The equation of a plane 16/05

SUMMARY

Having fixed Cartesian coordinates in space, an equation

$$ax + by + cz = d$$

determines a ***plane*** π perpendicular to the vector $\mathbf{n} = (a, b, c)$. The vector form of the equation is

$$\mathbf{n} \cdot \mathbf{v} = d,$$

where $\mathbf{v} = (x, y, z)$ represents an arbitrary point P of π.

The distance of a point P_0 from π equals

$$\frac{|\mathbf{v}_0 \cdot \mathbf{n} - d|}{\|\mathbf{n}\|} = \frac{|ax_0 + by_0 + cz_0 - d|}{\sqrt{a^2 + b^2 + c^2}}$$

where $\mathbf{v}_0 = (x_0, y_0, z_0)$, and π contains the origin iff $d = 0$.

Given two planes that are not parallel, their equations

$$\begin{aligned} \pi_1 : \quad & a_1 x + b_1 y + c_1 z = d_1 \\ \pi_2 : \quad & a_2 x + b_2 y + c_2 z = d_2, \end{aligned}$$

define a linear system with ∞^1 solutions depending on one free parameter t, and the intersection $\pi_1 \cap \pi_2$ is a ***line*** r.

The line r is parallel to the cross product $\mathbf{n}_1 \times \mathbf{n}_2 = (\alpha, \beta, \gamma)$, and the ***parametric equation*** of r is

$$(x, y, z) = (x_0 + \alpha t,\ y_0 + \beta t,\ z_0 + \gamma t), \quad t \in \mathbb{R},$$

where (x_0, y_0, z_0) is any fixed point in r.