What is a Particle?

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- 1. Simple Newtonian Particles
- 2. Relativistic Particles
- 3. Fields
- 4. Particles from Quantized Fields
- 5. Particles with Structure

1. Simple Newtonian Particles

- Particles are treated as pointlike. A particle with velocity v carries momentum and energy

$$P = mv\,, \quad E = \frac{1}{2}mv^2\,.$$

  Momentum and energy are conserved in particle collisions.
- Mass m is key parameter. Measured in gravitational balance against standard mass. (Can also use Newton's 2nd law, but you need to understand force acting.)
- Mass of a single particle is constant. If a particle decays and its mass changes, other particles must be emitted (e.g. muon decay to electron accompanied by neutrinos).
- Particles also have spin and electric charge (related to conservation laws of angular momentum, electric charge).

2. Relativistic Particles

- Particle mass m is still key parameter. Velocity v is always less than the speed of light c. We choose units $c = 1$.
- Momentum and energy still conserved in particle collisions, but these quantities depend differently on velocity (velocity can be measured independently by time of flight). Relativistic formulae are

$$P = m\gamma v \,, \quad E = m\gamma \,,$$

where

$$\gamma = (1 - v^2)^{-\frac{1}{2}} = 1 + \frac{1}{2}v^2 + O(v^4) \,.$$

- Important to note the Einstein relation

$$E^2 = P^2 + m^2 \,.$$

This follows from $\gamma^2 (1 - v^2) = 1$.

- At low velocities

$$P = mv + O(v^3)\,, \quad E = m + \frac{1}{2}mv^2 + O(v^4)\,.$$

- Novel thing is $E = m$, particle's rest energy, when $v = 0$. The mass m is large, hidden source of energy of a particle. Hard to see or exploit this. Chemical reactions just rearrange particles, so there's no change of rest energy.
- In nuclear decays, total mass decreases. Excess energy converts to kinetic energy of products. In black hole mergers, total black hole mass decreases, and excess energy emitted as gravitational waves.
- Particle pair production is possible in relativistic particle collisions, but requires sufficiently large energy.

3. Fields

- **(i) Coulomb field**: Present around an electrically charged particle. When particle moves, field moves with it but not instantaneously. There are time-dependent electric and magnetic fields, **E** and **B**.
- Maxwell equations relate time- and space-derivatives of **E** and **B** – requires a fundamental speed; Maxwell discovered that this is the speed of light, $c = 1$.
- Maxwell equations have EM (electromagnetic) wave solutions (simplified)

$$A \cos(kx - \omega t) \, ,$$

  where k is wave vector and ω is wave frequency.
- Key relation from Maxwell equations is

$$\omega = k \, .$$

- **(ii) Nuclear force field**: A pion field acts between protons and/or neutrons. The pion field π obeys the Klein-Gordon wave equation

$$\frac{\partial^2 \pi}{\partial t^2} - \nabla^2 \pi + M_0^2 \pi = 0 \, .$$

- Parameter $M_0 \sim 1\,\mathrm{fm}^{-1}$ has dimension of inverse length (inverse time), not mass.
- The equation has a static, Yukawa solution, falling off exponentially fast with distance R from the source proton or neutron,

$$\pi = \frac{A}{R} e^{-M_0 R} \, .$$

- The equation also has wave solutions $A \cos(kx - \omega t)$ as before, but now

$$\omega^2 = k^2 + M_0^2 \, .$$

4. Particles from Quantized Fields

- Need to introduce Planck's constant $\hbar$, with units energy $\times$ time (or energy $\times$ length, when $c = 1$). Numerically,

$$\hbar = 197.3 \, \mathrm{MeV} \, \mathrm{fm} \, .$$

- **(i) Photons**: Quantum states of an EM wave with wave vector k and frequency ω have quantized momentum and energy. The momentum and energy of one photon are

$$P = \hbar k \, , \quad E = \hbar \omega \, .$$

- Because $\omega = k$,

$$E = P$$

for a photon. This is the Einstein relation for a particle of zero mass. Therefore, a photon has zero mass and cannot be at rest.

- These formulae are verified in the photoelectric effect and Compton scattering, where photons interact with electrons.
- A classical EM wave is a coherent superposition of many photons, and carries the momentum and energy of these photons.
- **(ii) Pions**: Quantum states of the pion field are pion particles. Again $P = \hbar k$ and $E = \hbar\omega$, but here

  $$\omega^2 = k^2 + M_0^2 \,,$$

  so (multiplying by $\hbar^2$)

  $$E^2 = P^2 + \hbar^2 M_0^2 \,.$$

  This is the Einstein relation for a pion particle of mass $M = \hbar M_0$, which has the right dimensions for a mass.
- We see that in quantum field theory, the mass of a particle arises as a quantum mechanical effect. (This can be hidden if units are chosen where $\hbar = 1$.)

Provisional Summary

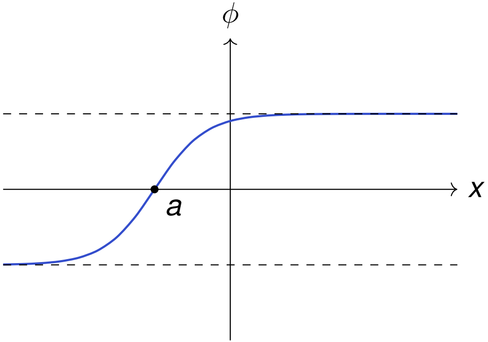
- In quantum field theory, particles are quantum states of fields. Particles obey the relativistic (Einstein) energy-momentum relation. There is a fundamental field for each fundamental particle.
- For particles like electrons and neutrinos with spin $\frac{1}{2}\hbar$, relevant wave equation is the Dirac equation. A Dirac field needs to be quantized.
- The field's algebraic structure determines the particle's spin. E.g. A photon has spin $\hbar$, because of the vector character of **E** and **B**, and because EM waves can be polarised; a pion has spin 0.
- Nonlinear field equations imply nonlinear evolution of classical waves. In quantum field theory, nonlinearity leads to particle scattering (particle interactions, including decays), and is essential for describing nature.

5. Particles with Structure

- A classical point particle has singularities in its matter density and electric charge density. Creates subtle problems in electromagnetic theory.
- A field has wave solutions (of definite frequency) with infinite extent. Quantized particle states have no structure.
- Both these particle models are unsatisfactory. Nonlinearity can come to the rescue!

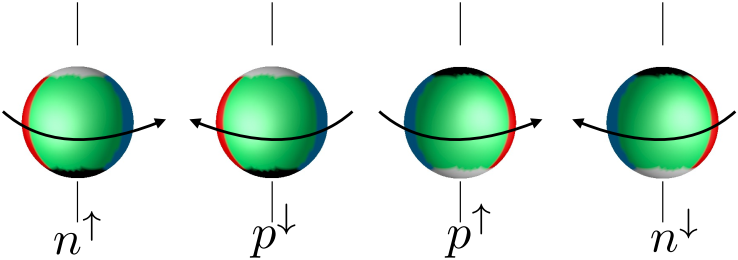
- Some particles have a clear structure, and a finite size. E.g. A proton has a measured charge radius, and measured matter radius. Both are approximately 1 fm.
- A field's wave equation can have an (inverse) length parameter M_0, but this is not enough to create structure.
- A combination of nonlinearity and a length scale are needed for a classical, particle-like solution. This can have a mass M, and energy-momentum obeying the Einstein relation. The mass M combines a nonlinear coupling parameter with M_0.

- **(i) Kink soliton in 1-dimension**: Field nonlinearity implies two vacua, related by symmetry. Kink interpolates between them – it has a topological stability.



- Field approaches each vacuum exponentially fast. The (classical) kink is one type of particle in this field theory. Quantized field oscillations around vacuum give a second type of particle.

- **(ii) Skyrmions in 3-dimensions**: Skyrmion is a solution of a nonlinear, pion field theory, with parameter M_0. Skyrmions represent proton/neutron sources for the pion field, and have finite energy (mass).
- A Skyrmion, like a kink, has a topological stability.
- Asymptotic field of a Skyrmion is a (triplet of) pion dipoles.
- Skyrmion is classical, and can be static or moving. Its rotational motion needs to be quantized. The quantum state distinguishes a proton from a neutron, and determines whether the spin state is up or down.
- There are also pion particles with mass $M = \hbar M_0$.



Classically spinning $B = 1$ Skyrmions, approximating p and n states [D. Foster and NSM]

Summary

- In quantum field theory, all particles are quantum states of fields. Particles with no known structure, e.g. electrons and photons, each need a fundamental field.
- Structured particles can be bound states of more fundamental particles, e.g. protons as bound states of quarks; pions as quark-antiquark states. This QCD picture is hard to implement theoretically, and unhelpful in studies of nuclei.
- An attractive alternative model is that the proton/neutron is a soliton – a Skyrmion – in the pion field. This is the approach of Effective Field Theory.
- In Skyrme theory, one field gives rise to several particle types: the spin 0 pions, and the spin $\frac{1}{2}\hbar$ proton and neutron.