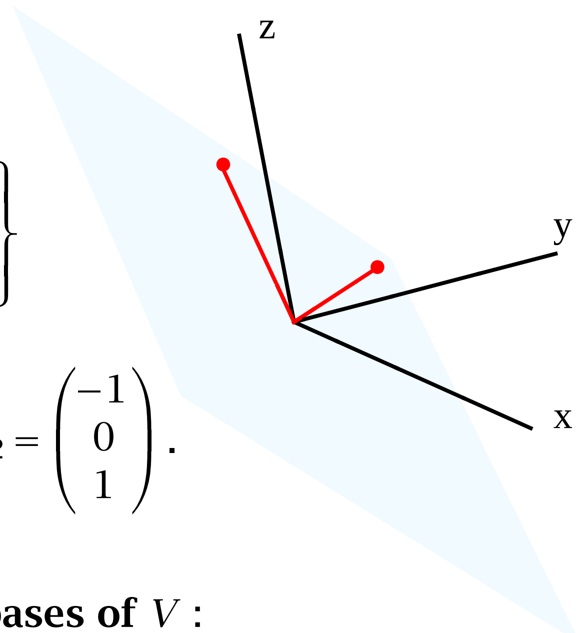
Bases of subspaces 05/04

Summary

A second way to define subspaces is via equations: the set of solutions X of any *homogeneous* linear system $AX = \mathbf{0}$ is a subspace. We can add solutions to get to new solutions, and multiply them by scalars.



Example. Let

$$V = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 : x + y + z = 0 \right\}$$

$$= \mathscr{L}\{\mathbf{u}_1, \mathbf{u}_2\}, \quad \mathbf{u}_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, \ \mathbf{u}_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}.$$

The set $\{\mathbf{u}_1, \mathbf{u}_2\}$ is one of many bases of V :

Definition. Given a subspace V of $\mathbb{R}^n$, a *basis* is a set of *linearly independent* vectors $\mathbf{v}_1, \ldots, \mathbf{v}_k$ that *generate* V in the sense that

$$V = \mathscr{L}\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}.$$

We shall see that any two bases have the same number of elements, called the *dimension* of the subspace V.