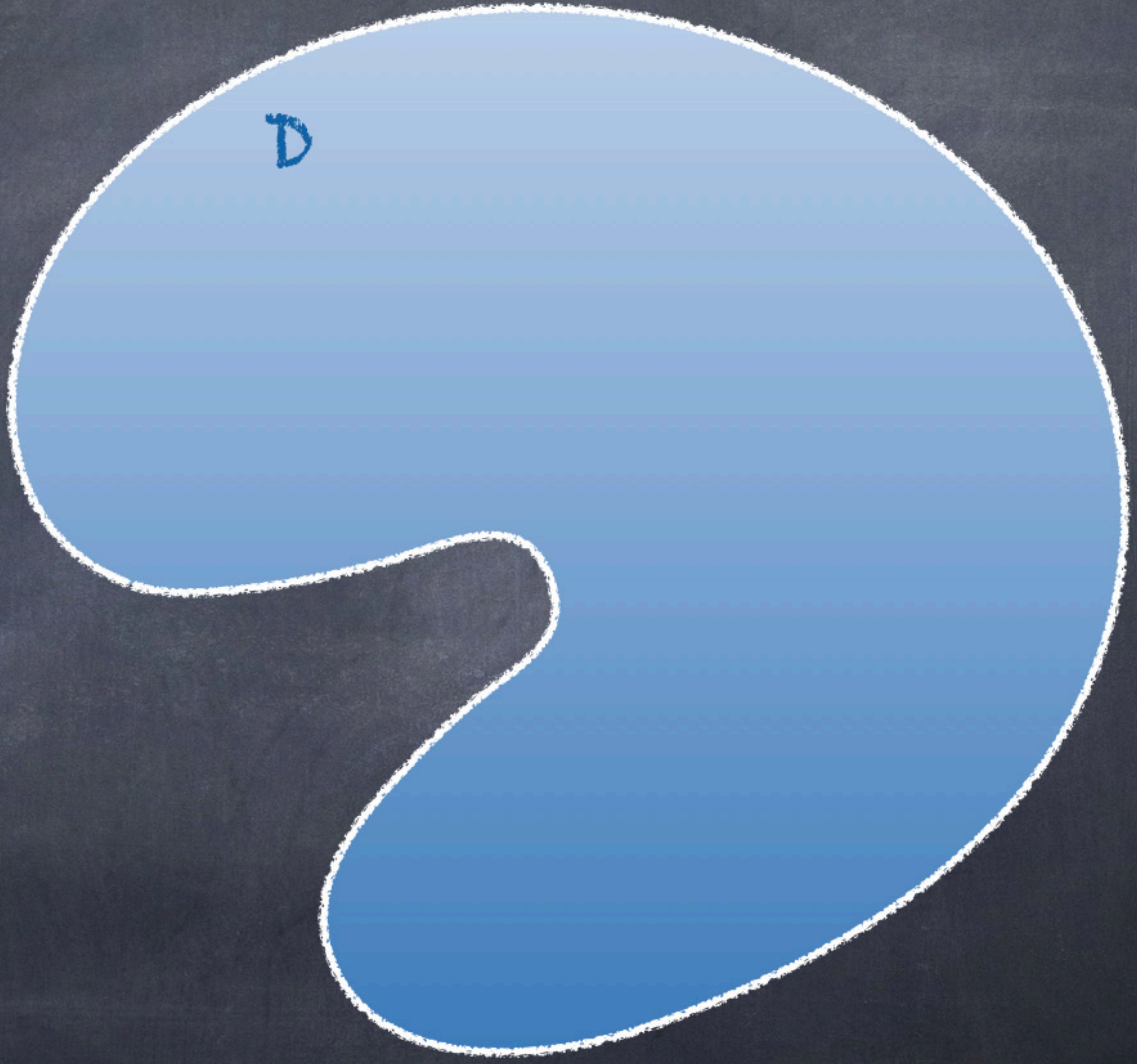
Measure the surface area of this shape:

$$\iint_D dxdy$$

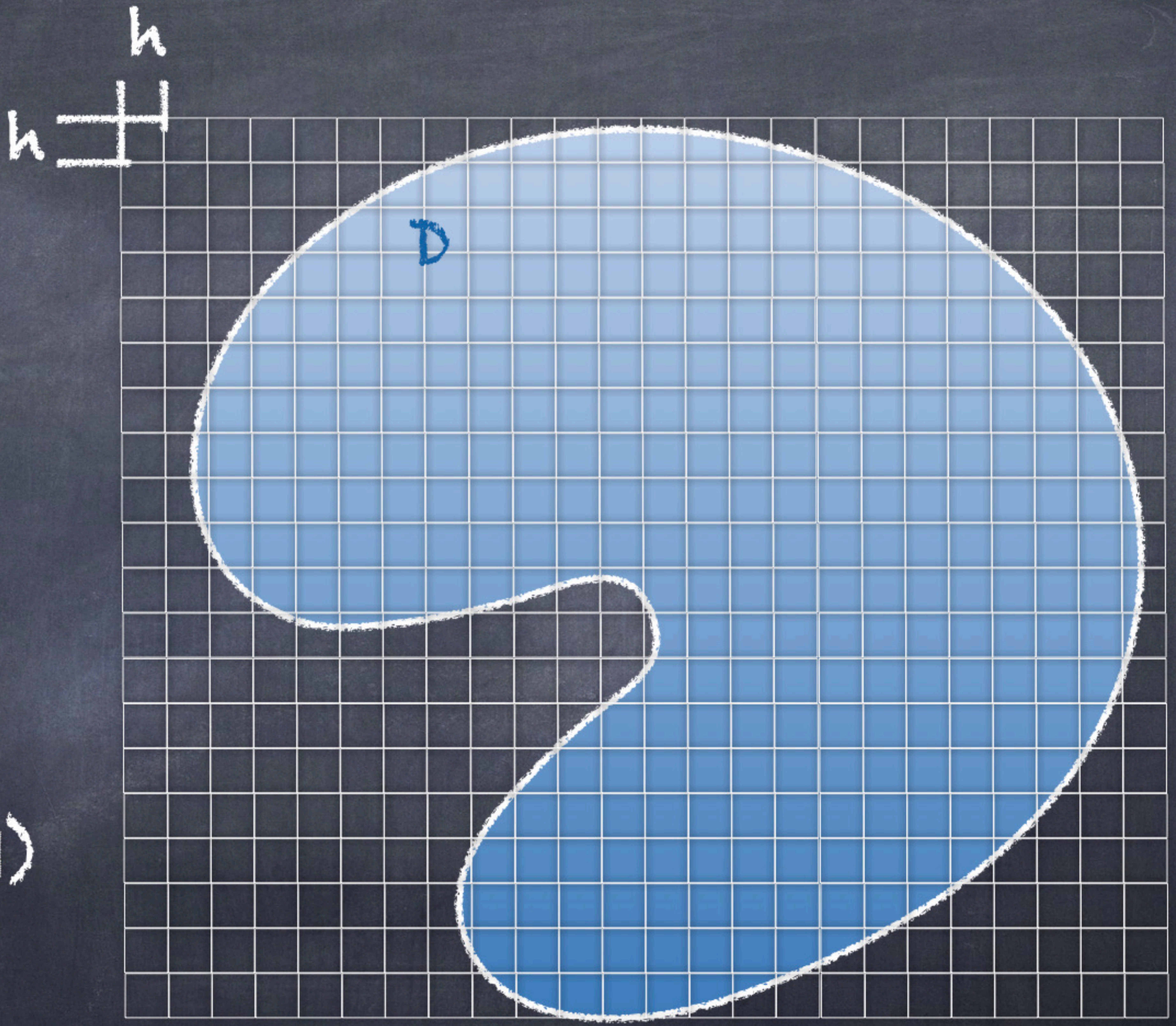
$$= \iint_{\mathbb{R}^2} 1_A dxdy$$



$$\iint_D dx\,dy$$

$$= \lim_{h \to 0} h^2 (\text{Number of } ■)$$

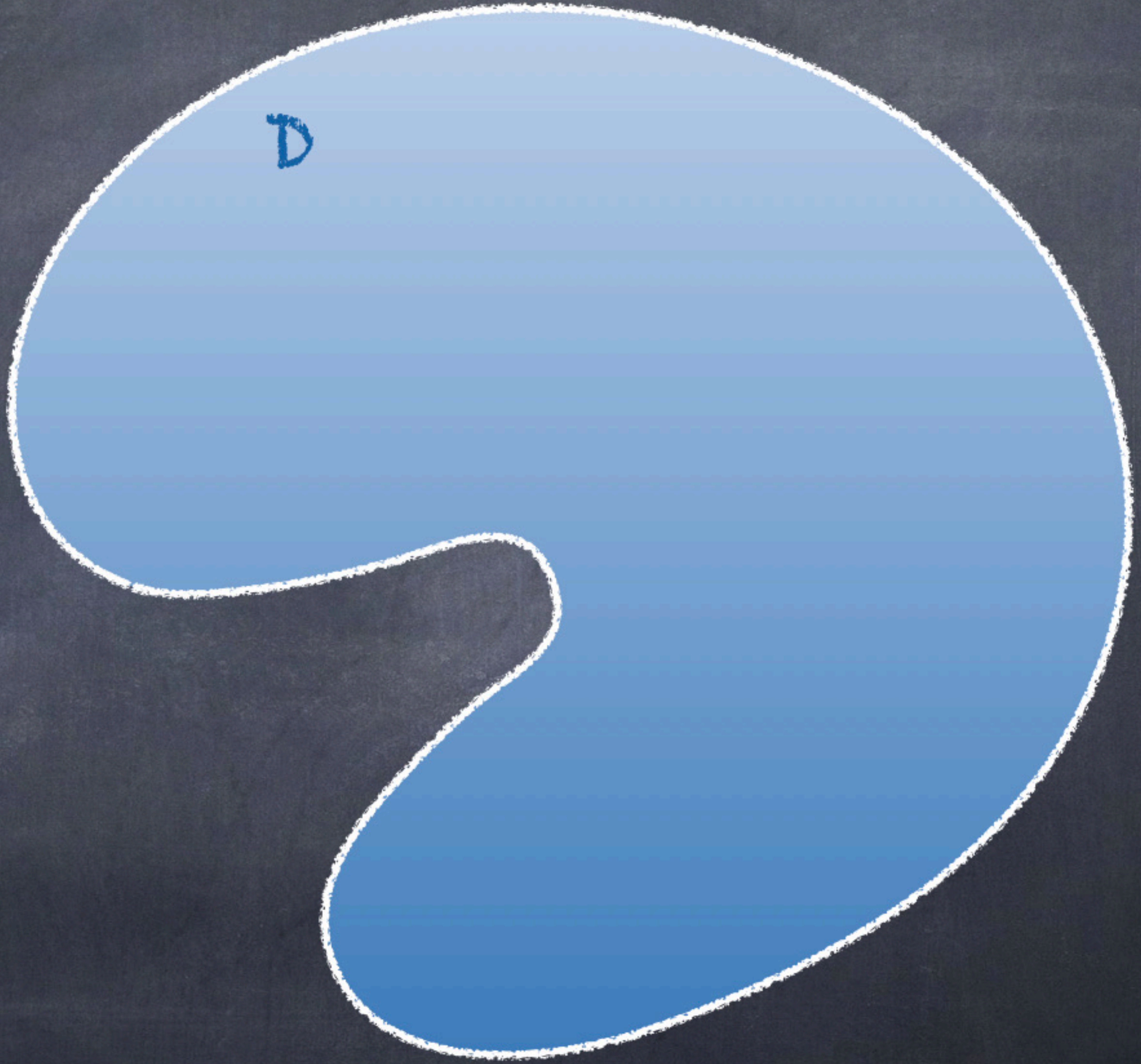
$$= \lim_{h \to 0} h^2 (\text{Number of } ◩, ■)$$



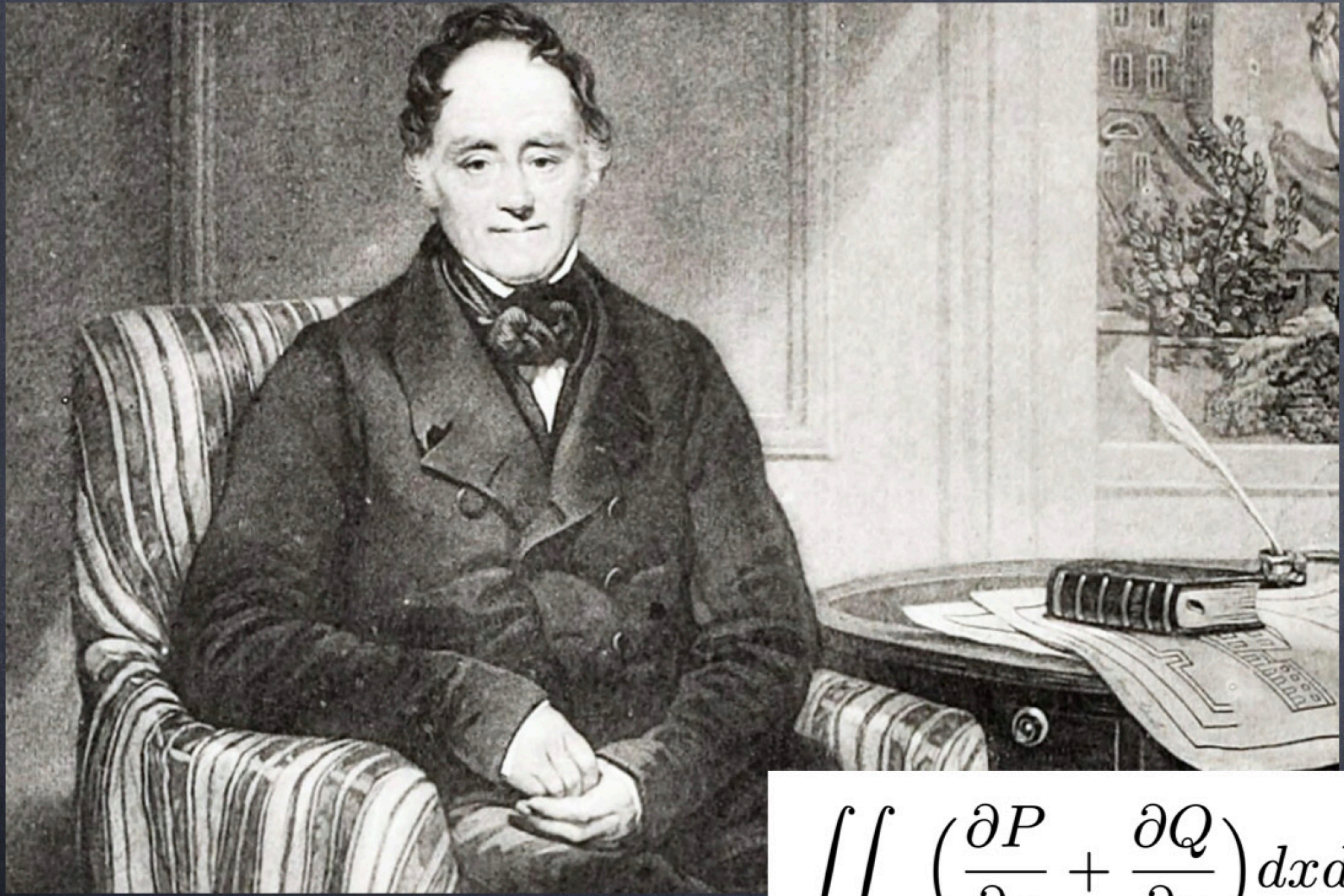
Measure the surface area of this shape:

Make a model using some homogeneous sheet material, then measure with a scale.

Use a planimeter!



George Green (1793-1841): Green's formula

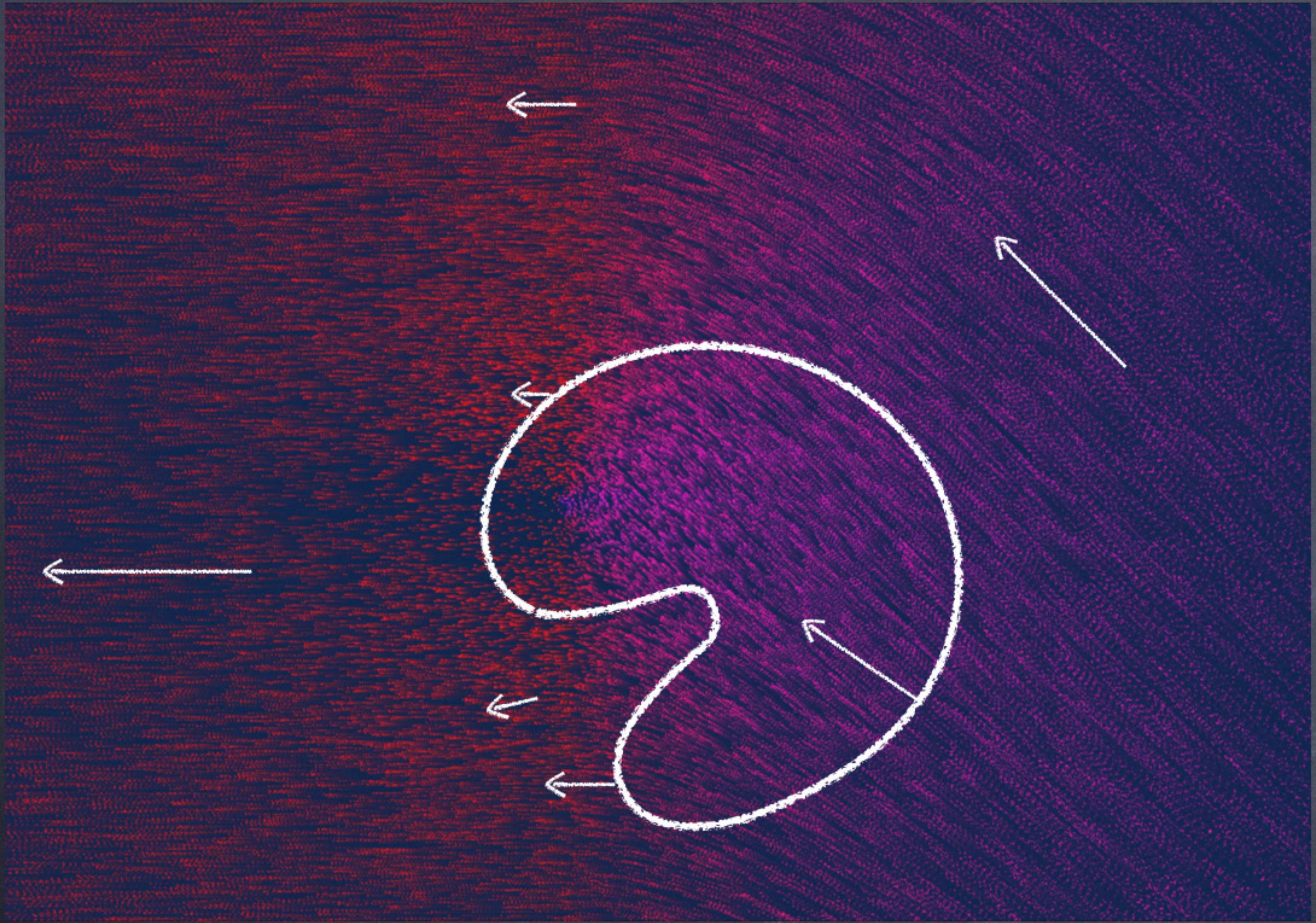


Divergence Theorem

Lagrange, Gauss, Ostrogradskii

Stokes, deRham

$$\iint_D \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y}\right) dxdy = \oint_{\partial D} (-Pdy + Qdx)$$

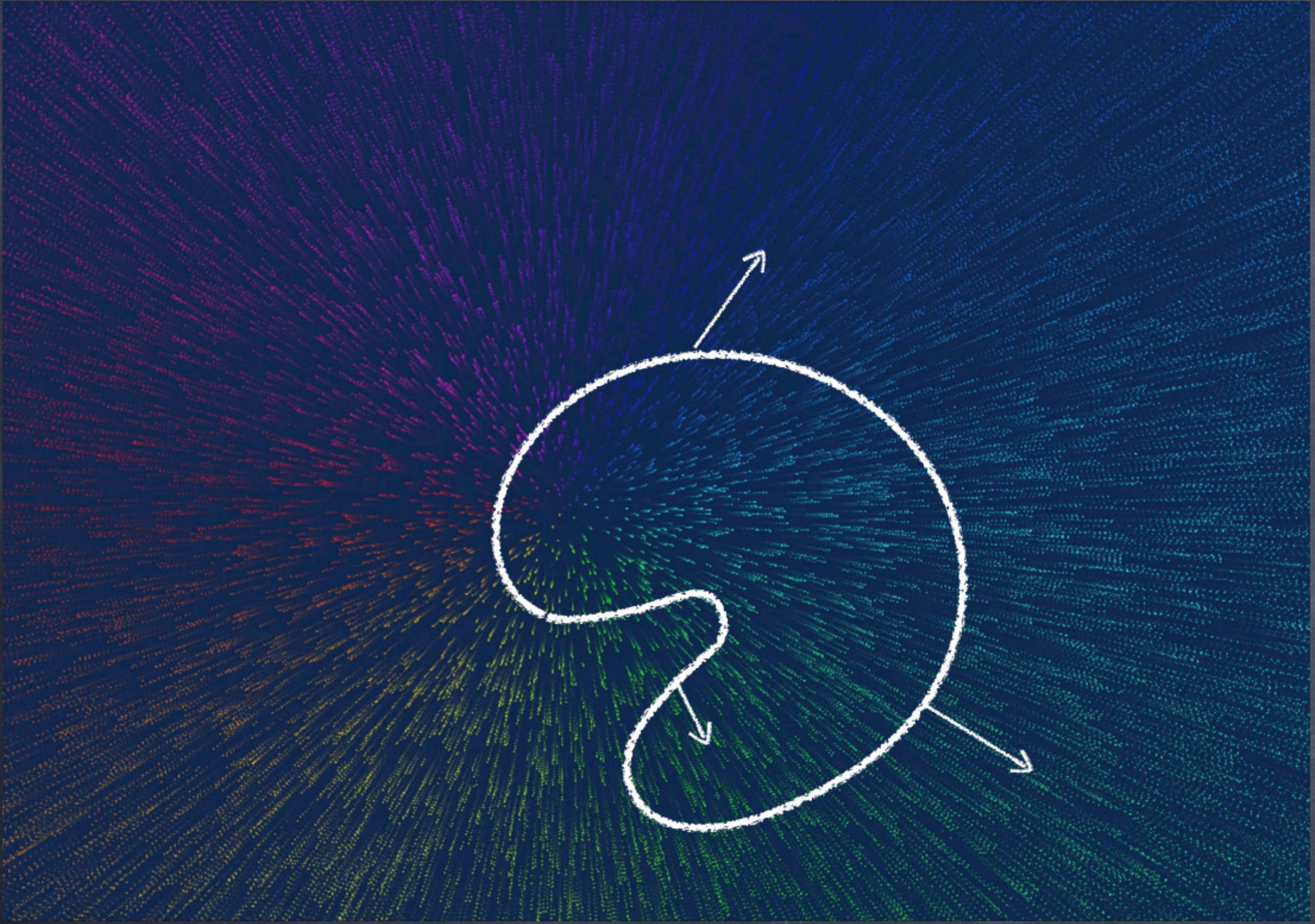


P(x, y)
Q(x, y) Vector field

Flow of a gas in the plane.

$$\iint_D \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y}\right) dxdy = \oint_{\partial D} (-Pdy + Qdx)$$

Creation inside D   Flow through ∂D



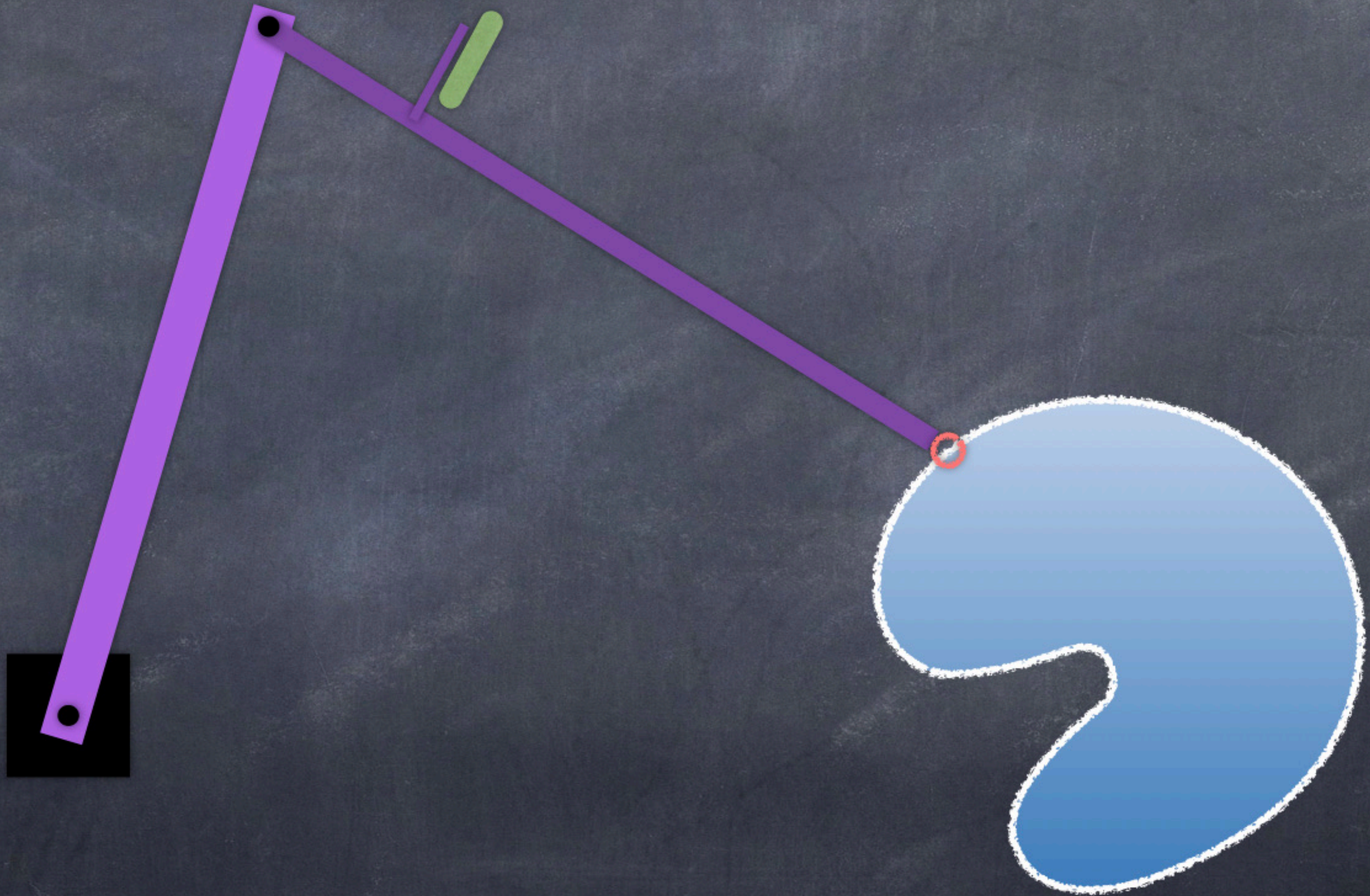
$P(x, y) = x$
$Q(x, y) = y$

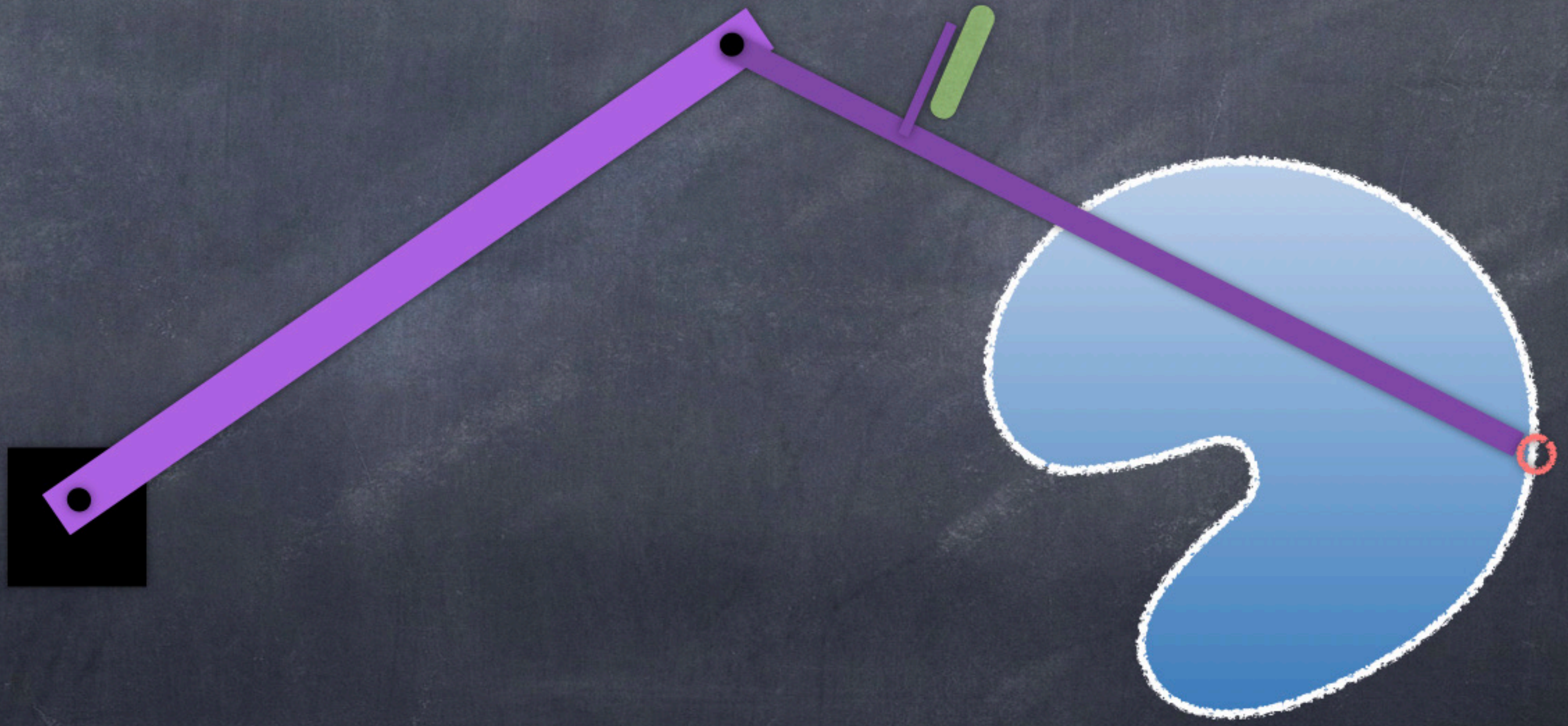
Flow of a gas in the plane.

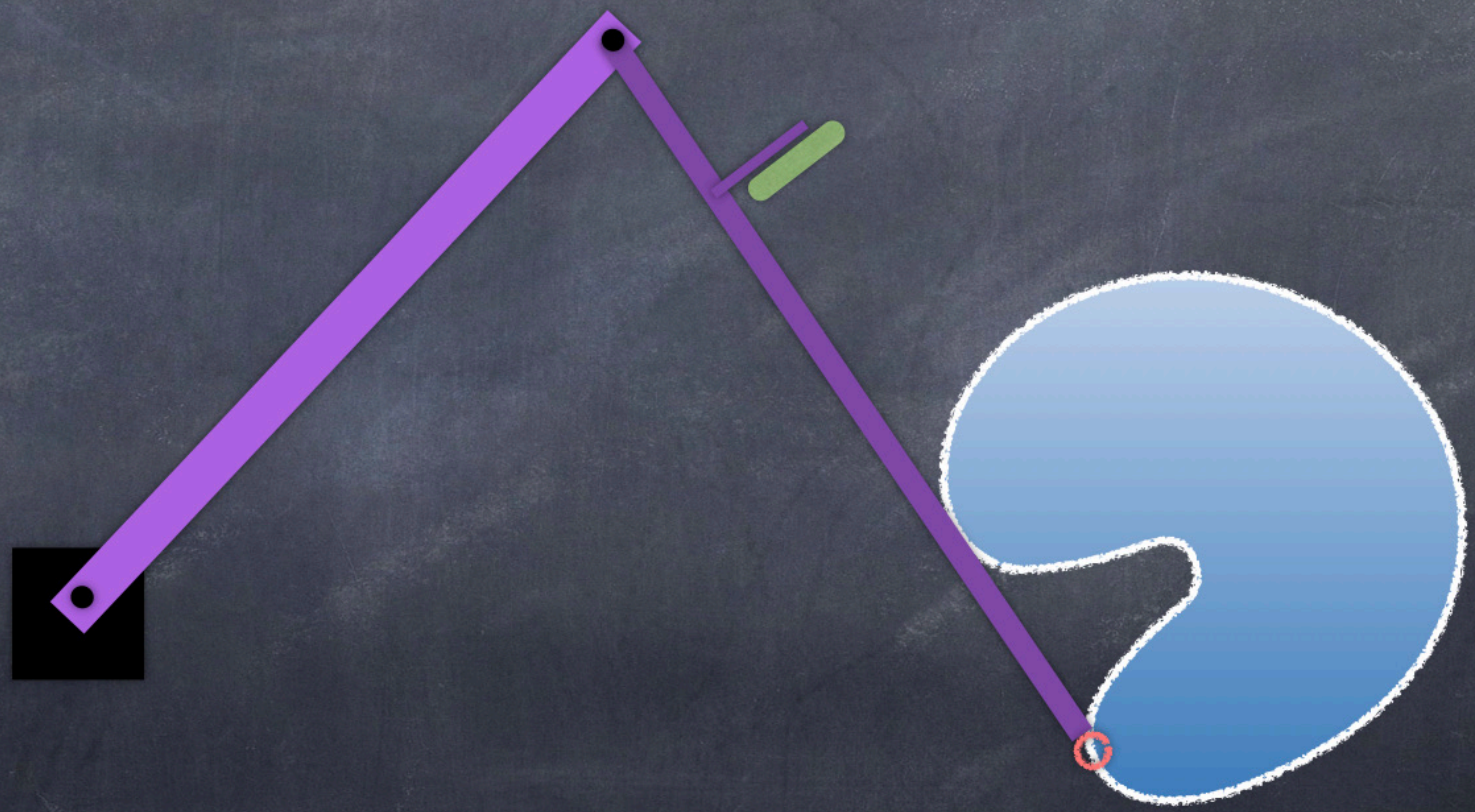
$ydx - xdy$

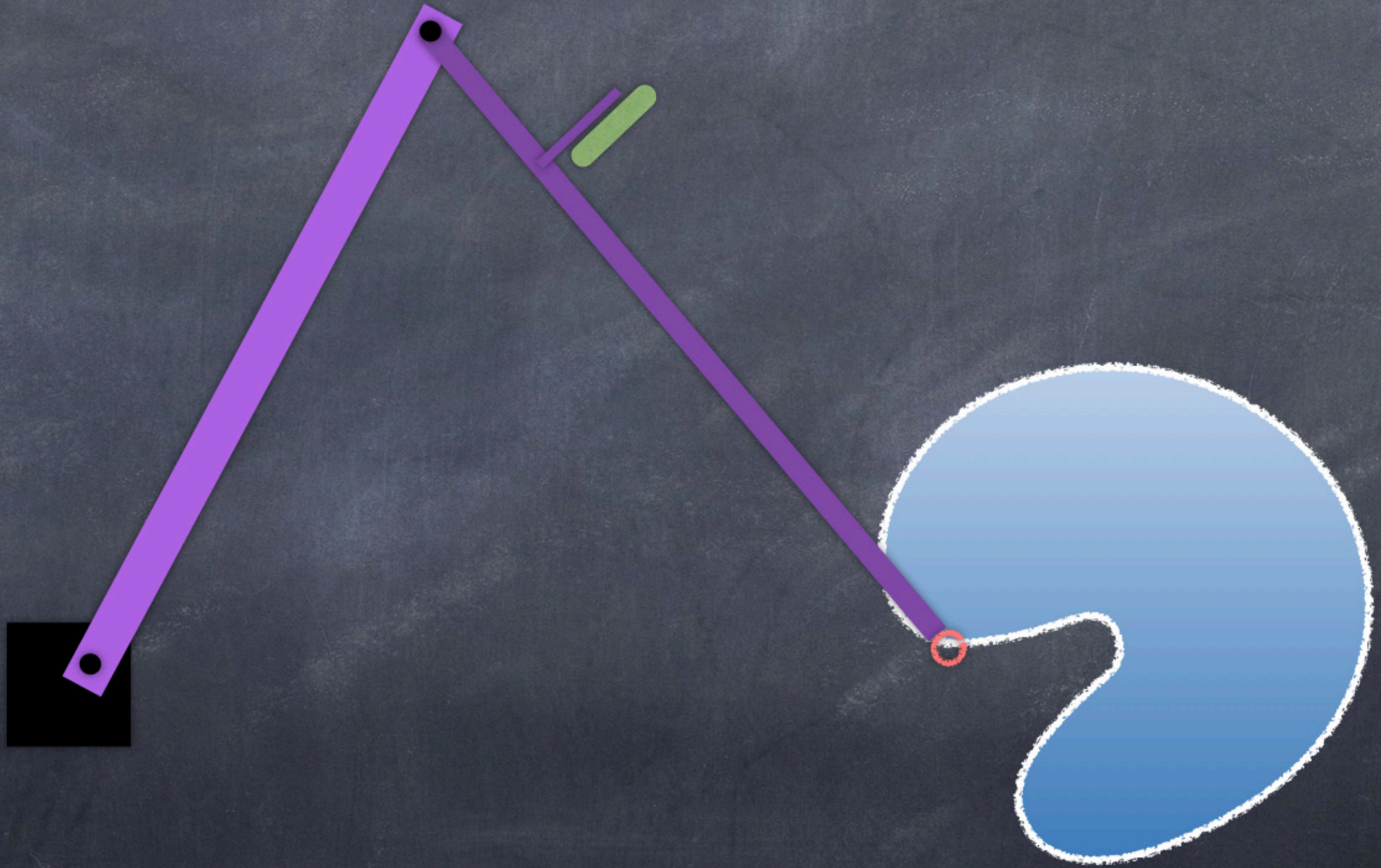
$$2\,\text{Area}(D) = \iint_D \Big(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y}\Big)dxdy = \oint_{\partial D}(-Pdy + Qdx)$$

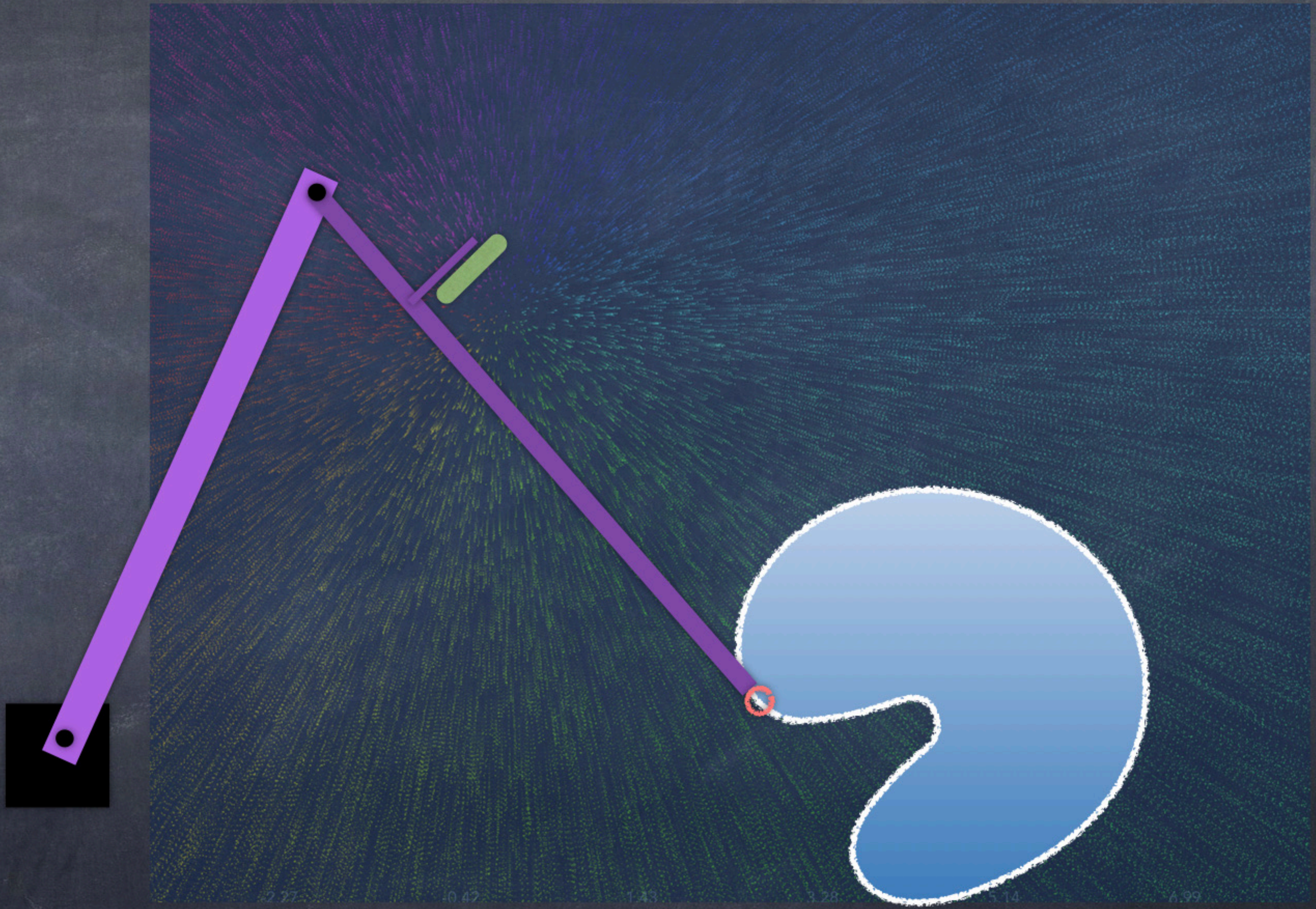
Creation inside D   Flow through ∂D

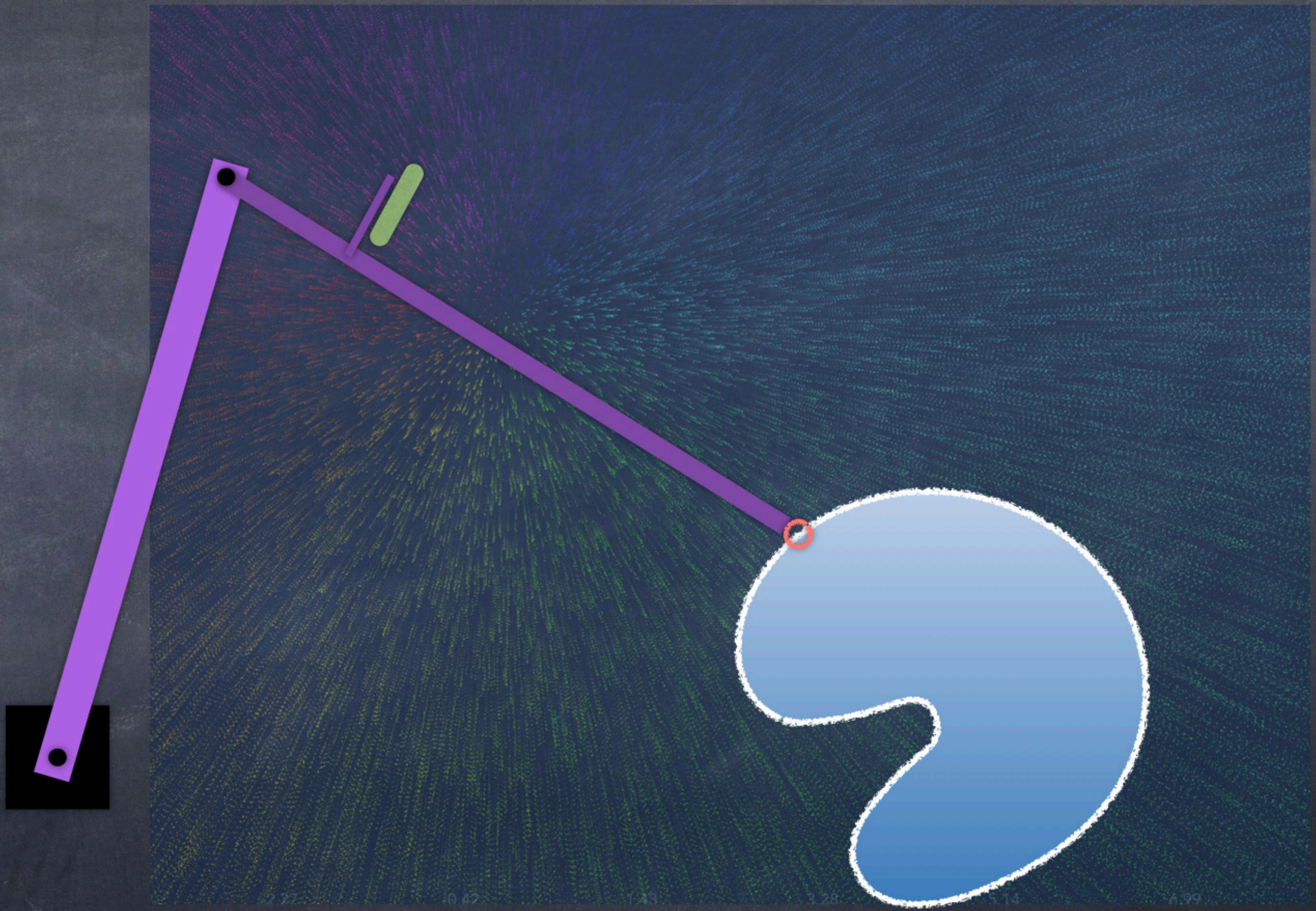












2.27
0.42
1.43
3.28
5.14
6.99

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\renewcommand{\baselinestretch}{1.2}


\include{Makros}


\date{\today}

\title{Notes}%
\author{Peter Jossen, \today}%

\begin{document}



\maketitle%


We place the basis of the planimeter at the origin in the Euclidean plane $\IR^2$, and denote by $m$ and $l$ the lengths of the guiding arm and the tracer arm respectively, as indicated in the following diagram.

\begin{center}
\includegraphics[width=0.5\textwidth]{PlanimeterScheme}
\end{center}

\noindent Denote by $c:[0,1]\to \IR^2$ a simple, differentiable curve enclosing the domain $D$. According to Green's Theorem, the surface area of $D$ is equal to
$$\mathrm{Area}(D)=\frac 12\int_\gamma (xdy - ydx) =\frac 12 \int_0^1 c_1(t)c_2'(t) - c_2(t)c_1'(t)dt =\frac 12 \int_0^1 r^2(t) \varphi'(t)dt$$



\end{document}



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TexNotes.pdf

1 of 1

Search

NOTES

ETER JOSSEN, FEBRUARY 22, 2025

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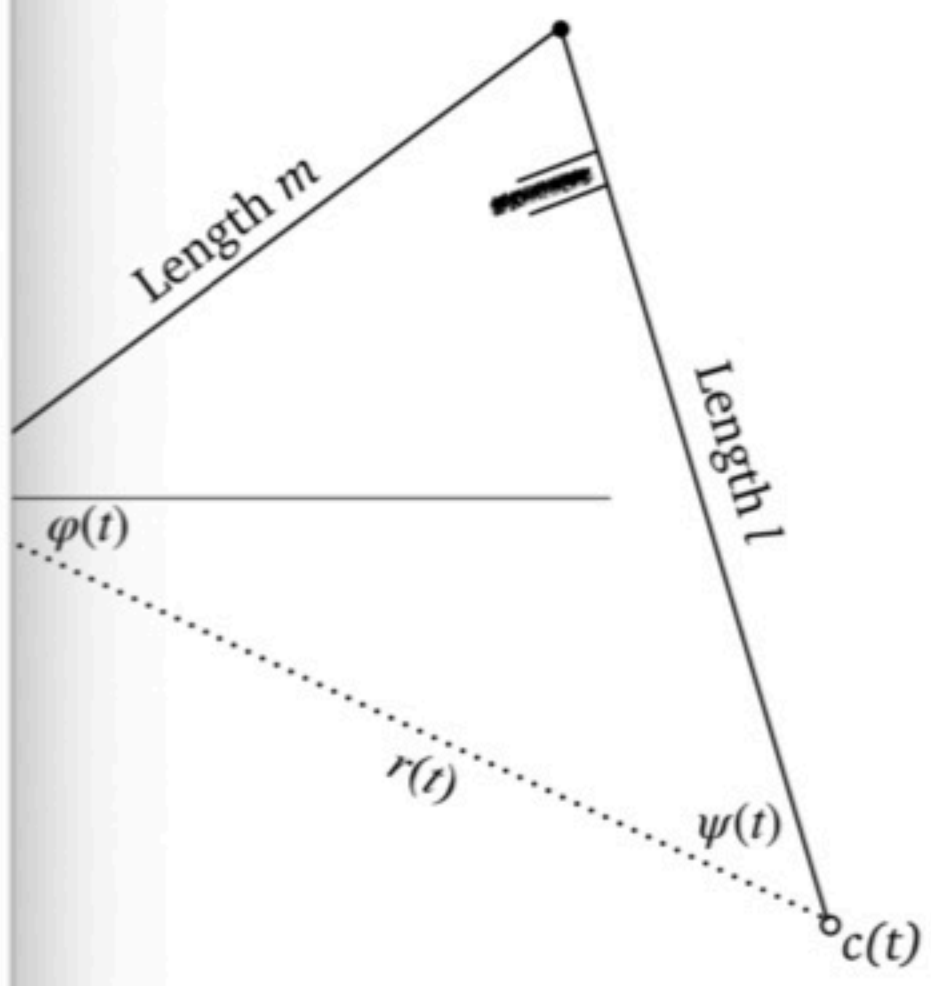

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a of D is equal to

$$dx) = \frac{1}{2}\int_0^1 c_1(t)c_2'(t) - c_2(t)c_1'(t)dt = \frac{1}{2}\int_0^1 r^2(t)\varphi'(t)dt$$

TexNotes.tex — Edited

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\date{\today}

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\end{document}

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TexNotes.pdf

Scale 156 Page 1 of 1 Search

NOTES

PETER JOSSEN, FEBRUARY 22, 2025

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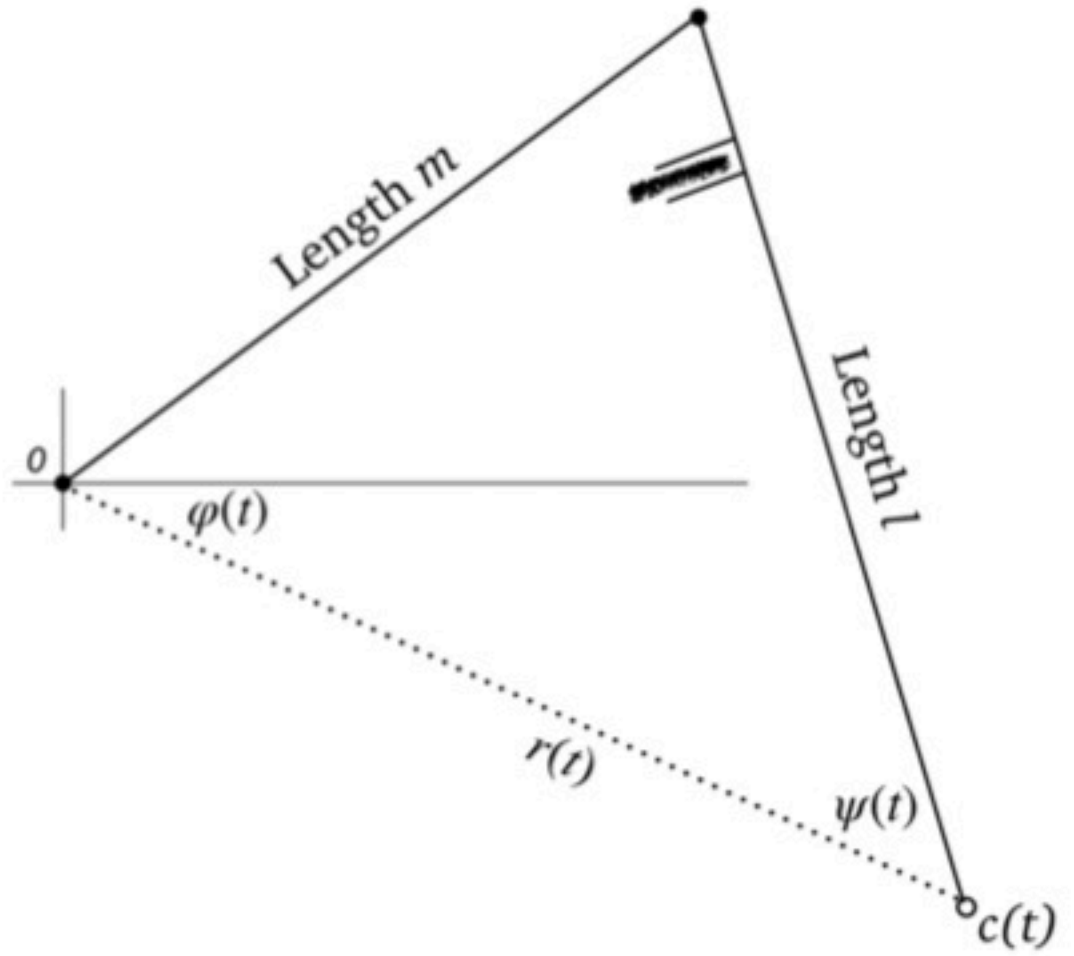

Denote by $c : [0,1] \longrightarrow \mathbb{R}^2$ a simple, differentiable curve enclosing the domain D. According to Green's Theorem, the surface area of D is equal to

$$\mathrm{Area}(D) = \frac{1}{2}\int_\gamma (xdy - ydx) = \frac{1}{2}\int_0^1 c_1(t)c_2'(t) - c_2(t)c_1'(t)dt = \frac{1}{2}\int_0^1 r^2(t)\varphi'(t)dt$$

“I had a feeling once about Mathematics – that I saw it all. Depth beyond depth was revealed to me – the Byss and Abyss. I saw – as one might see the transit of Venus or even the Lord Mayor's Show – a quantity passing through infinity and changing its sign from plus to minus. I saw exactly why it happened and why the tergiversation was inevitable but it was after dinner and I let it go.”