Parametric equation of a line 17/05

SUMMARY

A line r is determined by a vector $\mathbf{p}=(A,B,C)$ parallel to it and a point P_0 on it. If $\mathbf{v}_0=\vec{OP_0}=(x_0,y_0,z_0)$, a generic point P on r is

$$(x,y,z)=(x_0+At,\ y_0+Bt,\ z_0+Ct),\quad t\in\mathbb{R}.$$

Its position vector is

$$\vec{OP}=\mathbf{v}=\mathbf{v}_0+t\,\mathbf{p},$$

and so

$$\mathbf{v}\times\mathbf{p}=\mathbf{q}$$

where $\mathbf{p}$ and $\mathbf{q}=\mathbf{v}_0\times\mathbf{p}$ are fixed vectors such that $\mathbf{p}\cdot\mathbf{q}=0$.

Given two distinct lines r_1,r_2, there are three possibilities, namely they are

(i) *parallel* ($\mathbf{p}_1,\mathbf{p}_2$ are proportional), or

(ii) *incident* (r_1,r_2 intersect in a single point), or

(iii) *skew* (there is no plane that contains both r_1,r_2).

Given equations for r_1,r_2 one must first check that they are neither coincident nor parallel, and then solve equations to see whether or not there exists a point of intersection.

In (ii), the plane containing r_1,r_2 will have a normal vector

$$\mathbf{n}=\mathbf{p}_1\times\mathbf{p}_2.$$