Matrix addition and multiplication 14/03

SUMMARY

$\mathbb{R}^{m,n}$ is the set of matrices (arrays) of size $m \times n$.

Any matrix $A \in \mathbb{R}^{m,n}$ contains mn entries $a_{ij} \in \mathbb{R}$.

If $A, B \in \mathbb{R}^{m,n}$, their sum $A + B$ is defined by adding the entries in corresponding positions.

If $A \in \mathbb{R}^{m,n}$ and $\lambda \in \mathbb{R}$, to compute the product λA the number λ must multiply every element of A.

Example of special matrices:

$$(1 \quad 0 \quad -5 \quad 8) \in \mathbb{R}^{1,4}, \qquad \begin{pmatrix} 1 \\ 0 \\ -5 \\ 8 \end{pmatrix} \in \mathbb{R}^{4,1}$$

Both represent the vector $(1, 0, -5, 8) \in \mathbb{R}^4$.

They are related by the operation of transpose

$$(1 \quad 0 \quad -5 \quad 8)^{\mathrm{T}} = \begin{pmatrix} 1 \\ 0 \\ -5 \\ 8 \end{pmatrix}.$$

A matrix A is said to be symmetric if $A = A^{\mathrm{T}}$. Examples:

$$\begin{pmatrix} 1 & 8 \\ 8 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -7 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

The third is the ‘identity matrix’, the last the ‘zero matrix’.

One can define the ‘dot product’ between any two vectors of the same ‘length’. If

$$\mathbf{r} = (a_1 \ \cdots a_n), \qquad \mathbf{c} = \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix}$$

then

$$\mathbf{r} \cdot \mathbf{c} = a_1 b_2 + \cdots + a_n b_n = \sum_{i=1}^{n} a_i b_i .$$

Given matrices $A \in \mathbb{R}^{m,n}$ e $B \in \mathbb{R}^{n,p}$, let

$\mathbf{r}_1, \dots, \mathbf{r}_m$ be the rows of A, $\qquad \mathbf{c}_1, \dots, \mathbf{c}_p$ the columns of B.

Then

$$AB = (\mathbf{r}_i \cdot \mathbf{c}_j) \in \mathbb{R}^{m,p}$$

is the matrix whose entry in row i and column j is $\mathbf{r}_i \cdot \mathbf{c}_j$.

In particular, the matrix product $\mathbf{r}\,\mathbf{c}$ is the 1×1 matrix with entry $\mathbf{r} \cdot \mathbf{c}$.