Linear maps, kernels, images 28/04

SUMMARY

Let

$$f : U^n \longrightarrow V^m, \qquad g : V^m \longrightarrow W^p$$

be linear mappings (superscripts indicate dimensions of the vector spaces). Then the composite map

$$g \circ f : U^n \longrightarrow W^p$$

is linear, and its associated matrix is the product of the matrices associated to g and f:

$$A_{g \circ f} = A_g \cdot A_f.$$

Given a linear map $h : V \longrightarrow W$, the subsets

$$\operatorname{Ker} h \subseteq V \quad \textbf{and} \quad \operatorname{Im} h \subseteq W$$

are subspaces:

$\dim \operatorname{Ker} h$ **is called the 'nullity of h',**

$\dim \operatorname{Im} h$ **is called the 'rank of h' (and equals $\operatorname{rank} A_h$)**

Corollary of the rank-nullity theorem:

$$h \textbf{ is one-to-one} \iff \operatorname{rank} f = \dim V$$

$$h \textbf{ is onto} \iff \operatorname{rank} f = \dim W$$

$$h \textbf{ is bijective} \iff \operatorname{rank} f = \dim V = \dim W$$