Diagonalizable matrices 09/05

Summary

Let $A, P \in \mathbb{R}^{n,n}$ be square matrices. If the columns of P are eigenvectors of A then $AP = PD$. If these columns are also LI,

$$P^{-1}AP = D, \tag{1}$$

where D is a *diagonal* matrix with entries the eigenvalues in order. In this case, we say that A is *diagonalizable*.

Conversely, if (1) holds with D diagonal, then the columns of P will constitute a basis of eigenvectors of A.

The matrix A will be diagonalizable if and only if

$$\dim E_\lambda = \mathrm{mult}(\lambda)$$

for each root λ of $p(x) = \det(A - xI)$ of multiplicity > 1. Moreover, for P and D to be *real* matrices, all the roots of $p(x)$ must be *real*.

Examples: $A_1 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, $A_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

A_1 **has eigenvalues** $0, 0$ **but** $\dim E_0 = 1$, **so** P **does not exist.**

A_2 **has eigenvalues** $i, -i$, **so** $P^{-1}DP = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$ **for some** $P \in \mathbb{C}^{2,2}$.