Waves at King's

Zoe Wyatt

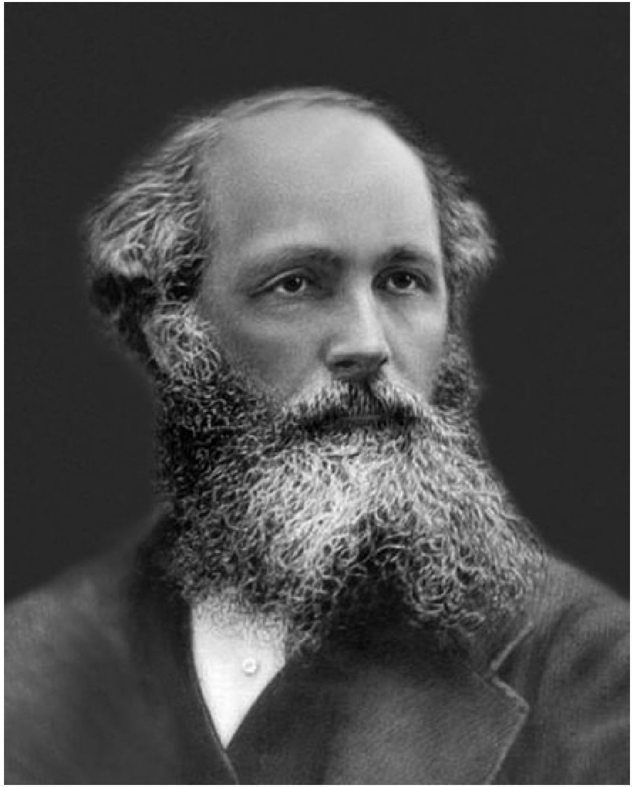
Talk Plan

- Electromagnetic waves and **Maxwell**
- Special Relativity and GPS
- Gravitational waves and **Bondi**

James Clerk Maxwell

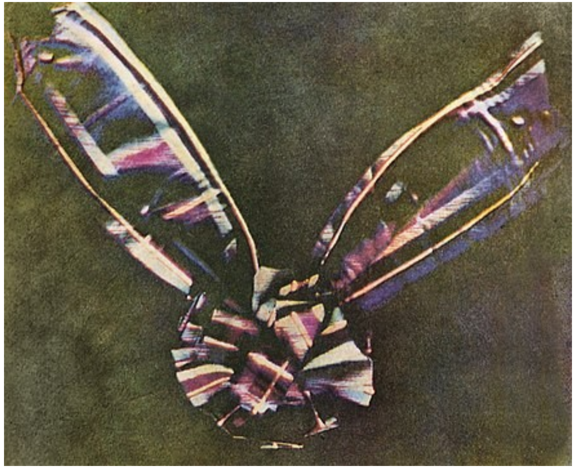
- 1831 – 1879

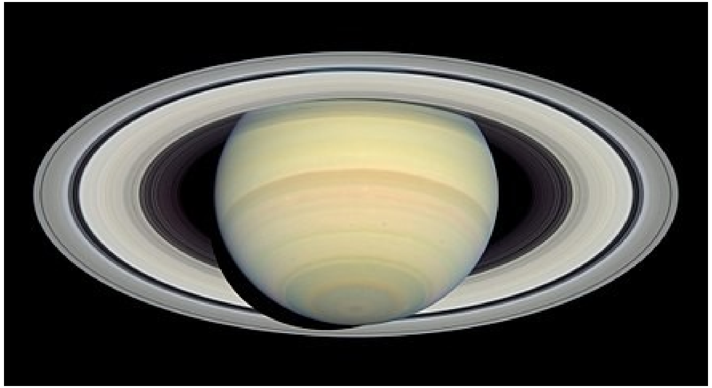
- Edinburgh → Cambridge → Marischal College (Aberdeen) → redundant → **King's** (1860-1865) → Glenlair → Cambridge (1871 --)



James Clerk Maxwell

- Electromagnetism (1855 - 1873)
- stability and matter type of Saturn’s rings (1859)
- Colour theory (1861, Royal Institution)
- kinetic theory and thermodynamics





Maxwell’s poems

An inextensible heavy chain

Lies on a smooth horizontal plane,

An impulsive force is applied at A,

Required the initial motion of K.

Let ds be the infinitesimal link,

Of which for the present we’ve only to think;

Let T be the tension, and T + dT

The same for the end that is nearest to B.

Let a be put, by a common convention,

For the angle at M ‘twixt OX and the tension;

Let Vt and Vn be ds‘s velocities,

Of which Vt along and Vn across it is;

Then Vn/Vt the tangent will equal,

Of the angle of starting worked out in the sequel.

….

Maxwell’s legacy

From a long view of the history of mankind, seen from, say, ten thousand years from now, there can be little doubt that the most significant event of the 19th century will be judged as Maxwell’s discovery of the laws of electrodynamics.

(Richard Feynman)

Einstein, when he visited the University of Cambridge in 1922, was told by his host that he had done great things because he stood on Newton's shoulders; Einstein replied:

No I don't. I stand on the shoulders of Maxwell.

Electricity and Magnetism

- Rocks have magnetic properties → compass needles
- 1700s: property called *electric charge* leading to force between charged particles

$$\mathbf{F} = \frac{Q_1 Q_2 (\mathbf{x_1} - \mathbf{x_2})}{4\pi\epsilon_0 |\mathbf{x_1} - \mathbf{x_2}|^3}$$

  (Coulomb's law)

- Electric charge is conserved: go from micro → macro view
- 1800s: batteries with electric currents that flow

Electricity and Magnetism

Coulomb's law

$$\mathbf{F} = \frac{Q_1 Q_2(\mathbf{x_1} - \mathbf{x_2})}{4\pi\epsilon_0|\mathbf{x_1} - \mathbf{x_2}|^3}$$

- Experiments on test particle with charge q, position x and velocity v show it experiences a force called the Lorentz force

$$\mathbf{F} = q\left[\mathbf{E}(t,\mathbf{x}) + \mathbf{v} \times \mathbf{B}(t,\mathbf{x})\right]$$

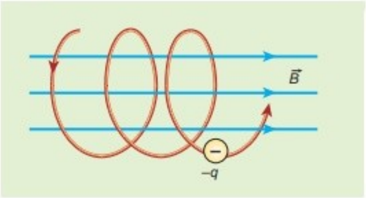


Figure 3.51 Helical path of the electron in a uniform magnetic field

- Coulomb (action at a distance) vs Lorentz force (local)
- Ie: test body responds to E, B at same point → fields are themselves created by the charge/current distribution.

Maxwell’s equations (1961)

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

Maxwell’s equations

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- Electric charge always → an electric field.
- No corresponding notion of magnetic charge
- time-dependent magnetic field → an electric field
- an electric current will produce a magnetic field, and a time-varying electric field can produce a magnetic field

Maxwell’s equations

Taking a curl of (M3) gives

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \times (\nabla \times \mathbf{E}) = -\frac{\partial}{\partial t}\nabla \times \mathbf{B}$$

Maxwell’s equations

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Taking a curl of (M3) gives

$$\nabla \times (\nabla \times \mathbf{E}) = -\frac{\partial}{\partial t}\nabla \times \mathbf{B}$$

use a vector calculus identity on the LHS and (M4) on the RHS

$$\nabla\nabla \cdot \mathbf{E} - \nabla^2 \mathbf{E} = -\mu_0 \frac{\partial \mathbf{J}}{\partial t} - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2}$$

Maxwell's equations

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use (M1) and rearrange to obtain

$$-\frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} + \nabla^2 \mathbf{E} = \frac{1}{\epsilon_0}\nabla\rho + \mu_0 \frac{\partial \mathbf{J}}{\partial t}$$

Maxwell's equations

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

take a curl of (M4) we obtain

$$\nabla \nabla \cdot \mathbf{B} - \nabla^2 \mathbf{B} = \mu_0 \nabla \times \mathbf{J} + \frac{1}{c^2}\frac{\partial}{\partial t}\nabla \times \mathbf{E}$$

Maxwell’s equations

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

take a curl of (M4) we obtain

$$\nabla\nabla \cdot \mathbf{B} - \nabla^2 \mathbf{B} = \mu_0 \nabla \times \mathbf{J} + \frac{1}{c^2}\frac{\partial}{\partial t}\nabla \times \mathbf{E}$$

using (M2) and (M3) gives

$$-\frac{1}{c^2}\frac{\partial^2 \mathbf{B}}{\partial t^2} + \nabla^2 \mathbf{B} = -\mu_0 \nabla \times \mathbf{J}$$

Maxwell's equations

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

$$\Rightarrow$$

$$-\frac{1}{c^2}\frac{\partial^2 \mathbf{B}}{\partial t^2} + \nabla^2 \mathbf{B} = -\mu_0 \nabla \times \mathbf{J}$$

$$-\frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} + \nabla^2 \mathbf{E} = \frac{1}{\epsilon_0}\nabla\rho + \mu_0 \frac{\partial \mathbf{J}}{\partial t}$$

- Inhomogeneous wave equations
- In vacuum → linear wave equation
- Maxwell's equations admit solutions describing waves propagating with speed c. → at the speed of light → light is an electromagnetic wave

Monochromatic plane waves

$$\mathbf{E} = \mathrm{Re}\left(\mathbf{E}_0 e^{-i\omega t + i\mathbf{k}\cdot\mathbf{x}}\right) \qquad \mathbf{B} = \mathrm{Re}\left(\mathbf{B}_0 e^{-i\omega t + i\mathbf{k}\cdot\mathbf{x}}\right)$$

where $\mathbf{E}_0$ and $\mathbf{B}_0$ are *complex* constants and ω and $\mathbf{k}$ are real constants.

Monochromatic plane waves

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First (M1) and (M2) reduce to

$$\mathrm{Re}\left(i\mathbf{E}_0 \cdot \mathbf{k} e^{-i\omega t + i\mathbf{k}\cdot\mathbf{x}}\right) = 0 \qquad \mathrm{Re}\left(i\mathbf{B}_0 \cdot \mathbf{k} e^{-i\omega t + i\mathbf{k}\cdot\mathbf{x}}\right) = 0$$

The only way of satisfying these equations for all $(t, \mathbf{x})$ is if

$$\mathbf{E}_0 \cdot \mathbf{k} = \mathbf{B}_0 \cdot \mathbf{k} = 0$$

Monochromatic plane waves

$$\mathbf{E} = \mathrm{Re}\left(\mathbf{E}_0 e^{-i\omega t + i\mathbf{k}\cdot\mathbf{x}}\right) \qquad \mathbf{B} = \mathrm{Re}\left(\mathbf{B}_0 e^{-i\omega t + i\mathbf{k}\cdot\mathbf{x}}\right)$$

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$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

Substituting (4.46) into (M3) gives

$$\mathrm{Re}\left(i\mathbf{k} \times \mathbf{E}_0 e^{-i\omega t + i\mathbf{k}\cdot\mathbf{x}}\right) = \mathrm{Re}\left(i\omega \mathbf{B}_0 e^{-i\omega t + i\mathbf{k}\cdot\mathbf{x}}\right)$$

and for this to hold for all $(t, \mathbf{x})$ we must have

$$\omega \mathbf{B}_0 = \mathbf{k} \times \mathbf{E}_0$$

Monochromatic plane waves

$$\mathbf{E} = \mathrm{Re}\left(\mathbf{E}_0 e^{-i\omega t + i\mathbf{k}\cdot\mathbf{x}}\right) \qquad \mathbf{B} = \mathrm{Re}\left(\mathbf{B}_0 e^{-i\omega t + i\mathbf{k}\cdot\mathbf{x}}\right)$$

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$$\nabla \cdot \mathbf{B} = 0$$

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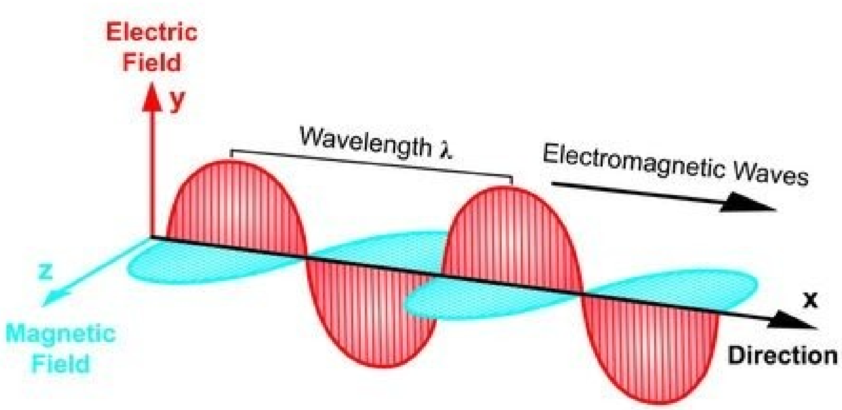
$$\omega = c|\mathbf{k}|$$

plane waves propagating with speed c in direction $\mathbf{k}$.

Monochromatic plane waves

$$\mathbf{E} = \mathrm{Re}\left(\mathbf{E}_0 e^{-i\omega t + i\mathbf{k}\cdot\mathbf{x}}\right) \qquad \mathbf{B} = \mathrm{Re}\left(\mathbf{B}_0 e^{-i\omega t + i\mathbf{k}\cdot\mathbf{x}}\right)$$



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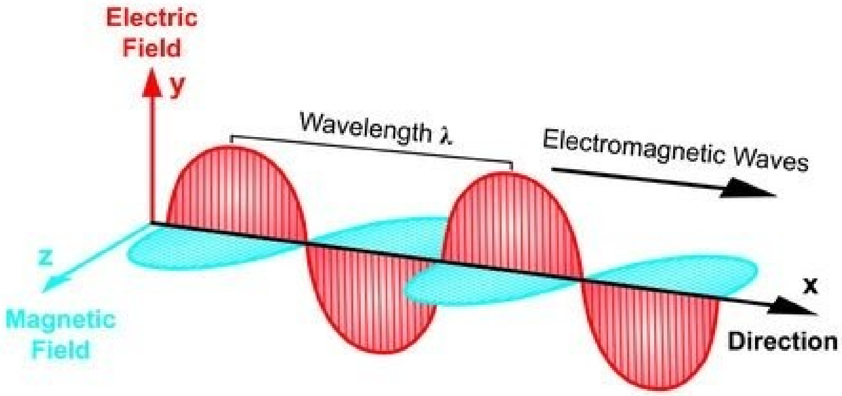
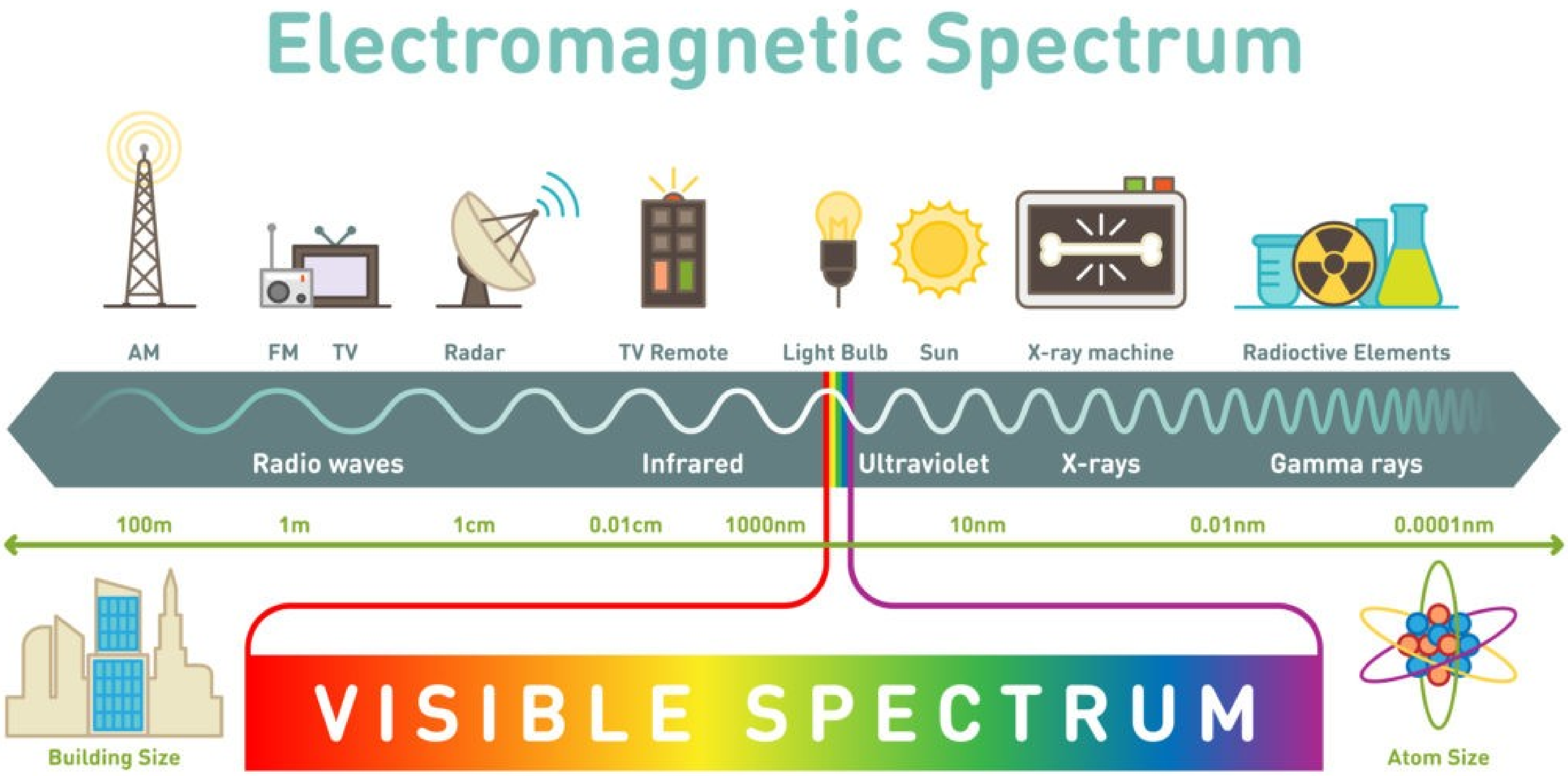


$$\mathbf{E}\cdot\mathbf{k} = \mathbf{B}\cdot\mathbf{k} = 0$$

$$\omega\mathbf{B} = \mathbf{k}\times\mathbf{E}$$

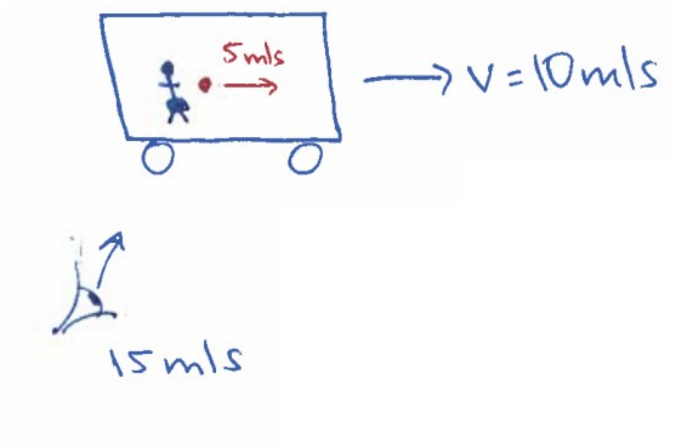
$$\lambda = \frac{2\pi}{k} = \frac{2\pi v}{\omega}$$


Electromagnetic Spectrum
AM
FM
TV
Radar
TV Remote
Light Bulb
Sun
X-ray machine
Radioctive Elements
Radio waves
Infrared
Ultraviolet
X-rays
Gamma rays
100m
1m
1cm
0.01cm
1000nm
10nm
0.01nm
0.0001nm
Building Size
VISIBLE SPECTRUM
Atom Size

Einstein’s thought experiment

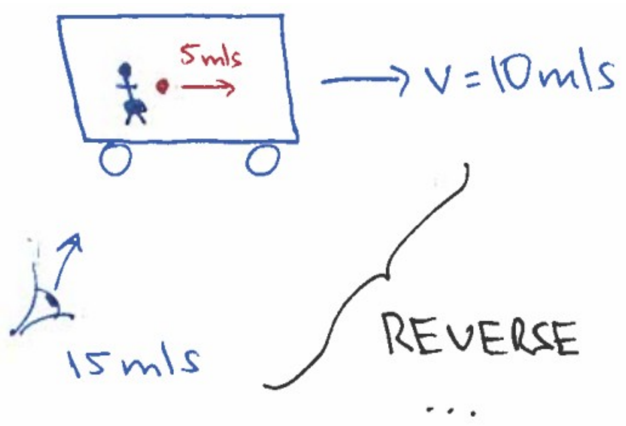
We all known how to add and subtract velocities.

If someone throws a ball at 5m/s on a train moving at 10 m/s, you (the eye) see the ball moving at the added velocity 15 m/s.



Einstein’s thought experiment

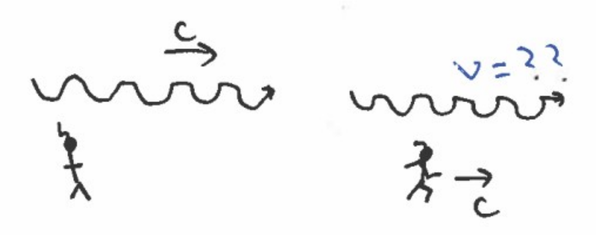
If, however, we also run at 15m/s, then the ball will appear stationary (we subtract the velocities)



Einstein’s thought experiment

What would happen if you ran next to a light beam while running at the speed of light?

What would you see? If velocities are “additive” would the light appear “stationary”?



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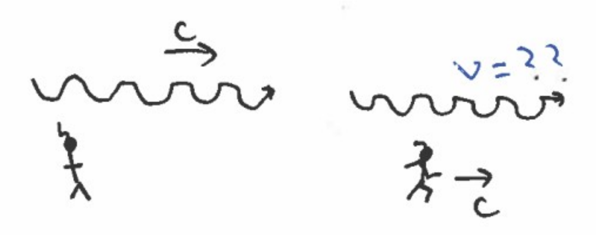


We cannot see a “stationary” solution however, since wave-like solutions to Maxwell’s equations move at the speed of light (which is non-zero!)

Special relativity

Maxwell’s equations are not invariant under Galilean transformations.

Michelson & Morley (1887) showed there is no preferred frame (aether).

Special relativity (1905) is Einstein’s way to overcome this.

Special relativity

Two postulates:

- The laws of physics are the same to every inertial observer
- The speed of light is constant and = c, in every inertial frame.

  Ie, no matter how fast the source of light and the observer are moving relative to each other

This leads to a new way to "add velocities".

$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}}$$

Time dilation

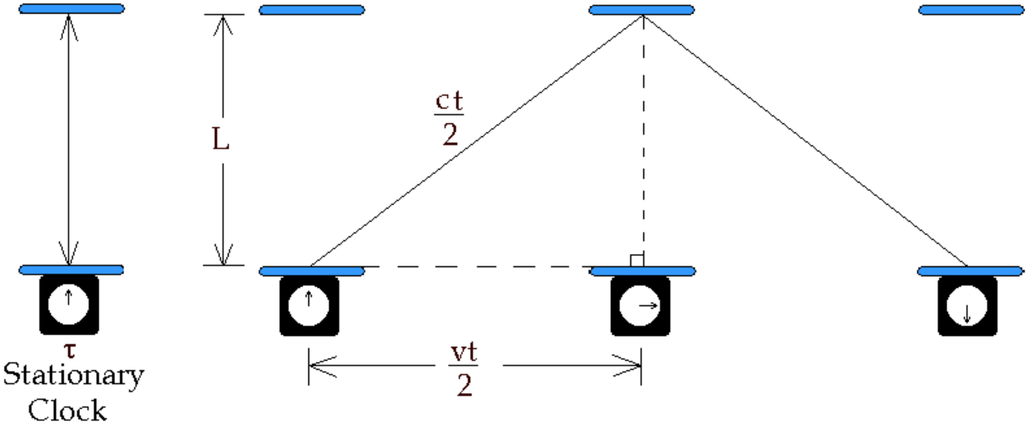
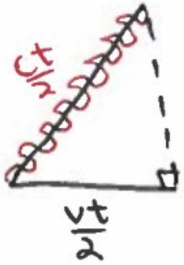


Figure 1

$2L = ct_0 \Rightarrow t_0 = \frac{2L}{c}$

In $\frac{t}{2}$ time, light moves

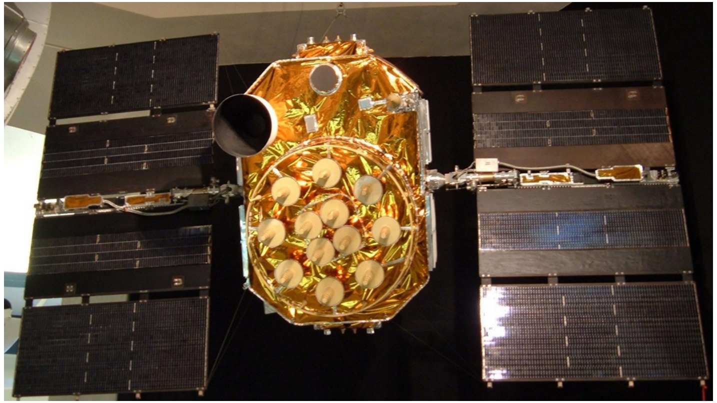


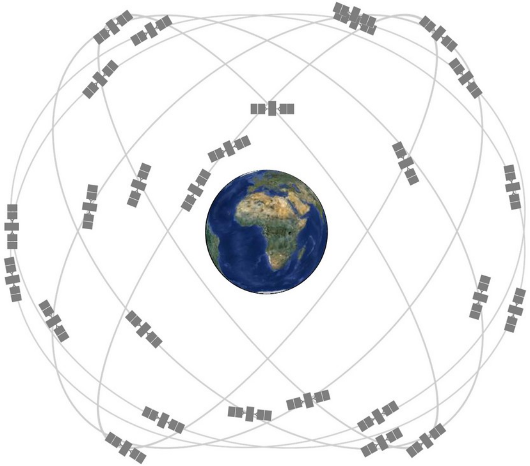
$L^2 + \left(\frac{vt}{2}\right)^2 = \left(\frac{ct}{2}\right)^2$

$\Rightarrow t^2 = \left(\frac{2L}{c}\right)^2 \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \Rightarrow t = \frac{t_0}{\sqrt{1-(v/c)^2}}$

Application to GPS

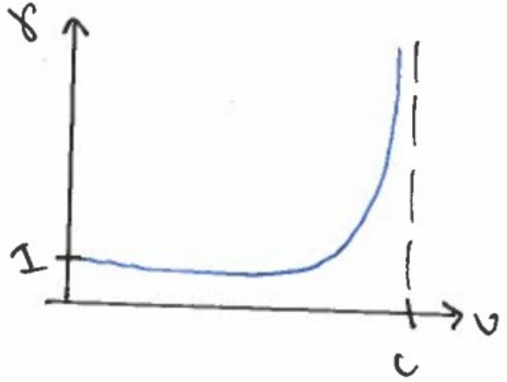
- 31 satellites
- Known radius 20km above sea level
- V = 3.874 km/s





Effect on GPS

"MOVING CLOCKS RUN SLOW"



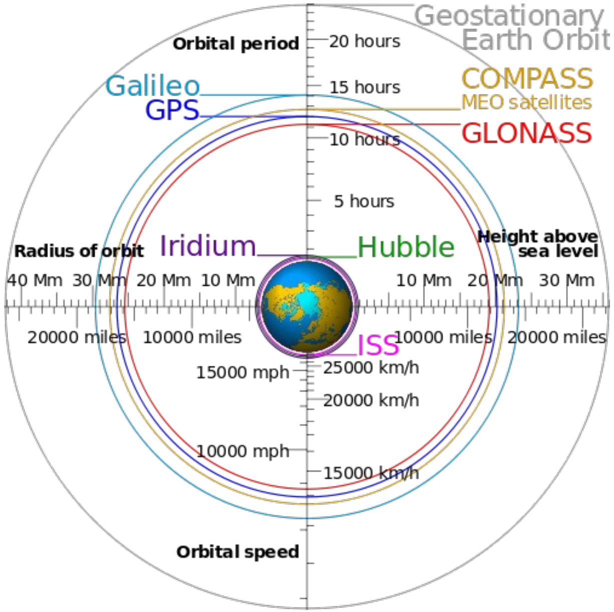
$$\sqrt{1-\frac{v^2}{c^2}} \approx 8.349 \times 10^{-11}$$

$$\Rightarrow \begin{pmatrix}\text{\# nanosec} \\ \text{per day}\end{pmatrix} \times \sqrt{1-\frac{v^2}{c^2}} \simeq 7214\text{ns}$$

- Satellite clock runs 7214 nanosecs slower than earth clock each day

Further effects on GPS

Gravitational effect only resolved by general relativity



Hermann Bondi

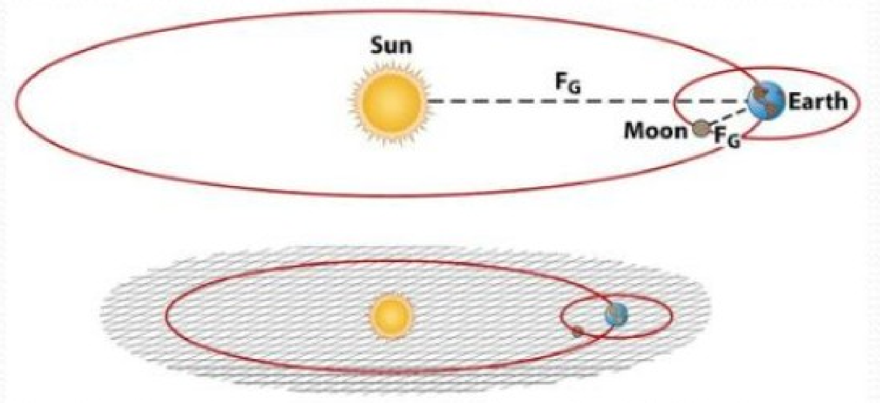
- 1919 - 2005
- Vienna → Cambridge → **King's** (1954 -1985) → retired but active
- 1950s – 70s, led a very active gravity group at King's. eg, Roger Penrose was a postdoc
- No experiments, gravity not a hot topic

General relativity

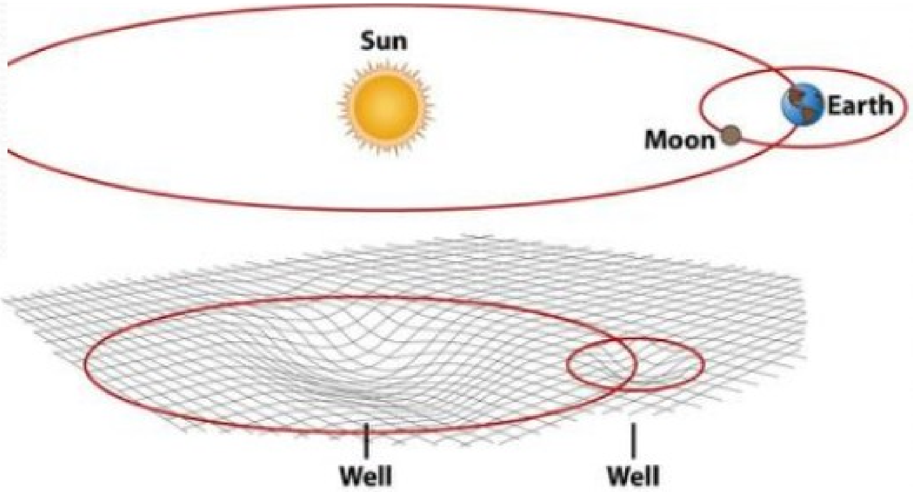
Theory of gravitation developed by Einstein between 1907 and 1915.

Key idea: the observed gravitational effect between masses results from their warping of spacetime.

Newton → GR



Newtonian universe: gravitational forces in a "flat" universe

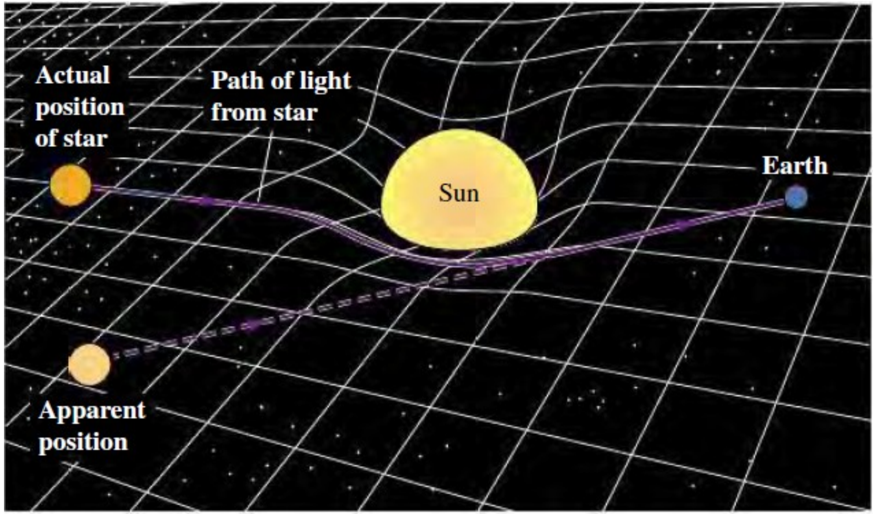


Einstein's universe: motion in a curved universe.

Predictions from general relativity

- In Newtonian theory, gravity only acts on massive particles

- In general relativity, gravity acts also on massless matter (light bending, 1919)

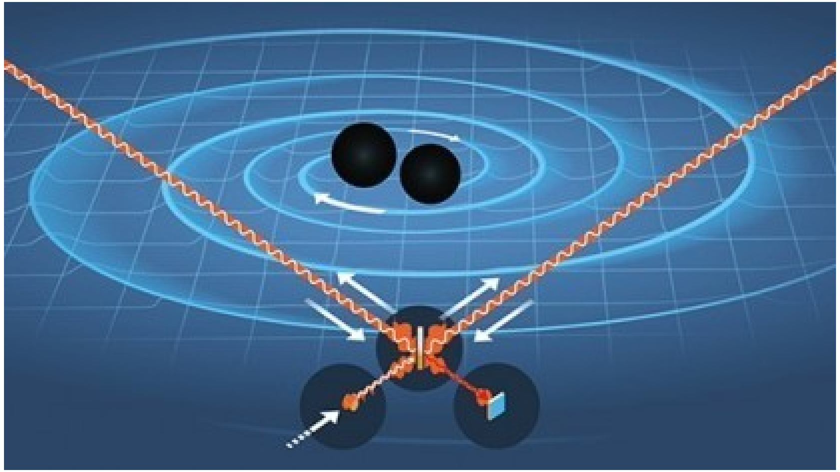


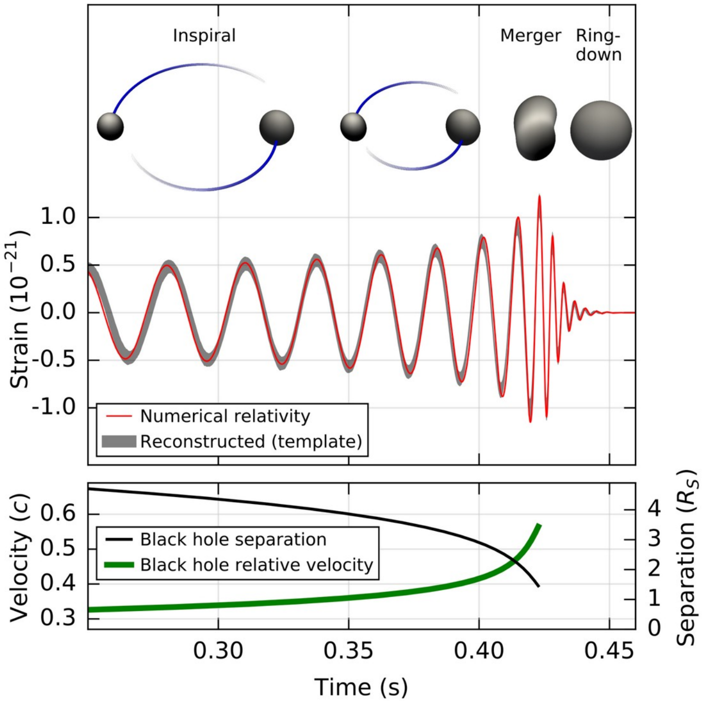
Predictions from general relativity

- Newtonian theory does not predict gravitational forces moving as waves, nor energy transfer from moving body
- General relativity does predict existence of gravitational waves and the transfer of energy by gravitational radiation (Einstein 1919, Bondi 1960)

Detection of gravitational waves

LIGO: 2016 detection, 2017 Nobel Prize





A **spacetime** is a $(1+n)-$dimensional manifold (M, g), with pseudo-Riemannian metric $g_{\mu\nu}$ satisfying

$$R_{\mu\nu} - \frac{1}{2} R \, g_{\mu\nu} + \Lambda \, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Spacetime tells matter how to move

Matter tells space how to curve

Minkowski metric

$$\eta = -c^2 dt^2 + dx^2 + dy^2 + dz^2$$

Looking at linear perturbations around this metric

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \ , \qquad |h_{\mu\nu}| << 1 \ .$$

Arrive at a wave equation for the spacetime perturbation

$$\Box h_{\mu\nu} = (-\partial_t^2 + c^2(\partial_x^2 + \partial_y^2 + \partial_z)^2)h_{\mu\nu} = 0$$

Subtle to understand coordinates (ie, show the wave is not just a gauge freedom)

Thanks!