parametrized by**

$$\boldsymbol{\sigma}(u,v) = (f(v)\cos u,\ f(t)\sin u,\ g(v)),$$

where $u \in [0, 2\pi)$ **and** $t = v \in [a,b]$.

Examples: 1. For the sphere, start with $x = f(t) = \cos t$ **and** $z = g(t) = \sin t$**. If we replace** $x = \cos t$ **by** $x = 2 + \cos t$ **we obtain a torus of revolution**

$$\boldsymbol{\sigma}(u,v) = ((2+\cos v)\cos u,\ (2+\cos v)\sin u,\ \sin v),$$

which satisfies $(x^2+y^2+z^2-5)^2 + 16z^2 = 16$.

2. Starting with the line $f(t) = t,\ g(t) = mt$ **(**$m > 0$ **fixed) gives a cone. Taking** $f(t) = 1,\ g(t) = t$ **gives a cylinder.**

3. Taking the hyperbola $x^2 - z^2 = 1$ **with**

$$f(t) = \cosh t = \tfrac{1}{2}(e^t + e^{-t}), \qquad g(t) = \sinh t = \tfrac{1}{2}(e^t - e^{-t})$$

gives the hyperboloid of one sheet

$$\boldsymbol{\sigma} = (\cosh v\cos u,\ \cosh v\sin u,\ \sinh v).$$

Replacing $z = \sinh v$ **with** $z = v$ **gives the catenoid**

$$\boldsymbol{\sigma} = (\cosh v\cos u,\ \cosh v\sin u,\ v)$$

illustrated: