Bases and linear maps 18/04

SUMMARY

Let V be a vector space of dimension n, so it has a basis $\{\mathbf{v}_1, \dots, \mathbf{v}_n\}$. We know that there exists an isomorphism

$$\mathbb{R}^n \longleftrightarrow V$$

that allows us to regard vectors in V as rows of a matrix with n columns, and prove the

Theorem.
(i) If m vectors are LI in V then $m \leqslant n$.
(ii) If p vectors span V then $p \geqslant n$.

In particular, any basis of V has n elements and it is enough to check one of the two conditions.

Let $\{\mathbf{e}_1, \dots, \mathbf{e}_n\}$ be the canonical basis of $\mathbb{R}^{n,1}$. If f is the linear mapping defined by a matrix $A \in \mathbb{R}^{m,n}$ then $f(\mathbf{e}_1)$ is determined by the first column $A\mathbf{e}_1$ of A, and so on.

Conversely, given *any* linear mapping $g: V \to W$ between vector spaces, we shall use the images $g(\mathbf{v}_j)$ of elements of a basis $\{\mathbf{v}_1, \dots, \mathbf{v}_n\}$ of V to construct column-by-column a matrix B representing g.