Linear systems of equations 21/03

SUMMARY

Here is a linear system of m equations in n unknowns:

$$\left\{\begin{array}{rcl} a_{11}x_1+\cdots+a_{1n}x_n &=& b_1\\ a_{21}x_1+\cdots+a_{2n}x_n &=& b_2\\ &\cdots& \\ &\cdots& \\ a_{m1}x_1+\cdots+a_{mn}x_n &=& b_m. \end{array}\right.$$

It is equivalent to the single matrix equation

$$AX=B \quad \text{(also written } A\mathbf{x}=\mathbf{b}\text{)} \tag{1}$$

where

$$A=(a_{ij})\in\mathbb{R}^{m,n}$$

is the matrix of assigned coefficients visible on the left (so $i=1,\dots,m$ and $j=1,\dots,n$),

$$X=(x_j)\in\mathbb{R}^{n,1}$$

is the column of unknowns (variables), and

$$B=(b_i)\in\mathbb{R}^{m,1}$$

is the column of assigned numbers on the right.

A *solution* is merely a column vector X satisfying (1); it represents one set of values of $x_1,\dots,x_n$ satisfying all m equations simultaneously. The system is said to be *inconsistent* (or the equations *incompatible*) if there is no such solution.

The system is *homogeneous* if $B=0$. This always has the *trivial solution* $X=0$, meaning $x_1=\cdots=x_n=0$. There may or may not be other solutions.