Conics and Quadrics 13/06

A *conic* $\mathscr{C}$ is the set of points (x, y) in $\mathbb{R}^2$ determined by an equation of the form

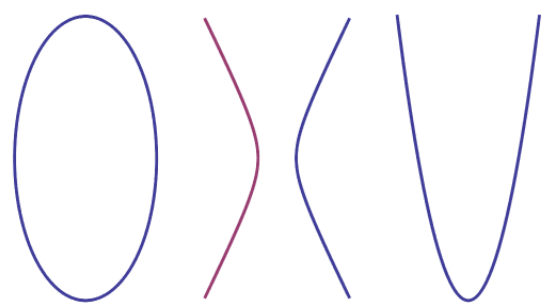
$$\underbrace{ax^2 + 2bxy + cy^2}+2dx + 2ey + f = 0,$$

where $a, \dots, f$ are real numbers (with a, b, c not all zero). There are 8 essentially different types of conics!

Let $S = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$ **and** $T = \begin{pmatrix} a & b & d \\ b & c & e \\ d & e & f \end{pmatrix}$.

If $\mathscr{C}$ has more than one point and does not contain a line, it is an ellipse, parabola or hyperbola and $\det T \neq 0$. In this case,

- $\mathscr{C}$ **is an ellipse** $\Leftrightarrow$ $\det S > 0$
- $\mathscr{C}$ **is a hyperbola** $\Leftrightarrow$ $\det S < 0$
- $\mathscr{C}$ **is a parabola** $\Leftrightarrow$ $\det S = 0$



If $\det T = 0$ then

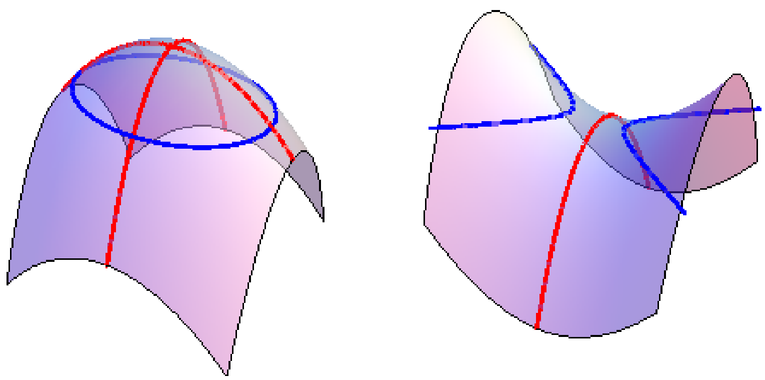
- $\det S > 0 \;\Rightarrow\; \mathscr{C}$ **is a point.**
- $\det S < 0 \;\Rightarrow\; \mathscr{C}$ **is two incident lines.**
- $\det S = 0 \;\Rightarrow\; \mathscr{C}$ **is two parallel lines, or one, or $\emptyset$.**

A ***quadric*** $\mathcal{Q}$ **is the locus of points** (x, y, z) **in** $\mathbb{R}^3$ **satisfying an equation of the form**

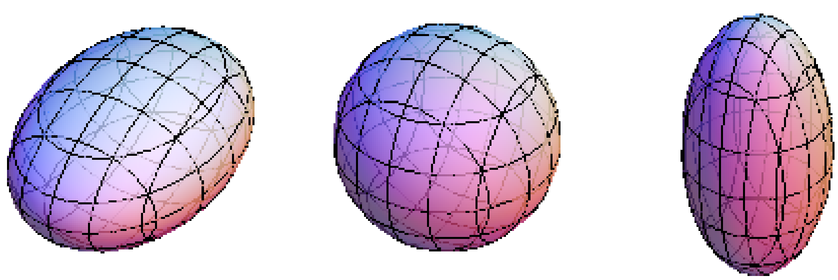
$$\underbrace{ax^2+by^2+cz^2+2dyz+2ezx+2fxy}+2gx+2hy+2kz+\ell=0$$

(with not all a, b, c, d, e, f **zero). There are 15 types of quadrics!**

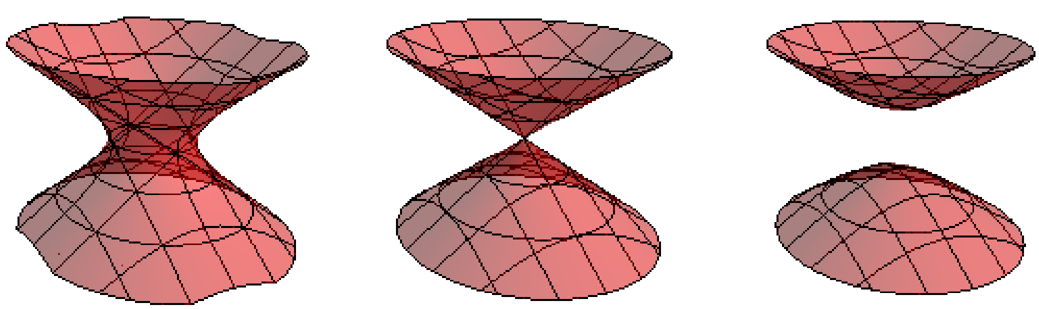
Elliptic and hyperbolic paraboloids $z = ax^2 \pm by^2$ **:**



Ellipsoids $ax^2 + by^2 + cz^2 = 1$ **with** $a, b, c > 0$ **:**



Hyperboloids and cone $ax^2+by^2-cz^2 = -\ell$ **with** $a, b, c > 0$ **:**



8 types of cylinder over a conic including $\emptyset$**, finally a point!**