Curves in space 19/05

SUMMARY

A curve in space is described by a mapping

$$\begin{array}{rcl} \gamma\colon I & \to & \mathbb{R}^3 \\ t & \mapsto & (x(t), y(t), z(t)). \end{array}$$

We usually require the components to be differentiable, and

$$\gamma'(t) = (x'(t), y'(t), z'(t))$$

is the *velocity* at 'time' t. If $\gamma'(t) \neq \mathbf{0}$ it represents a vector *tangent* to the image at $\gamma(t)$.

Example. If $\gamma(t) = (x_0 + At,\ y_0 + Bt,\ z_0 + Ct)$ then

$$\gamma'(t) = (A, B, C) = \mathbf{p}$$

is a constant vector parallel to the line $r = \mathrm{Im}(\gamma)$.

If $z(t)$ is identically zero (or absent), γ is a *plane curve*. In this case, it can also be described by one implicit equation relating x and y.

Example. If

$$\gamma(t) = \Big(\frac{1-t^2}{1+t^2},\ \frac{2t}{1+t^2}\Big)$$

then $x^2 + y^2 = 1$ and $\mathrm{Im}(\gamma)$ is a circle (with $t = \tan\frac{\theta}{2}$.)