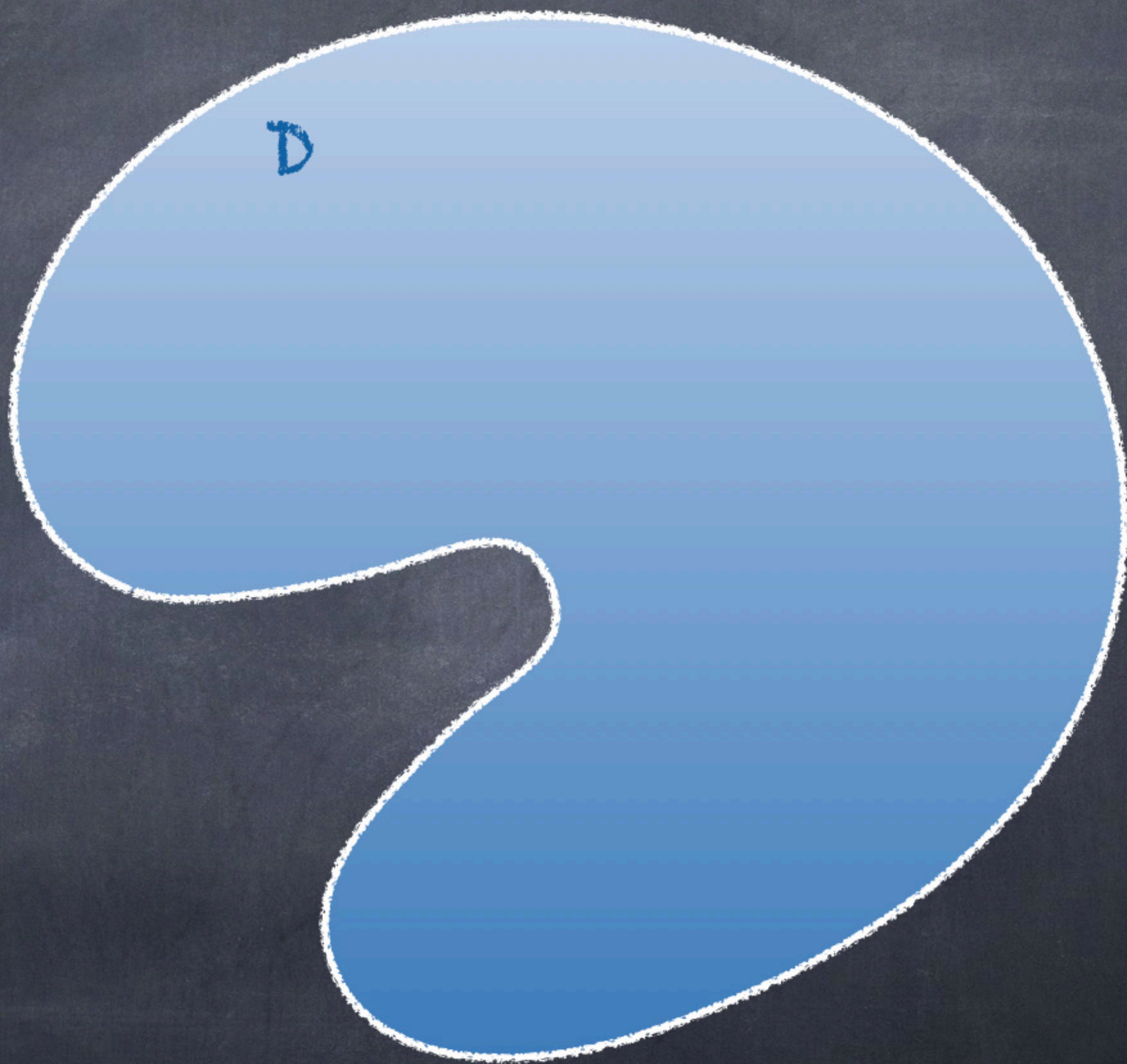


Measure the surface
area of this shape:

$$\iint_D dx dy$$

$$= \iint_{\mathbb{R}^2} 1_A dx dy$$

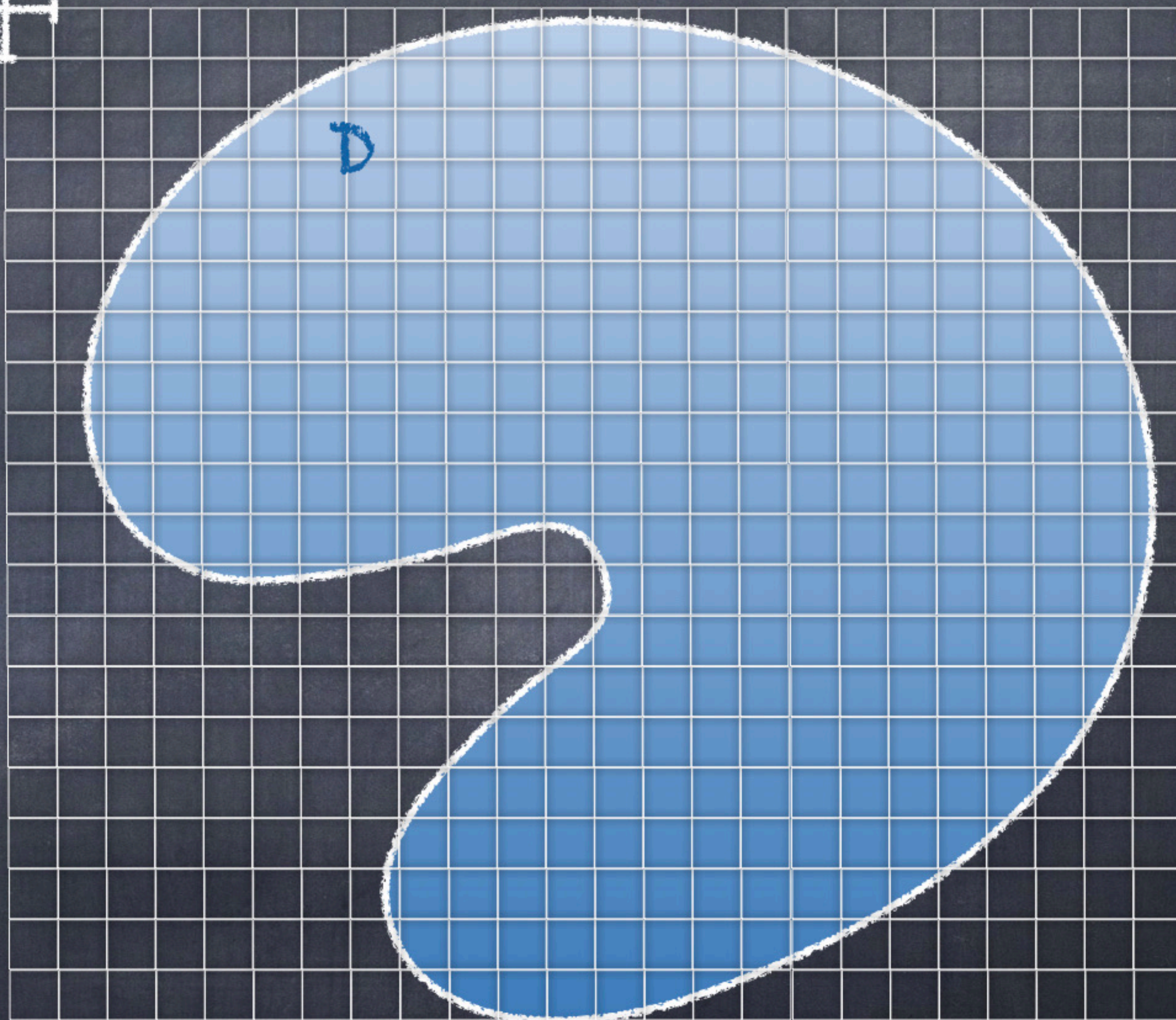


$$\iint_D dx dy$$

$$= \lim_{h \rightarrow 0} h^2 (\text{Number of } \blacksquare)$$

$$= \lim_{h \rightarrow 0} h^2 (\text{Number of } \begin{array}{|c|} \hline \square \\ \hline \end{array}, \blacksquare)$$

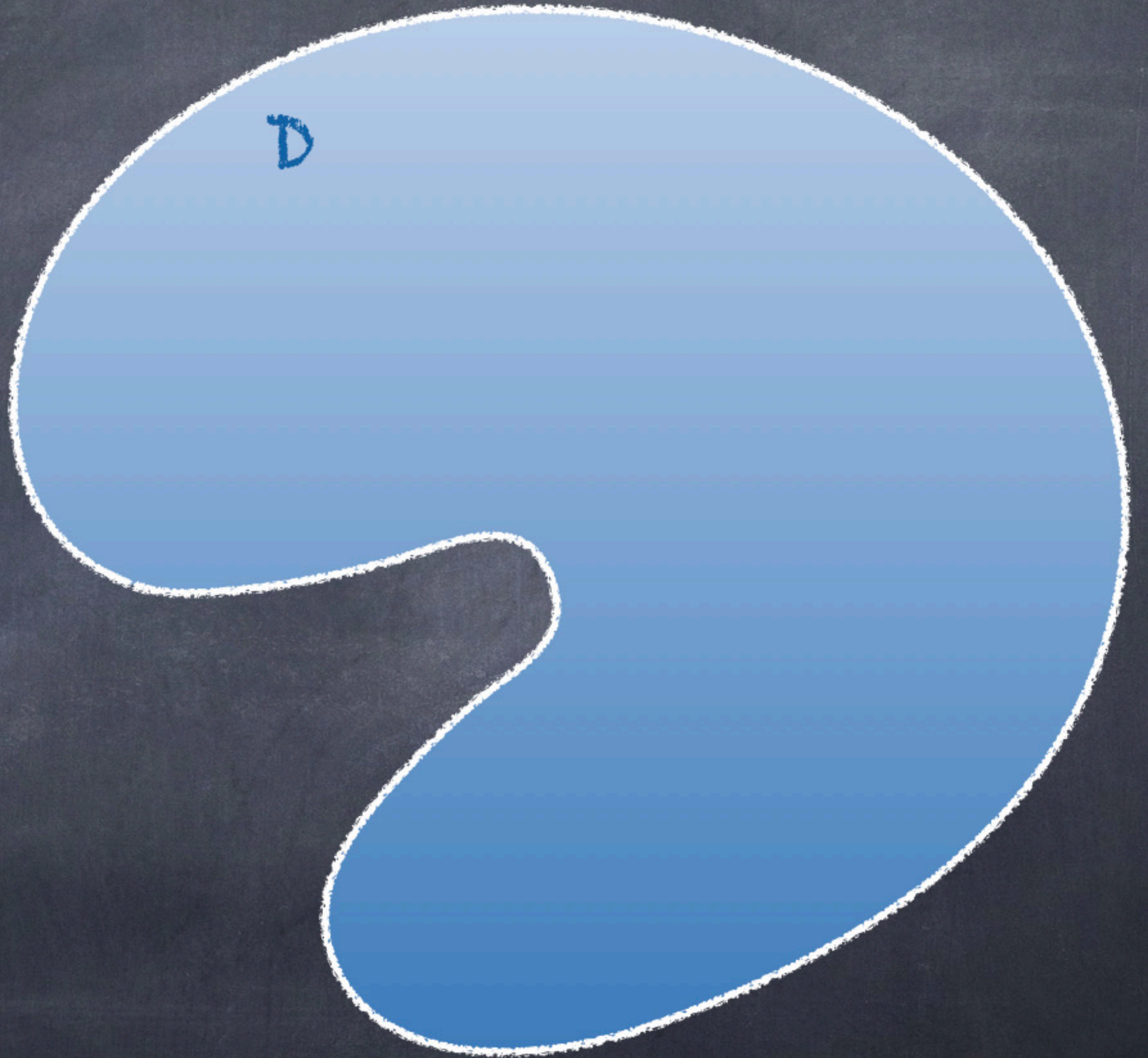
h
h



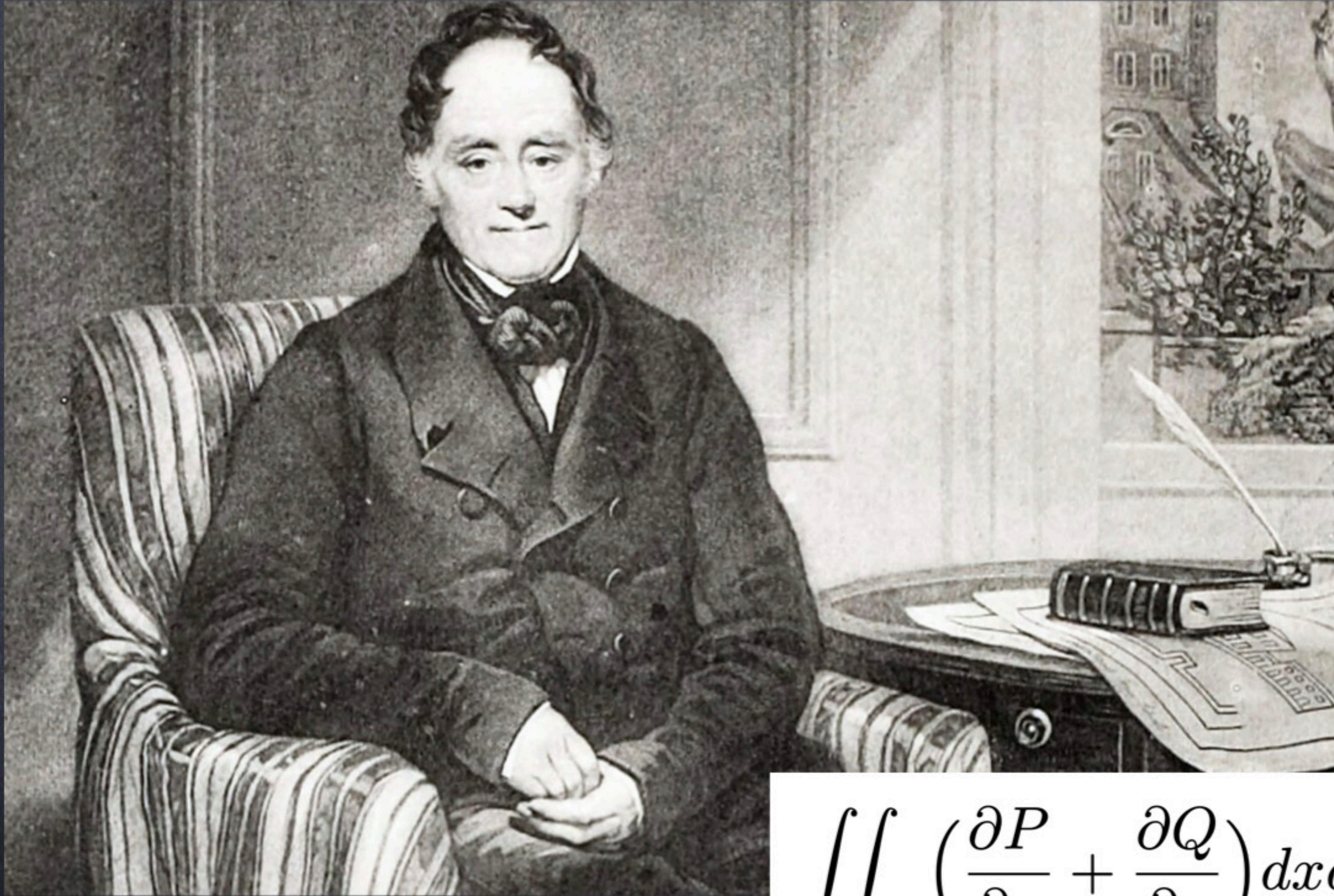
Measure the surface
area of this shape:

Make a model using
some homogeneous
sheet material, then
measure with a scale.

Use a planimeter!



George Green (1793-1841): Green's formula



Divergence Theorem

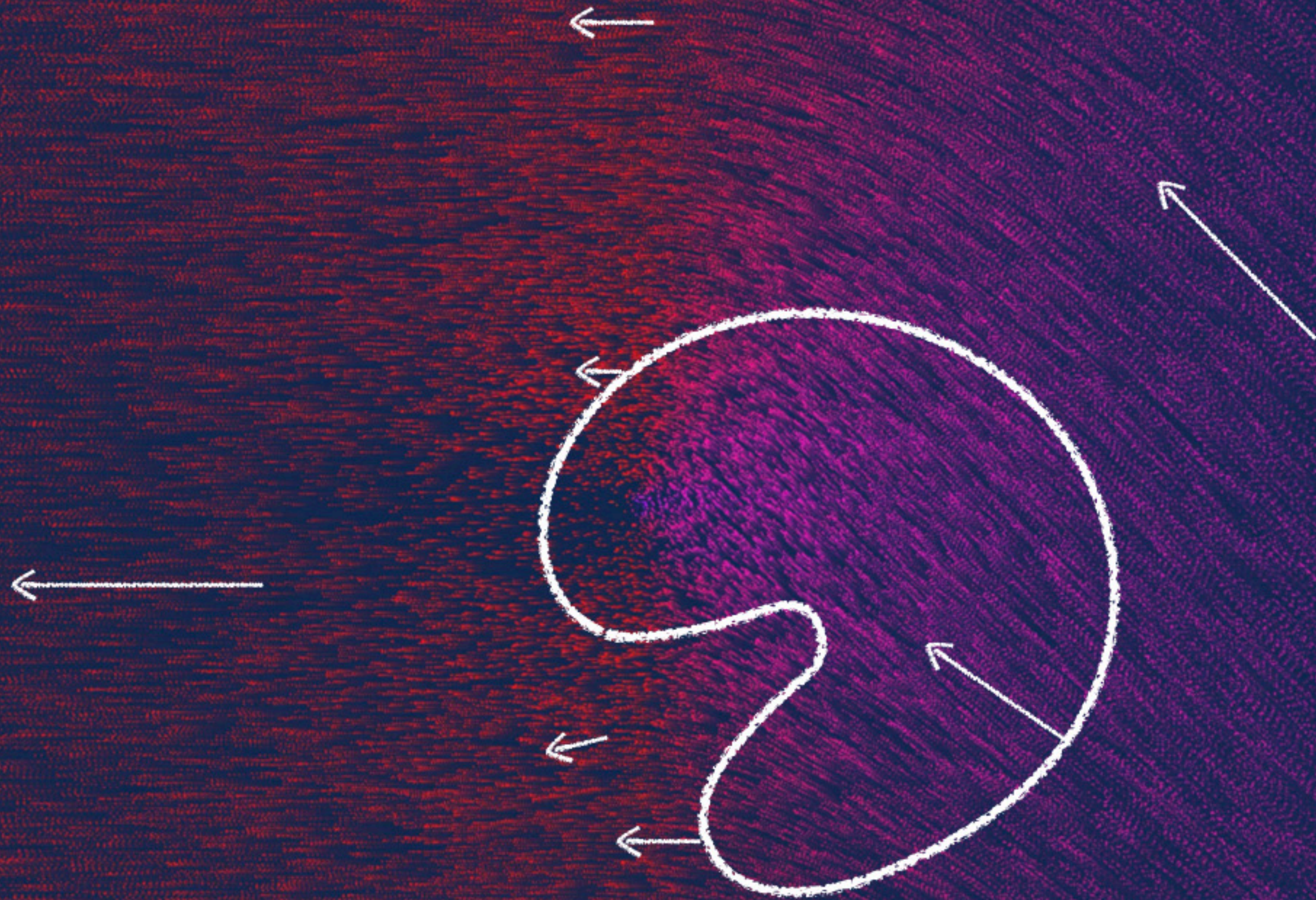
Lagrange, Gauss,
Ostrogradskii

Stokes, deRham

$$\iint_D \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} \right) dx dy = \oint_{\partial D} (-P dy + Q dx)$$

$P(x, y)$
 $Q(x, y)$ vector field

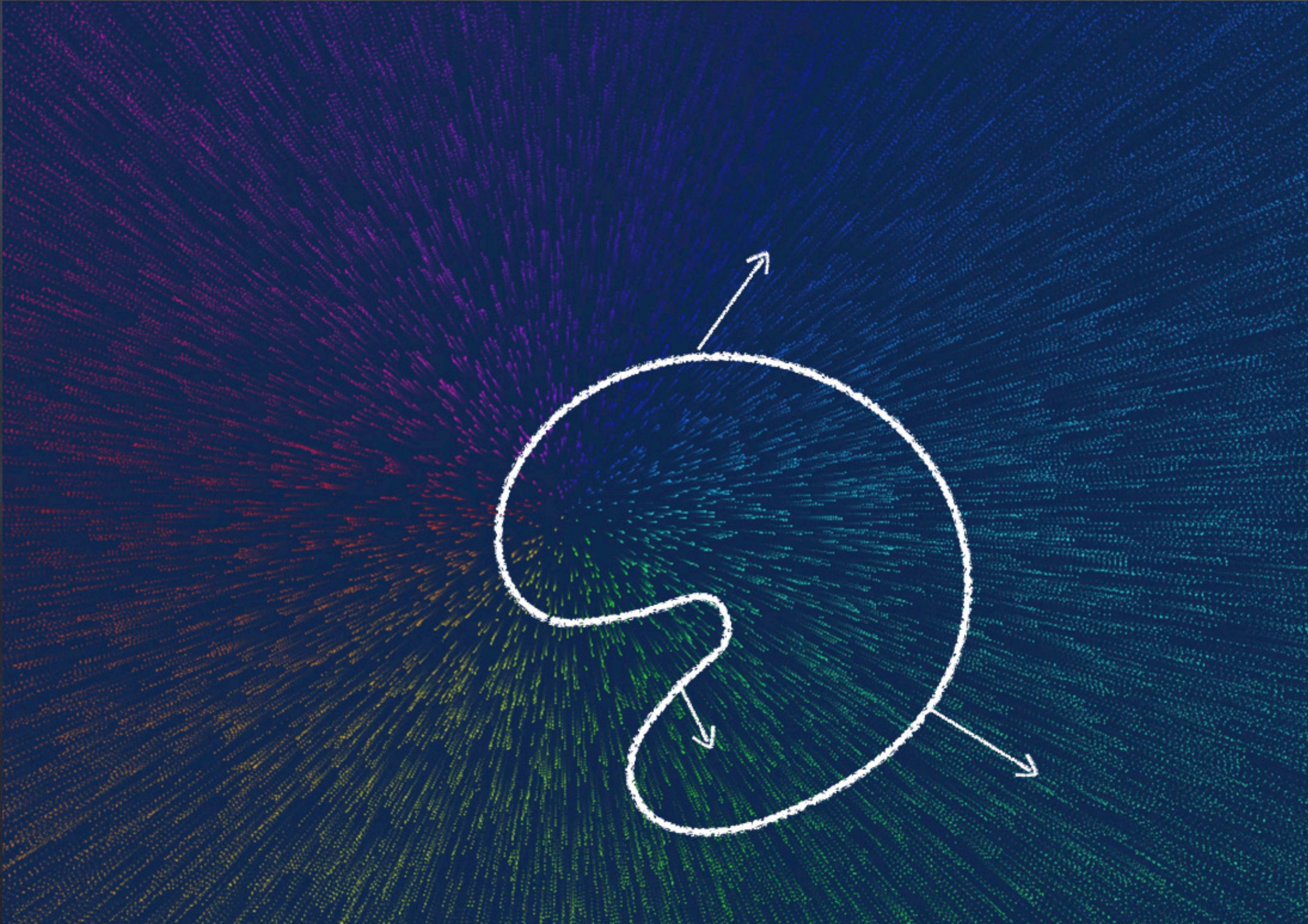
Flow of a gas in
the plane.



$$\iint_D \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} \right) dx dy = \oint_{\partial D} (-P dy + Q dx)$$

Creation inside D

Flow through ∂D



$$P(x, y) = x$$

$$Q(x, y) = y$$

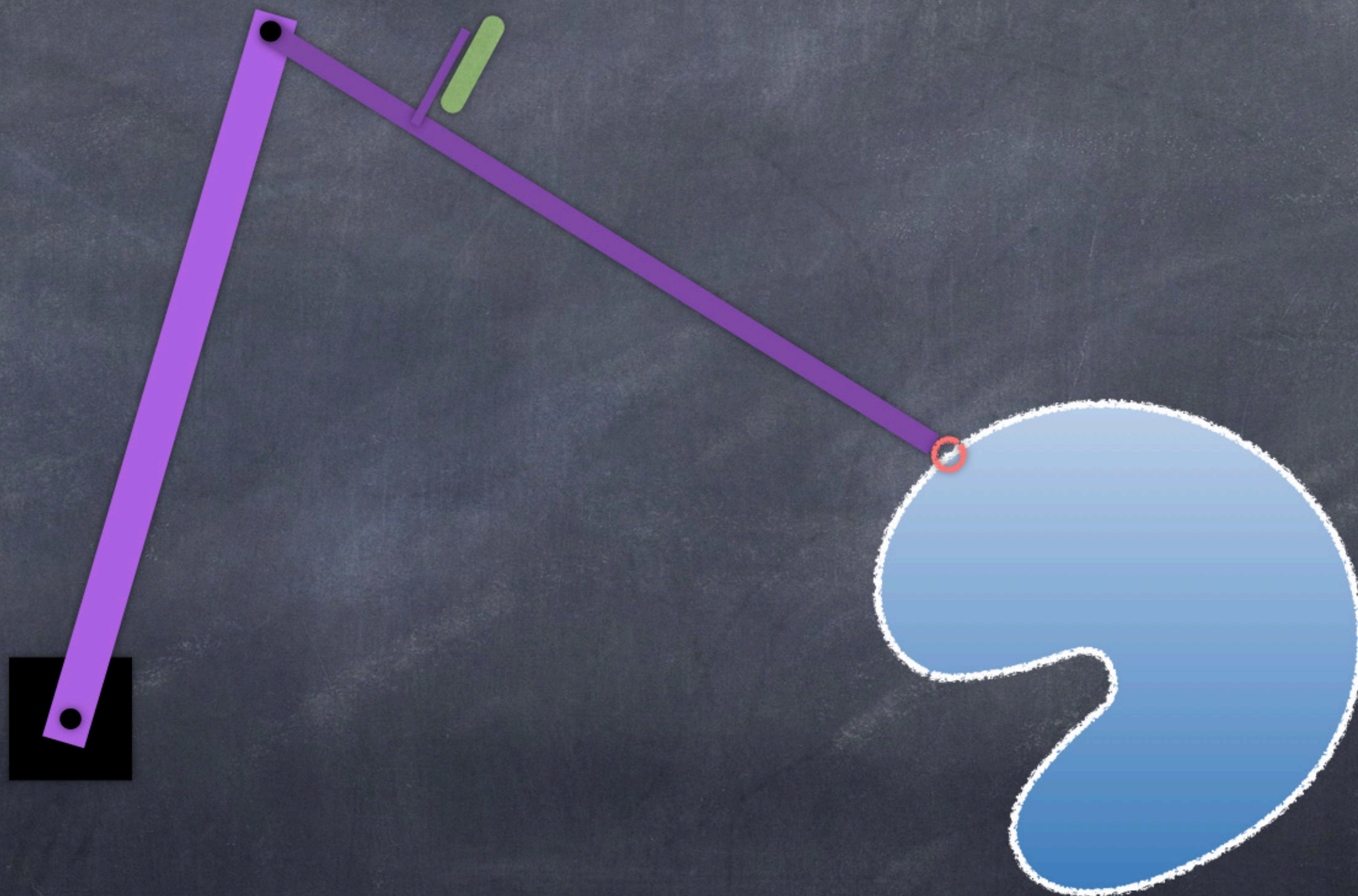
Flow of a gas in the plane.

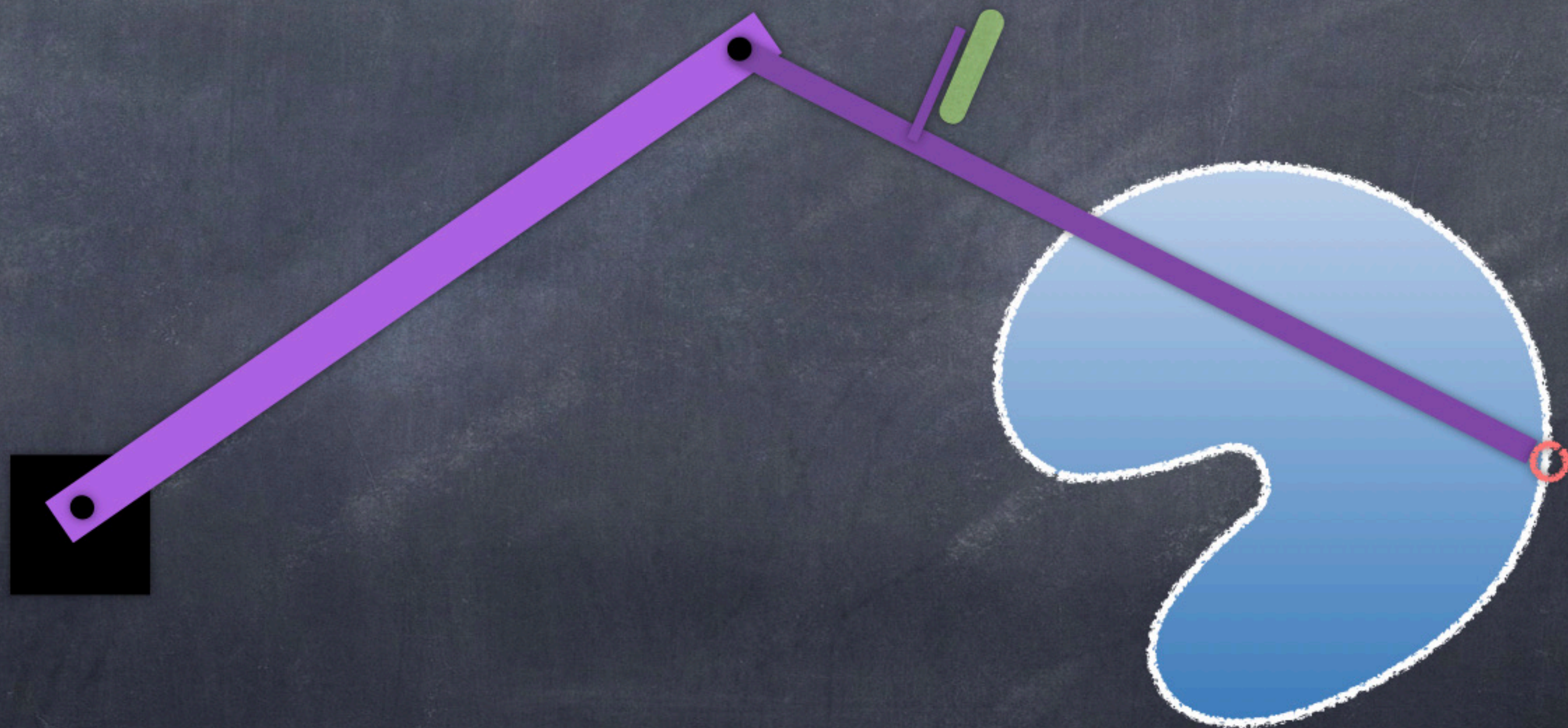
$$ydx - xdy$$

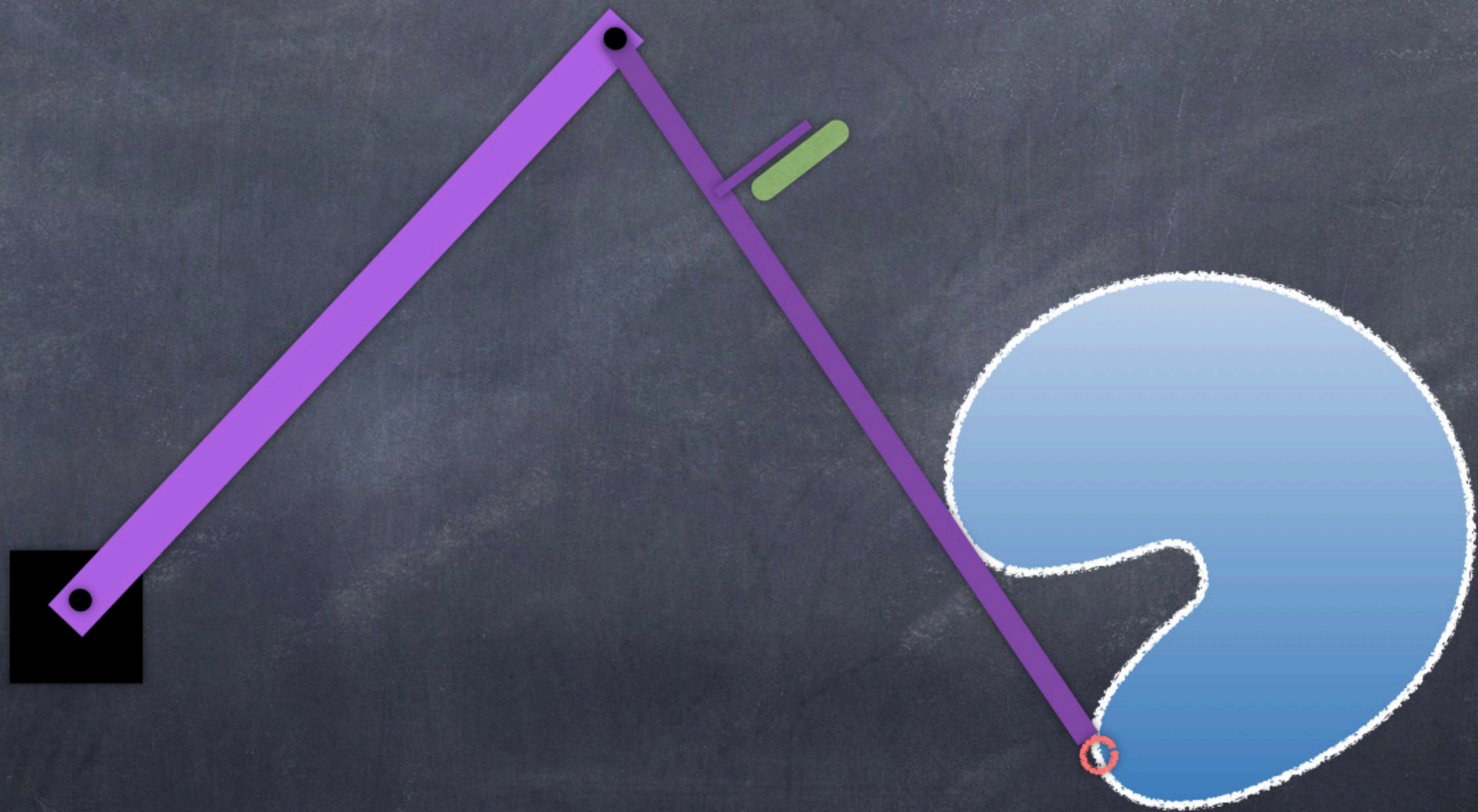
$$2 \text{ Area}(D) = \iint_D \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} \right) dx dy = \oint_{\partial D} (-P dy + Q dx)$$

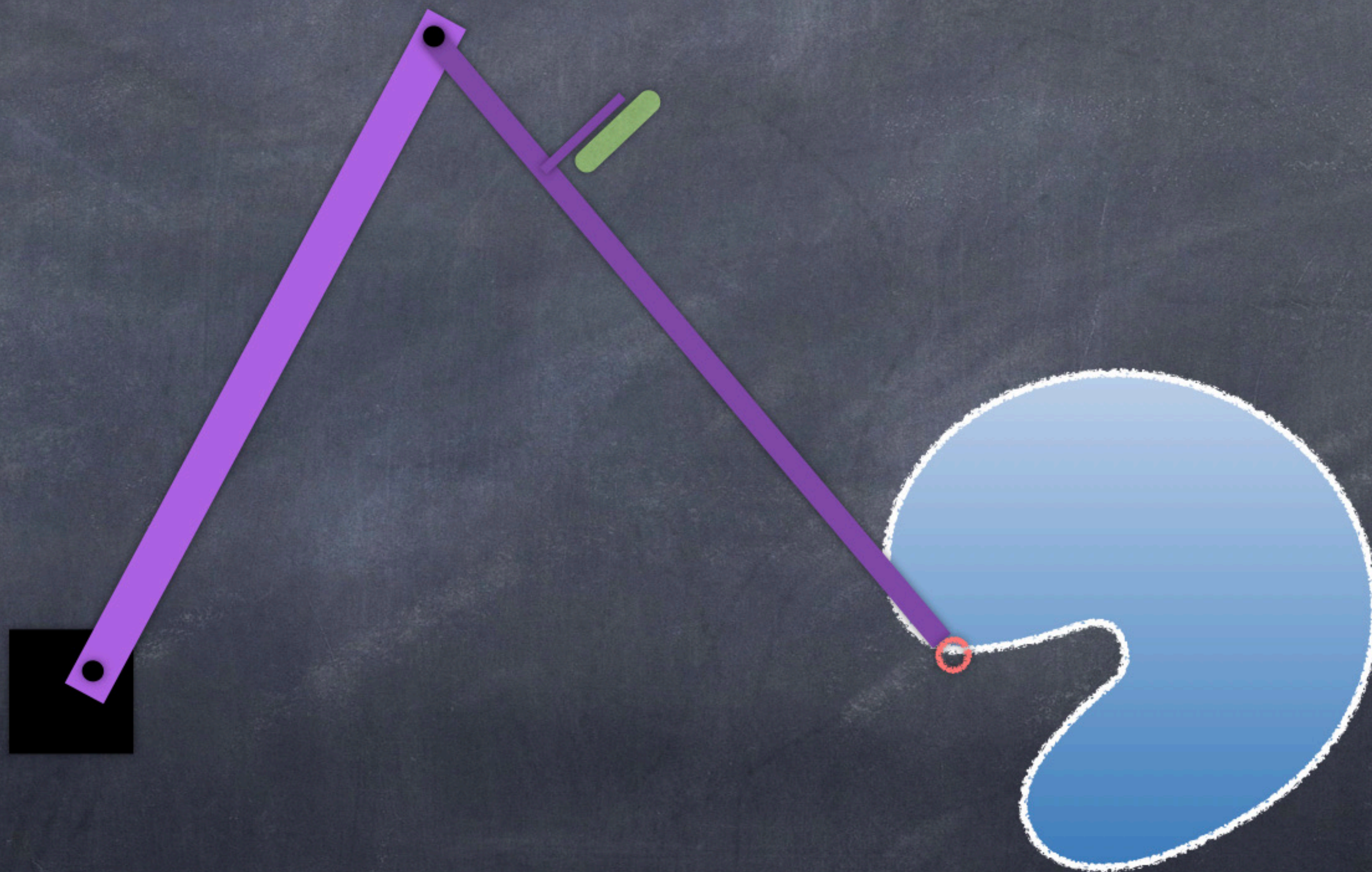
Creation inside D

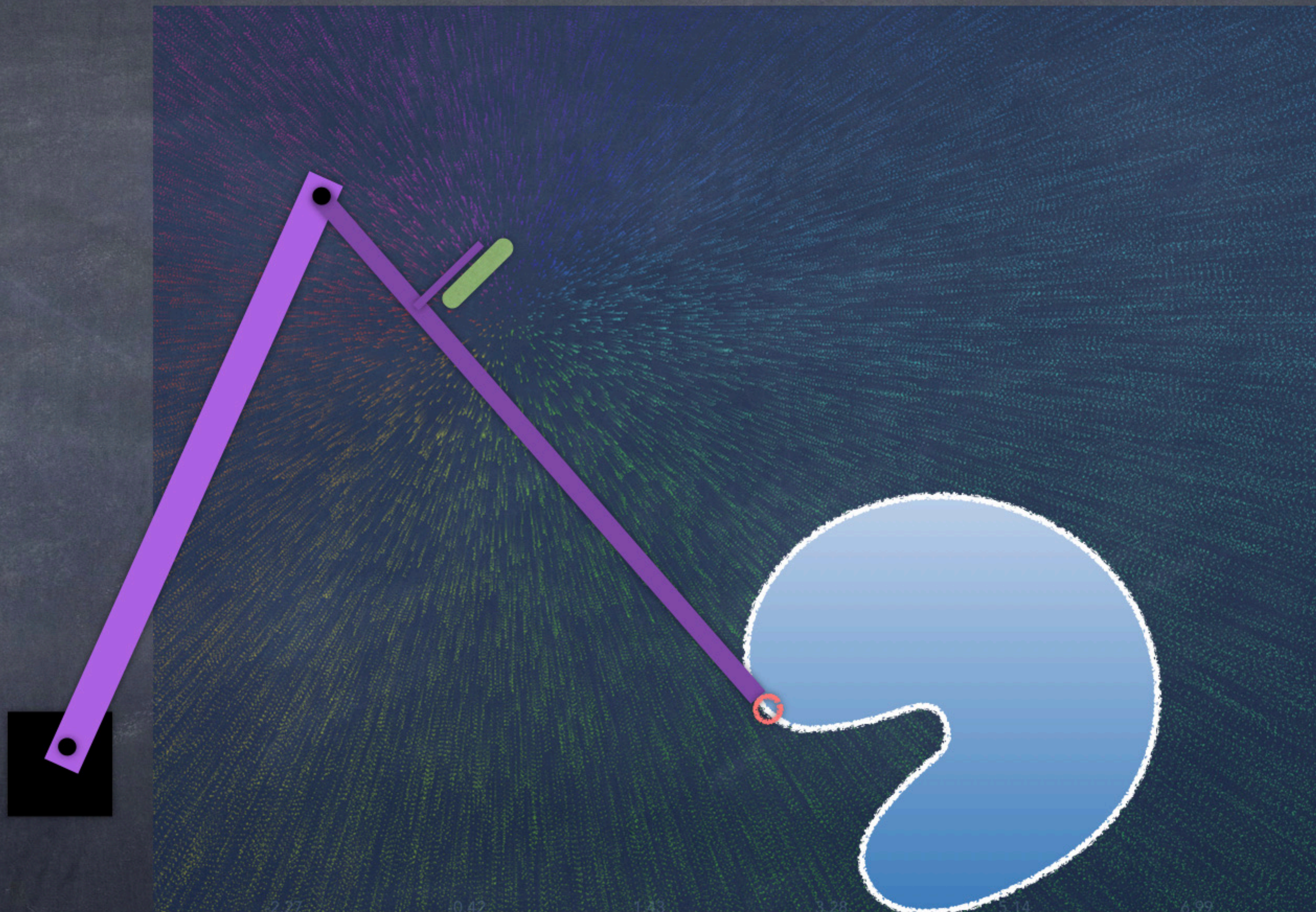
Flow through ∂D

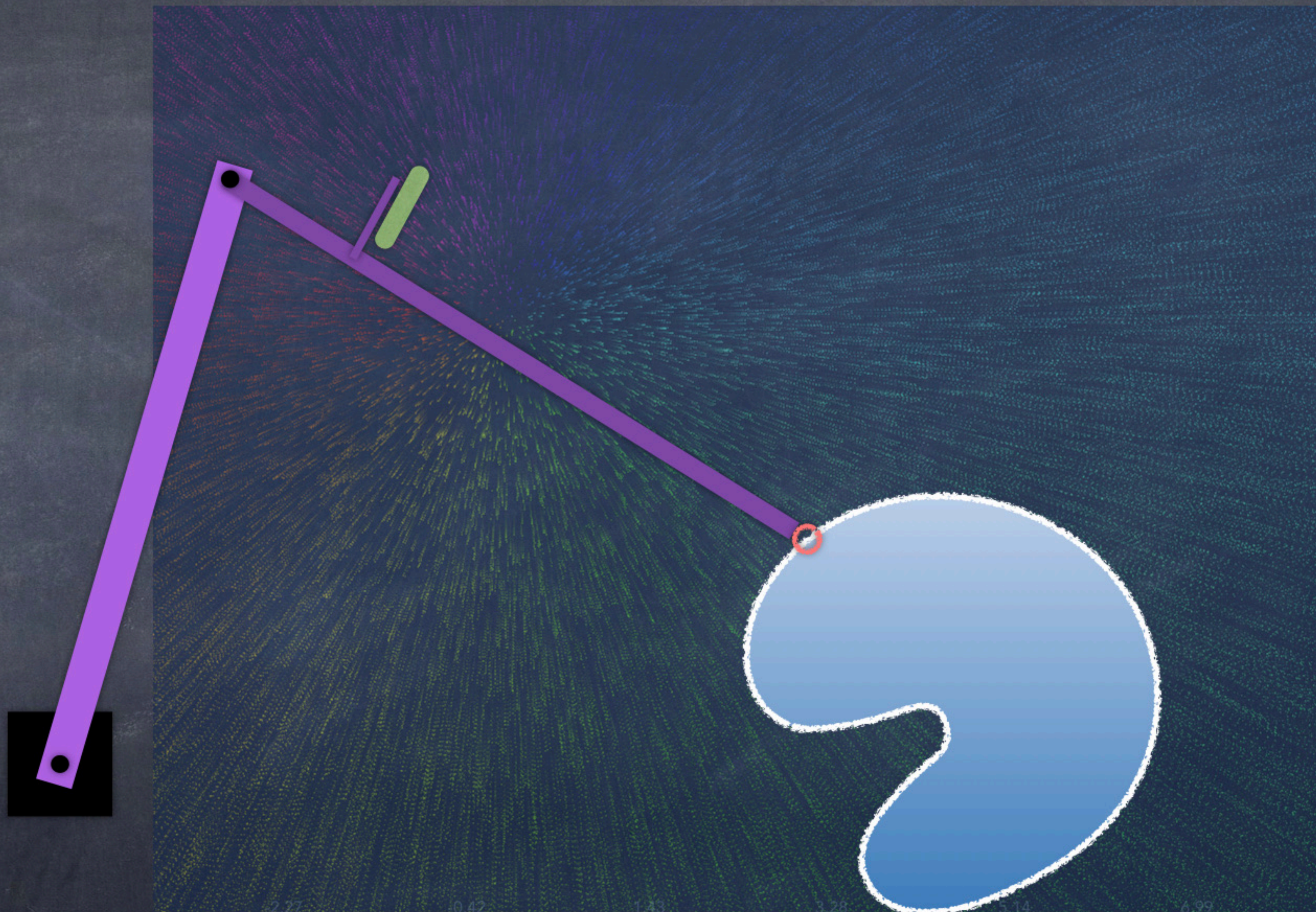













```

return new ArrayList<WordCount>() {
    @Override
    public boolean add(WordCount wc) {
        return true;
    }
};

```

```
36 \include{Makros}
```

39 \date{\today}

```
41 \title{Notes}%
```

42 \author{Peter Jossen, \today}%

```
44 \begin{document}
```

48 \maketitle%

We place the basis of the planimeter at the origin in the Euclidean plane \mathbb{R}^2 , and denote by m and l the lengths of the guiding arm and the tracer arm respectively, as indicated in the following diagram.

53 \begin{center}

```
\includegraphics[width=0.5\textwidth]{PlanimeterScheme}
```

```
55 \end{center}
```

Denote by $\gamma: [0, 1] \rightarrow \mathbb{R}^2$ a simple, differentiable curve enclosing the domain D . According to Green's Theorem, the surface area of D is equal to

$$\text{Area}(D) = \frac{1}{2} \int_{\gamma} (x dy - y dx) = \frac{1}{2} \int_0^1 c_1(t) c_2'(t) - c_2(t) c_1'(t) dt = \frac{1}{2} \int_0^1 r^2(t) \varphi'(t) dt$$

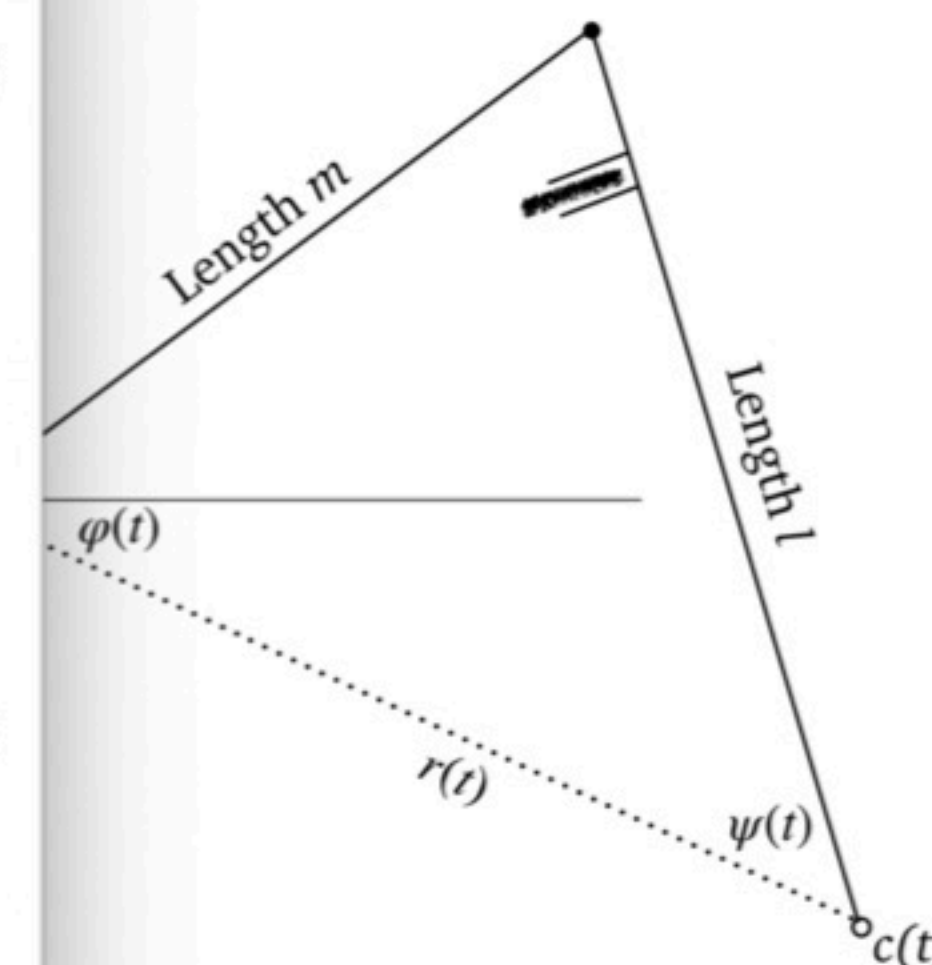
```
62 \end{document}
```

56 \vspace{14mm}

NOTES

ETER JOSSEN, FEBRUARY 22, 2025

meter at the origin in the Euclidean plane \mathbb{R}^2 , and denote by m rm and the tracer arm respectively, as indicated in the following



ple, differentiable curve enclosing the domain D . According to
a of D is equal to

$$dx) = \frac{1}{2} \int_0^1 c_1(t)c_2'(t) - c_2(t)c_1'(t)dt = \frac{1}{2} \int_0^1 r^2(t)\varphi'(t)dt$$


```
TexNotes.tex — Edited
Typeset LaTeX Macros Tags Labels Templates
34
35
36 \include{Makros}
37
38
39 \date{today}
40
41 \title{Notes}%
42 \author{Peter Jossen, \today}%
43
44 \begin{document}
45
46
47
48 \maketitle%
49
50
51 We place the basis of the planimeter at the origin in the Euclidean plane  $\mathbb{R}^2$ 
the guiding arm and the tracer arm respectively, as indicated in the following diagram.
52
53 \begin{center}
54 \includegraphics[width=0.5\textwidth]{PlanimeterScheme}
55 \end{center}
56
57 \noindent Denote by  $c:[0,1] \rightarrow \mathbb{R}^2$  a simple, differentiable curve enclosing the
Theorem, the surface area of  $D$  is equal to
58 
$$\mathrm{Area}(D) = \frac{1}{2} \int_{\gamma} (x dy - y dx) = \frac{1}{2} \int_0^1 c_1(t) c_2'(t) - c_2(t) c_1'(t) dt$$

59
60
61
62 \end{document}
63
64
65
66 \vspace{14mm}
```

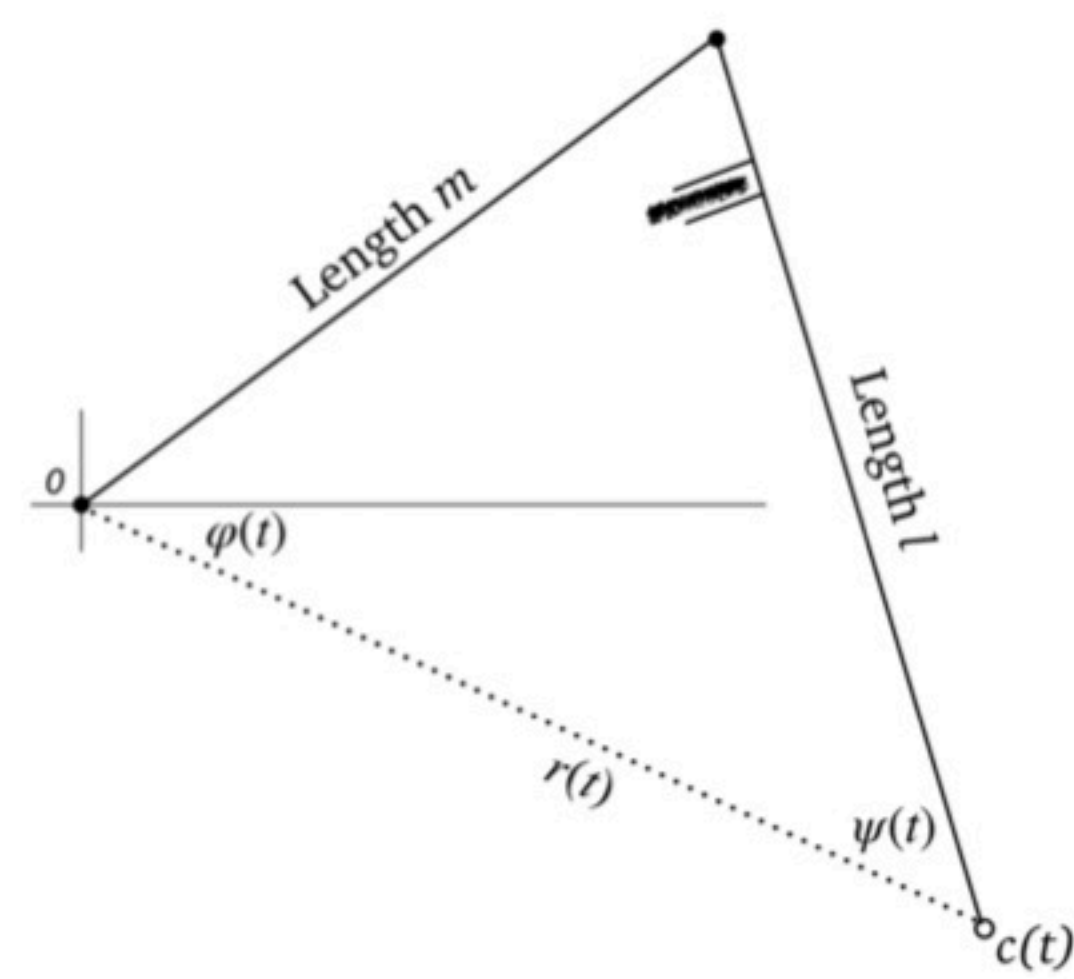
TexNotes.pdf

Scale 156 Page 1 of 1

NOTES

PETER JOSSEN, FEBRUARY 22, 2025

We place the basis of the planimeter at the origin in the Euclidean plane \mathbb{R}^2 , and denote by m and l the lengths of the guiding arm and the tracer arm respectively, as indicated in the following diagram.



The diagram shows a planimeter with a guiding arm of length m and a tracer arm of length l . The origin o is at the base of the guiding arm. The angle between the guiding arm and the horizontal is $\varphi(t)$. The angle between the tracer arm and the line segment from o to $c(t)$ is $\psi(t)$. The point $c(t)$ is the end of the tracer arm.

Denote by $c : [0, 1] \rightarrow \mathbb{R}^2$ a simple, differentiable curve enclosing the domain D . According to Green's Theorem, the surface area of D is equal to

$$\mathrm{Area}(D) = \frac{1}{2} \int_{\gamma} (x dy - y dx) = \frac{1}{2} \int_0^1 c_1(t) c_2'(t) - c_2(t) c_1'(t) dt = \frac{1}{2} \int_0^1 r^2(t) \varphi'(t) dt$$



“I had a feeling once about Mathematics – that I saw it all. Depth beyond depth was revealed to me – the Byss and Abyss. I saw – as one might see the transit of Venus or even the Lord Mayor's Show – a quantity passing through infinity and changing its sign from plus to minus. I saw exactly why it happened and why the tergiversation was inevitable but it was after dinner and I let it go.”