Eigenspaces 05/05

SUMMARY

Let A be a square matrix. If λ is a root of the characteristic polynomial $p(x) = \det(A - xI)$, the associated ***eigenspace*** is

$$E_\lambda = \mathrm{Ker}(A - \lambda I).$$

Its dimension equals $n - r(A - \lambda I)$, and is no greater than the ***multiplicity*** of λ as a root of $p(x)$.

One can find the eigenvectors associated to λ, and a basis of E_λ, by row-reducing $A - \lambda I$ so as to solve the homogeneous linear system $(A - \lambda I)\mathbf{v} = \mathbf{0}$.

If $A \in \mathbb{R}^{n,n}$ has n ***distinct*** real eigenvalues then the associated eigenvectors are LI and therefore form a basis of $\mathbb{R}^n$. But this can happen in other cases; for example, when A is a multiple of the identity matrix, ***any*** basis of $\mathbb{R}^n$ consists of eigenvectors.

It is known that if, more generally, $A \in \mathbb{R}^{n,n}$ is symmetric then the roots of $p(x)$ are all real, and there is a basis of eigenvectors.