Holographic dualities

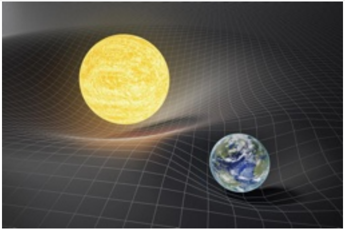
A strange feature of quantum gravity

Silvia Georgescu

Cumberland Lodge KCL

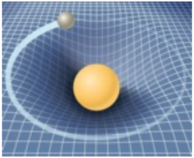
22 February 2025

General relativity



gravity = geometry

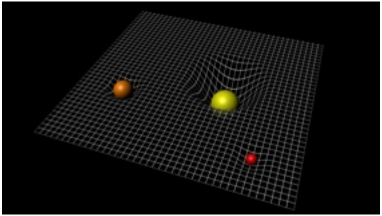
- The presence of matter makes the spacetime to curve
- The motion of objects in spacetime is determined by the geometry
- Geometry: metric $\Rightarrow$ notion of distance



General relativity

Einstein's equations: geometry and matter

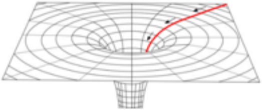
$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$



- General relativity passed many experimental tests and made correct predictions. Recent example: **gravitational waves**
- It provides the framework for **cosmology**: evolution in time of our universe

General relativity

- Einstein’s equations admit singular solutions called **black holes**
- The singularities are hidden behind **event horizons**
- Black holes have strong gravitational fields: once you passed the event horizon, you cannot escape and you will reach the singularity



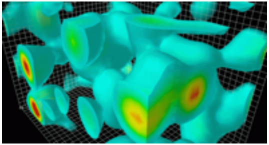
Quantum physics

- Quantum mechanics describes physics at very small scales: intrinsically probabilistic
- The physical quantities that we measure are modeled by Hermitian operators
- In the absence of measurement, quantum systems can be in superposition of states

- Quantum mechanics treats time and space differently $\Rightarrow$ in tension with the theory of special relativity: time intervals and space distances are not absolute, but distance in spacetime is
- Quantum mechanism and special relativity were incorporated into a consistent theory called **quantum field theory**

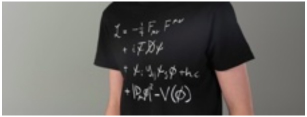
Quantum field theory

- The fundamental concept: **field**, which fills the entire spacetime. A classical field is a function of spacetime. Then, the field is quantized: its coefficients when we expand it into a complete basis of functions are promoted to be operators
- Particles appear as excitations of the quantum fields



- Quantum field theory describes three out of the four fundamental forces in nature: electromagnetism, the weak and the strong nuclear force (the fourth one is gravity)
- The great success of quantum field theory was to unify these three fundamental forces into a single consistent quantum theory called the **Standard model** of particle physics

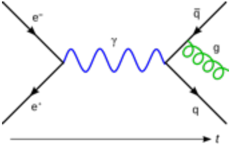
Quantum field theory



- The Standard model describes electrons, neutrinos, quarks, photos, gluons, etc (all the elementary particles)
- It passed many experimental tests and made correct predictions. A famous recent example: the existence of the Higgs boson, which was detected at CERN in 2012, about 40y after its theoretical prediction

Quantum field theory

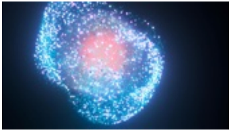
- In quantum field theory we compute **scattering amplitudes**: probabilities that particles in some initial states will be found in some other final states after interaction ⇒ describe **interactions** among particles
- Feynman diagrams & algorithmic way to compute scattering amplitudes



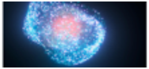
- Naively applying the rules: infinite results. How can a probability be infinite???

The development of quantum electrodynamics.
1937 (colourised).

Quantum field theory

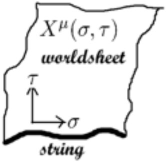


- The naive computations were not in terms of measurable quantities: “bare" vs physical
- A technique to treat the infinities: **renormalization**
- Incredible agreement with experiments
- Not any QFT is renormalizable: Standard model is, but gravity is not
- General relativity should be understood as an effective theory (valid only in some regime). It should be a limit of a theory of **quantum gravity**

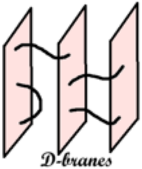
 $+$  $= ???$

String theory

- In string theory the fundamental objects are **strings**: 1d objects that can be open or closed
- When they propagate in time they describe a 2d surface, the **string worldsheet**
- 2d field theory on the worldsheet: particles are excitations



- Higher-dimensional extended objects: **D-branes**. Open strings attach to them.



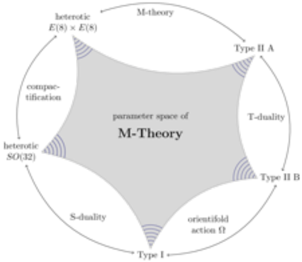
String theory

- String theory: very constrained. One of the consistency requirements implies classical gravity at low energies $\Rightarrow$ we can reproduce Einstein's equations from string theory
- The same kind of consistency requirement: string theory makes sense only in **10 dimensions**
- Are we really sure we live in 4 spacetime dimensions? String theory: we actually live in 10 dimensions, but 6 of them are compact and very small, so we cannot access them
- The idea that physics in 4d originates from simpler physics in higher dimensions is not new: Kaluza and Klein came with this idea when they tried to unify classical gravity and electromagnetism:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + e^{2\phi}(A_\nu dx^\nu + dx^5)^2$$

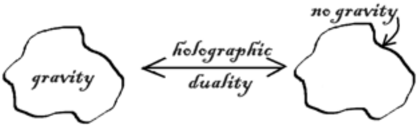
- In string theory, the same Kaluza-Klein reduction is used. It is not known how to compactify in order to obtain the Standard model

String theory



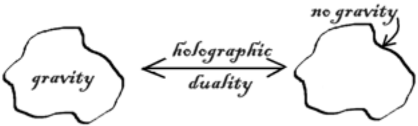
- Current formulation of string theory is limited: five consistent string theories plus M-theory, but each description is valid only in some regime
- Within string theory we have **dualities** which map one description to the other, such that we can think of these “theories" as being limits of one single theory
- String theory is a consistent theory in which we can test ideas about quantum gravity in general: for example **holography**

Holography



- In quantum gravity: the idea that gravity is emergent from some degrees of freedom that live in a lower dimensional spacetime
- **Holographic dualities**: equivalences between theories of quantum gravity in spacetimes with d dimensions and theories without gravity in spacetimes with less dimensions
- What does it mean that two theories are equivalent? The same physical, observable quantities that we can compute in one, we can also compute in the other one, although the two may seem very different
- There exist concrete examples of holographic dualities (see later)
- The mapping between the two equivalent theories: **holographic dictionary**

Holography



- Holography is useful: **weak-strong dualities**. When one side is weakly-interacting (= we know how to do computations), the other one is strongly-interacting (= we do not know how to do computations in general)
- A bit of history: the idea of holography came from black hole physics:

$$S_{BH} = \frac{A_{horizon}}{4G_N}$$

- *S* counts the number of quantum microstates: effectively the theory which describes microscopically the black hole is lower dimensional
- t'Hooft: quantum field theories behave like string theories in some limit
- Physical quantities in theories of gravity effectively are conserved quantities in lower dimensional theories. Beautiful example by Brown-Henneaux: symmetries of gravity theories in some spacetimes called “AdS" match the symmetries of some field theories called “CFT"

What is AdS?

- AdS (anti-de Sitter) is a d-dimensional spacetime that can be obtained by starting from a $d+1$-dimensional flat spacetime with metric:

$$ds^2 = -dX_1^2 - dX_2^2 + \sum_{i=3}^{d+1} dX_i^2$$

and restricting to the points which satisfy:

$$-dX_1^2 - dX_2^2 + \sum_{i=3}^{d+1} dX_i^2 = -\ell^2$$

where ℓ is a constant.

- It is maximally symmetric
- It has constant, negative scalar curvature. It is a solution of Einstein's equations in vacuum with negative cosmological constant

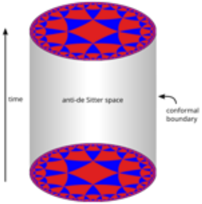
What is AdS?

- In Poincare coordinates:

$$ds^2 = \ell^2 \left(\frac{dr^2}{r^2} + r^2 dx_\mu dx^\mu \right)$$

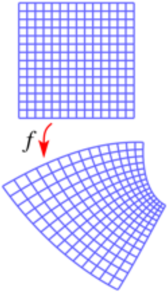
where r is the radial coordinate and x^μ are $d-1$ dimensional flat coordinates $\Rightarrow$ a flat spacetime at each fixed value of r

- Conformal boundary at $r \to \infty$

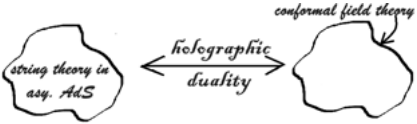


What is CFT?

- A CFT (conformal field theory) is a field theory which is invariant under **conformal transformations** = transformations which preserve the metric un to rescalings $g'_{\mu\nu} = \Omega(x) g_{\mu\nu}$ (preserve angles, but not necessarily distances)
- In flat space $d > 2$: translations, rotations, dilatations, special conformal transformations
- In 2d flat space any transformations $z \to f(z), \bar{z} \to \bar{f}(\bar{z})$ are conformal: infinite dimensional conformal symmetry algebra



The *AdS*/*CFT* correspondence



- **AdS/CFT**: family of holographic dualities between string theory in asy. AdS spacetimes and CFTs in one less dimension (“living on the conformal boundary")
- They are derived from string theory using **decoupling limits**
- Consider a system of D-branes, open strings attached to them and closed strings. Closed string excitations can move everywhere in spacetime, but the open string excitations are localized on the D-branes.

The *AdS/CFT* correspondence

- 3 type of interactions: open-open, closed-closed, open-closed
- If we consider very low energies, only massless excitations can be created. Formally, we restrict to low energies by taking:

$$\alpha' \to 0 \qquad\qquad \alpha' = l_s^2$$

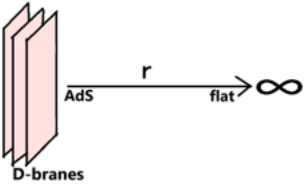
  which means that we look at length scales much larger than the string scale $\leftrightarrow$ energy scales much smaller than the string energy scale
- In this low energy limit, closed-closed and open-closed interactions are neglijable
- Decoupling limit: the physics on the D-branes decouples from the bulk, in particular from gravity

decoupling limit

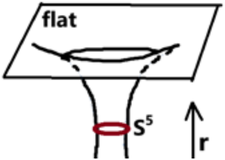
$\Downarrow$

free gravity in the bulk + *non-gravitational theory on the D-branes* (CFT)

The AdS/CFT correspondence



- How does AdS enter the story? We need to consider the same system from a different perspective
- D-branes are massive: they modify the nearby geometry. In the presence of the D-branes the geometry is flat far away and generically AdS near them (“in the near horizon")
- Example: for D3-branes $AdS_5 \times S^5$



The *AdS*/*CFT* correspondence

- If we consider the same low energy limit, we restrict again to massless particles, but the way we measure energy/mass is modified because of the geometry. From infinity: massless particles in the bulk, but also particles of any mass in the near-horizon region
- We can ignore again the interactions between massless particles in the bulk and also interactions between massless particles in the bulk and particles in the near-horizon region

decoupling limit

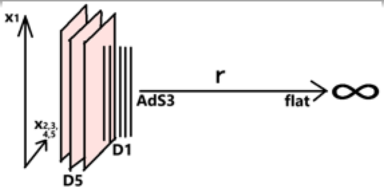
$\Downarrow$

free gravity in the bulk + *gravity in the near-horizon AdS region*

- Putting the two together: the *AdS*/*CFT* correspondence:

free gravity in the bulk + *gravity in the near-horizon AdS region* $\Longleftrightarrow$ *free gravity in the bulk* + *non-gravitational theory on the D-branes* (CFT)

An example: AdS_3/CFT_2 from string theory



- The D1-D5 system: p D1-branes and k D5-branes:

$$ds^2 = \frac{1}{\sqrt{f_1 f_5}}(-dt^2 + dx_1^2) + \sqrt{f_1 f_5}(dr^2 + r^2 d\Omega_3^2) + \sqrt{\frac{f_1}{f_5}} \sum_{i=2}^{5} dx_i^2$$

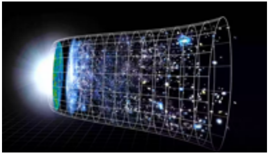
where

$$f_1 = 1 + \frac{\alpha' p g_s}{r^2 v} \qquad f_5 = 1 + \frac{\alpha' k g_s}{r^2}$$

- The decoupling limit: $\alpha' \to 0$, $\frac{r}{\alpha'} = fixed$
- We obtain $AdS_3 \times S^3 \times T^4$:

$$ds^2 = \ell^2 \left(\frac{dr^2}{r^2} + r^2(-dt^2 + dx_1^2) \right) + \ell^2 d\Omega_3^2 + \sqrt{\frac{p}{vk}} \sum_{i=2}^{5} dx_i^2$$

Holography beyond AdS

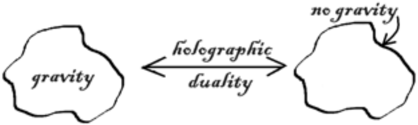


- Our universe is not asymptotically AdS:

$$\Lambda > 0$$

- If holography is a fundamental feature of quantum gravity, it should also apply to our universe $\Rightarrow$ we should try to understand **holography beyond AdS**
- Very challenging open problem: no examples from string theory of de Sitter holography or flat space holography
- The non-AdS holographic dualities obtained from string theory: difficult to study (non-local fields theories, theories of strings or other extended objects decoupled from gravity...)
- Top-down vs bottom-up approaches?

Holography beyond AdS



- We learnt a lot about quantum gravity from holography, but not about quantum gravity in our universe.
- Strong reasons to believe that holography is a general feature of quantum gravity, that extends beyond the AdS/CFT dualities
- We hope to learn more about holography beyond AdS and try to see what it can teach us about black holes, about cosmology, about gravity in general.

Thank you very much for your attention!