Diagonalizing symmetric matrices 10/05

SUMMARY

A square matrix S is called *symmetric* if $S^T = S$.

A square matrix P is called *orthogonal* if $P^T P = I$.

Let $S \in \mathbb{R}^{n,n}$ be a symmetric matrix. It is a theorem that

- **the roots of $p(x) = \det(S - xI)$ are all *real*;**
- **eigenvectors $\mathbf{v}_1, \mathbf{v}_2$ associated to *distinct* eigenvalues λ_1, λ_2 are orthogonal: $\mathbf{v}_1 \cdot \mathbf{v}_2 = 0$;**
- **S is always diagonalizable.**

It follows that one can choose an

- ***orthonormal* basis of $\mathbb{R}^n$ consisting of eigenvectors of S;**
- ***orthogonal* matrix P such that $P^{-1}SP = P^T SP$ is diagonal.**