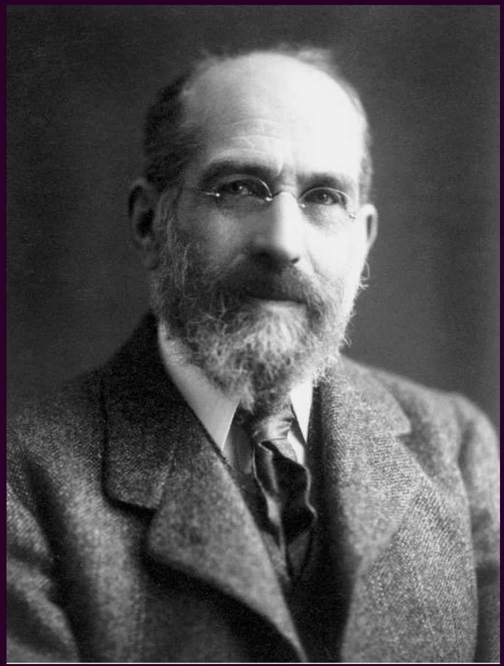
# CAN ONE HEAR THE SHAPE OF A DRUM?

JEAN LAGACÉ

ARTHUR SCHUSTER!

1882



# MARK KAC:

SIMPLIFY THE QUESTION:

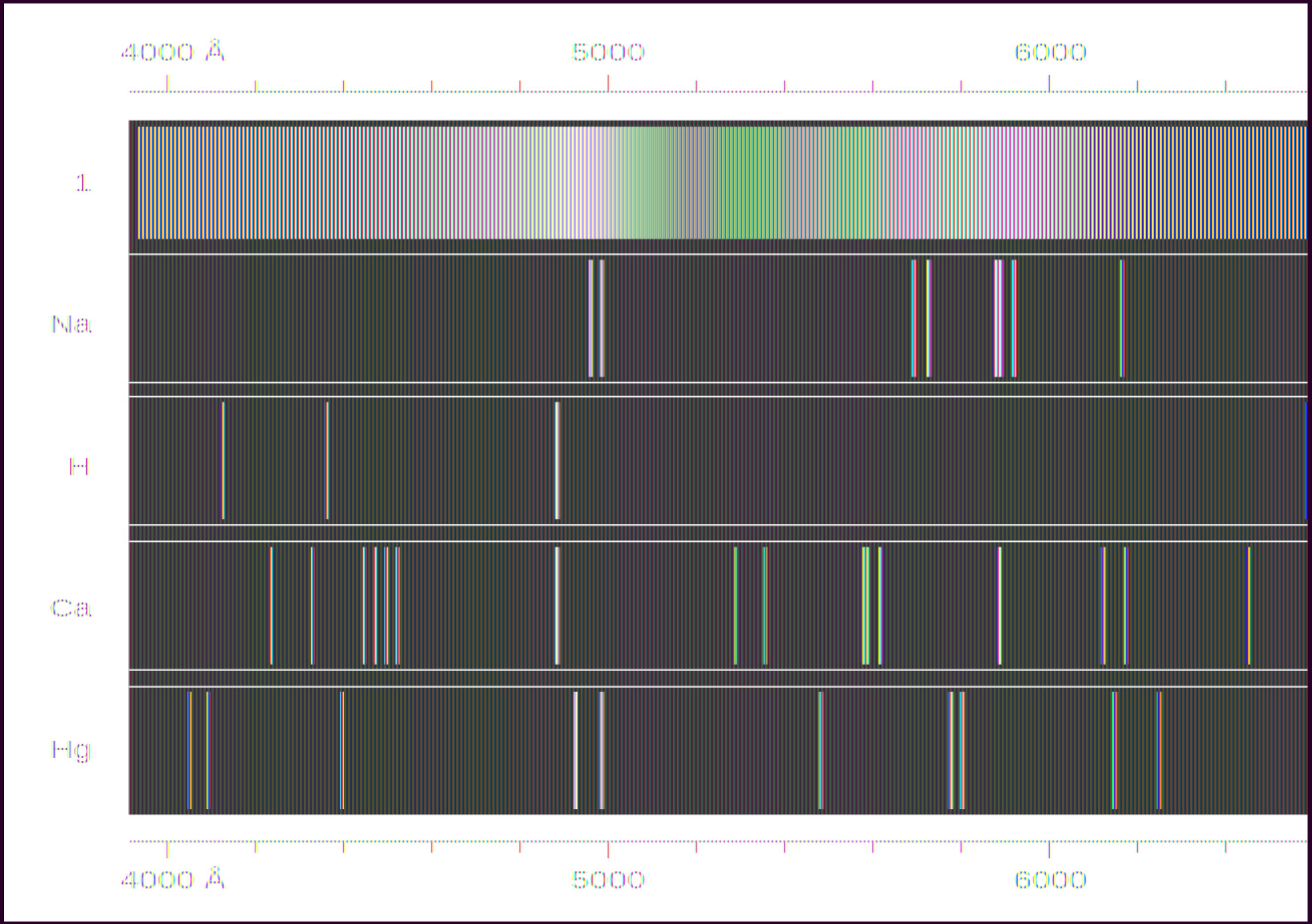
HOW ABOUT THE SHAPE OF A DRUM?

# SOUND → SHAPE

FREQUENCIES

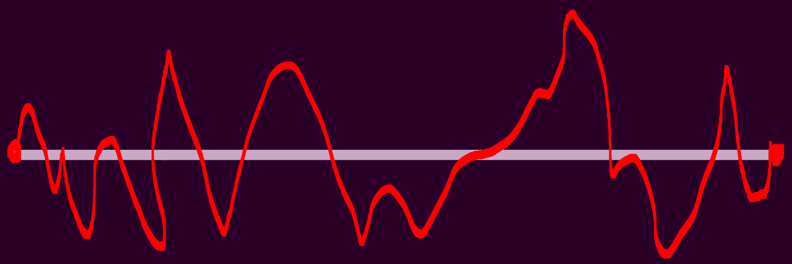
SIGNATURE

# ATOMIC SPECTROSCOPY



# MODELISING THE SOUND

SOUND IS



VIBRATION

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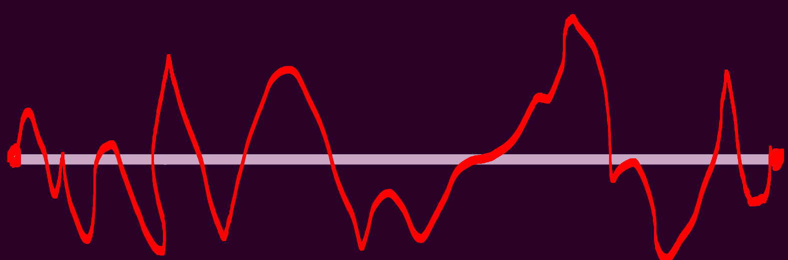
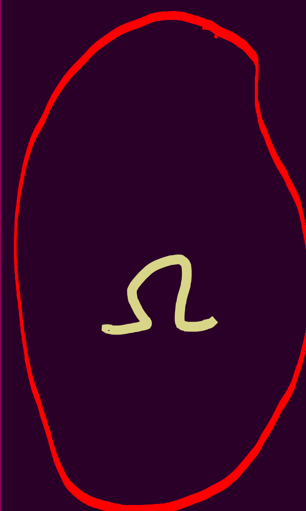
SOUND IS



VIBRATION

WAVE EQUATION

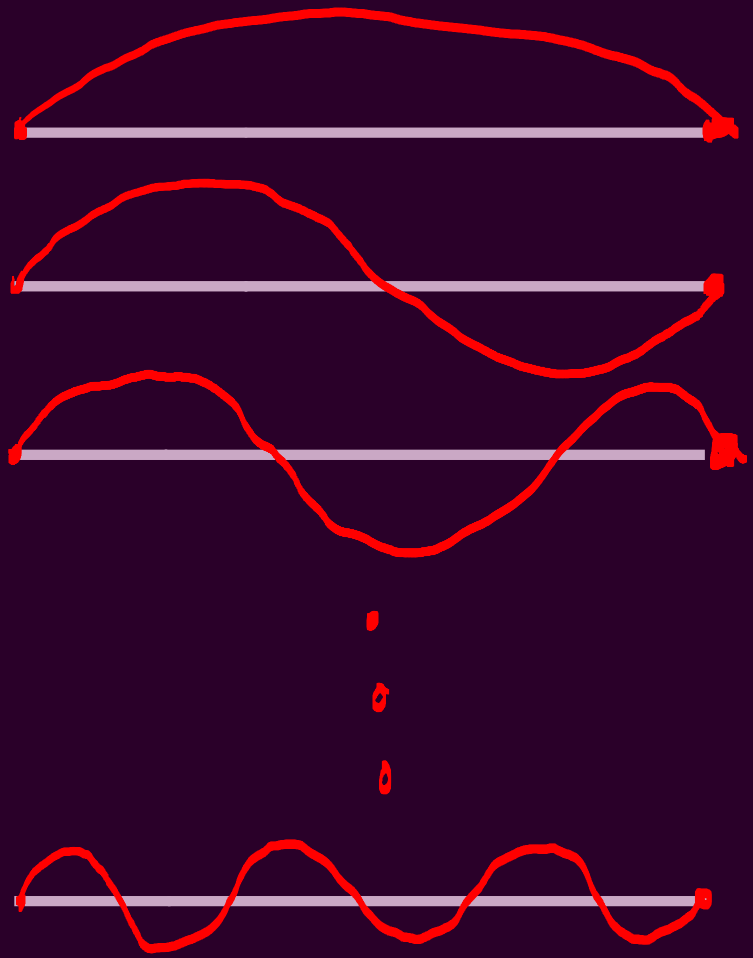



$$\frac{\partial^2 u}{\partial t^2} = \overbrace{\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}}^{\Delta}$$

$$u(t,x) = 0, \quad x \in \partial\Omega$$

$$u(0,x) = f(x)$$

# SUPERPOSITION OF WAVES



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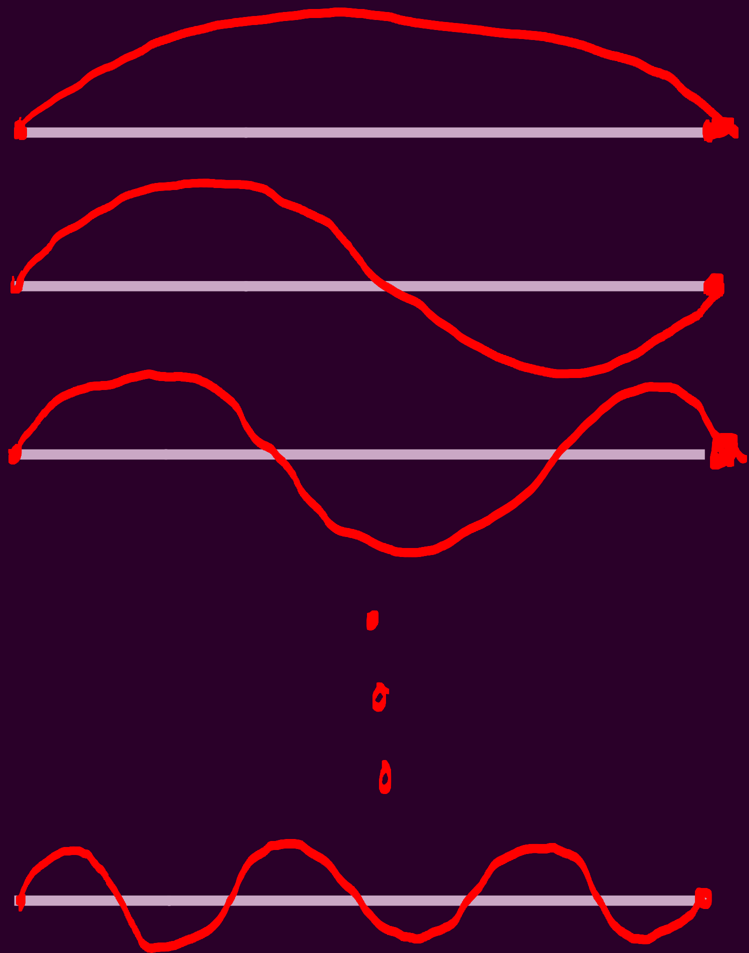


EIGENVALUE EQUATION

$$-\Delta u = \lambda u \quad \text{in } \Omega$$

$$u = 0 \quad \text{on } \partial\Omega.$$

$$\lambda_1(\Omega) < \lambda_2(\Omega) \le \lambda_3(\Omega)$$

$$\dots \nearrow \infty$$

$$\lambda_1(\Omega) < \lambda_2(\Omega) \leq \lambda_3(\Omega) \ldots$$

Shape of $\Omega$

# WARM-UP: HEARING THE SHAPE OF A STRING

$\Omega = [0, L]$

$-u'' = \lambda u \quad \text{on } [0, L]$

$u(0) = u(L) = 0$

SOLUTIONS:

$\sin\left(\frac{\pi n x}{L}\right)$

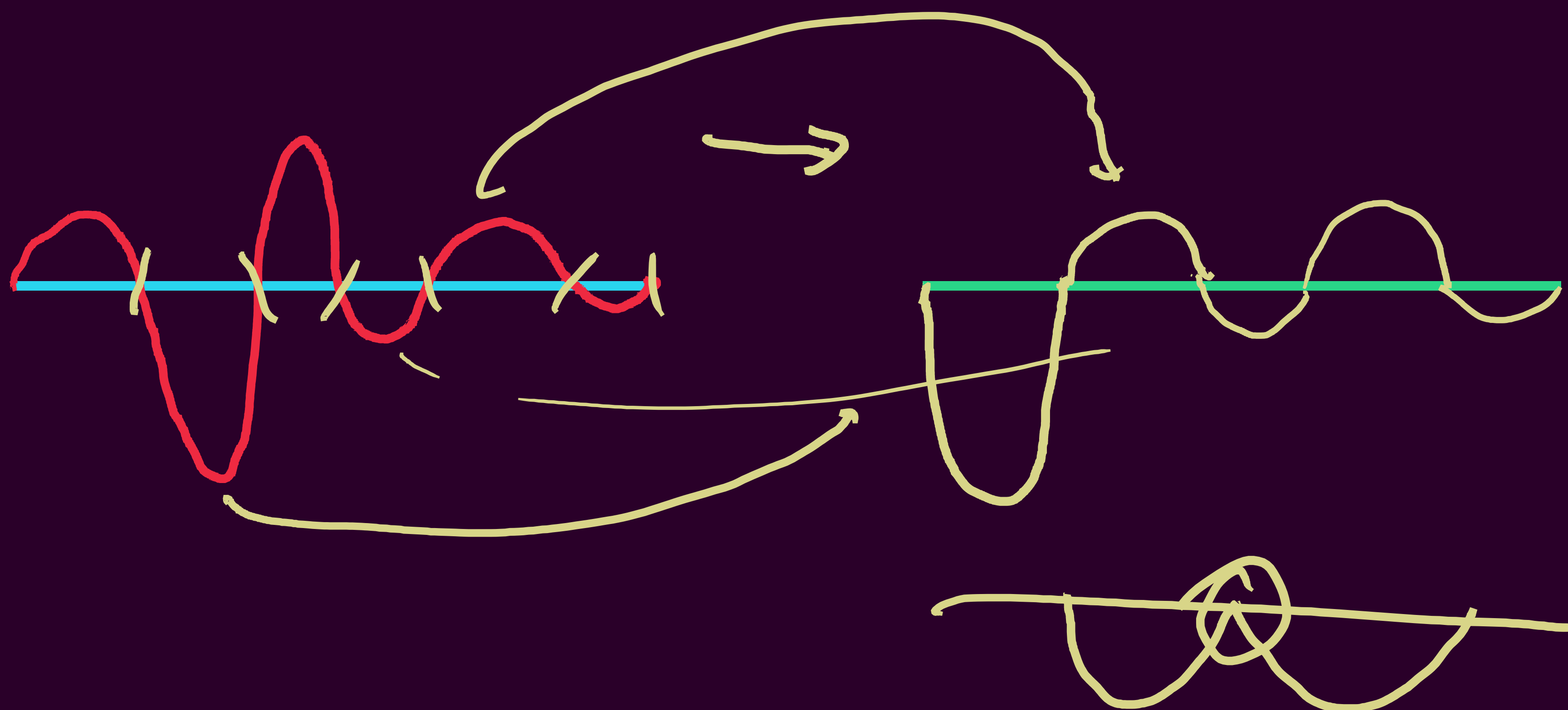
$$\lambda_n = \frac{\pi^2 n^2}{L^2}$$

$$\lim_{n \to \infty} \sqrt{\frac{\lambda_n}{\pi^2 n^2}} = L$$

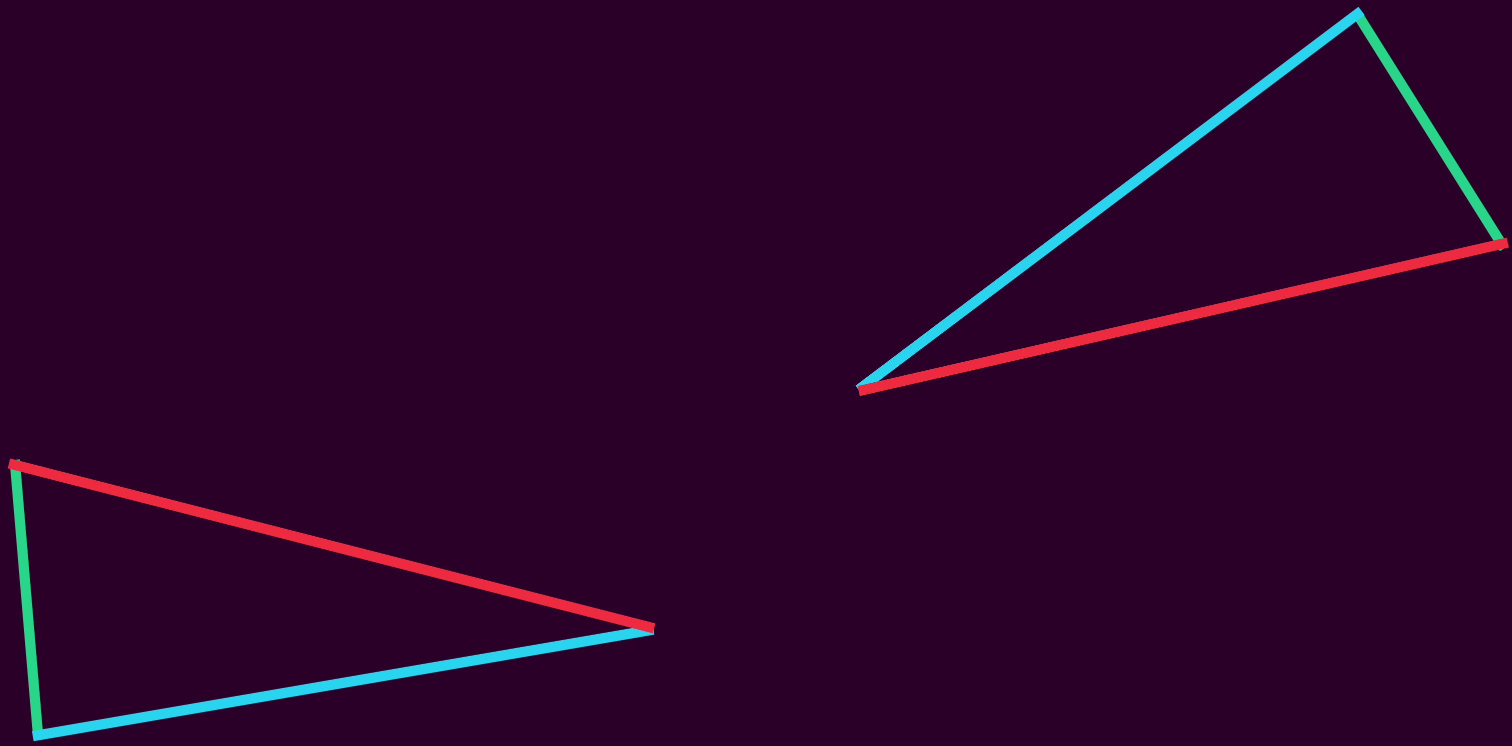
# NOW ONTO DRUMS

- NO NICE FORMULA
- LESS RIGIDITY IN SPECTRUM.
- MORE FLEXIBILITY IN THE ALLOWED SHAPES.

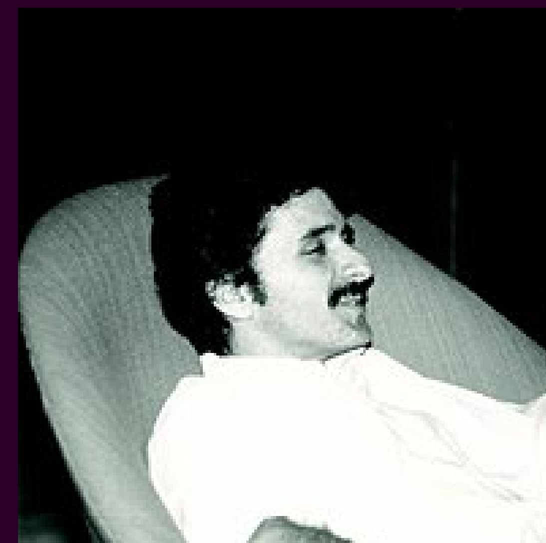
# TRANSPLANTATION

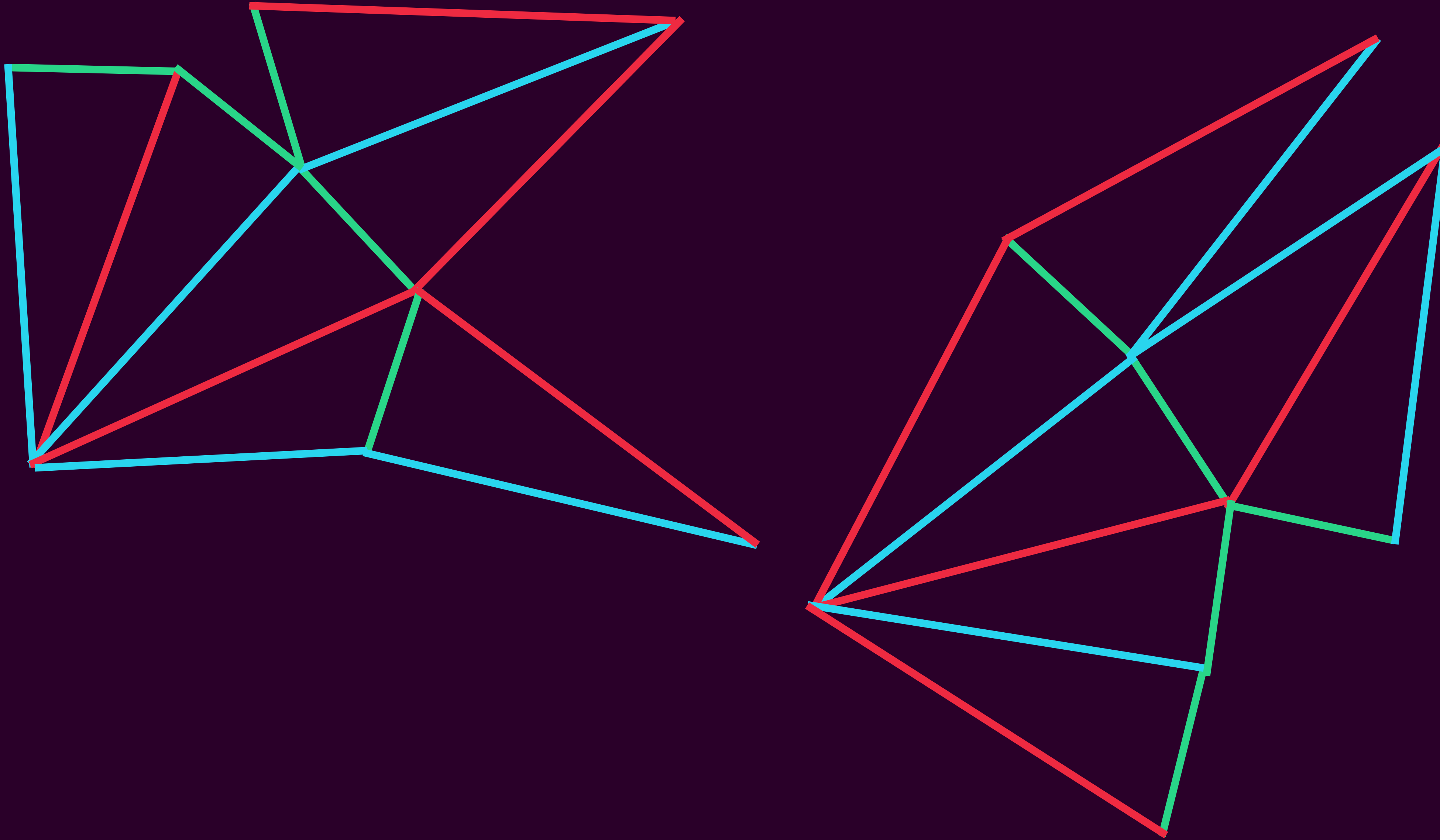


# Building Blocks



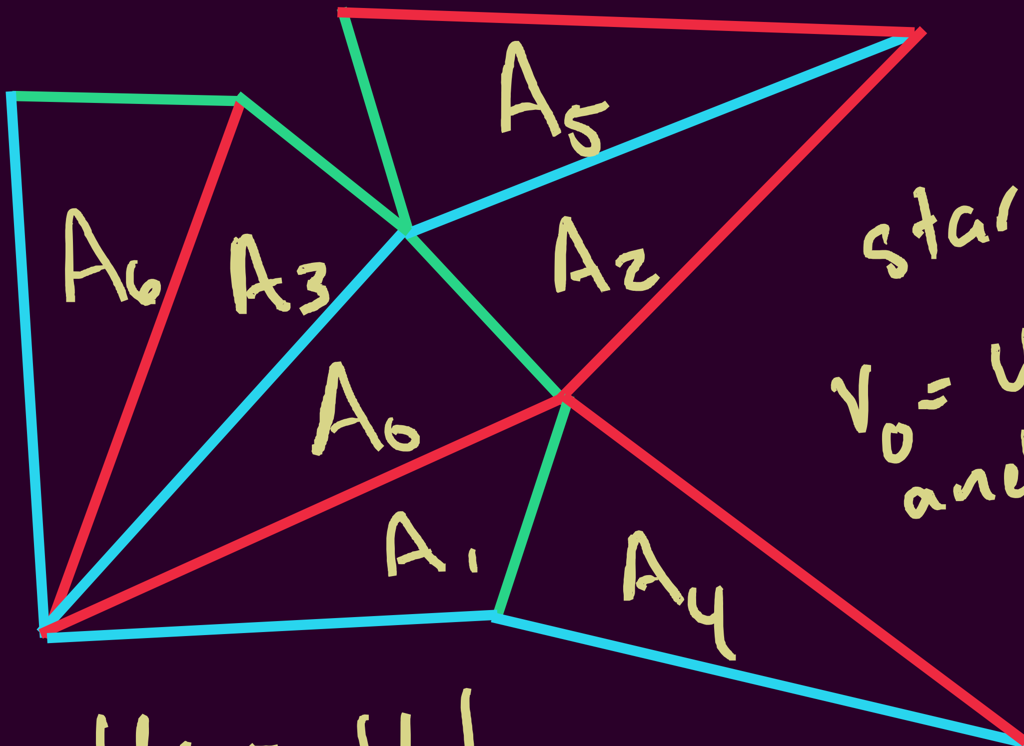

Sunada triple:
"almost
conjugate
WASH. U. MATH
WASH. U. MATH





# TRANSPLANTATION RULES

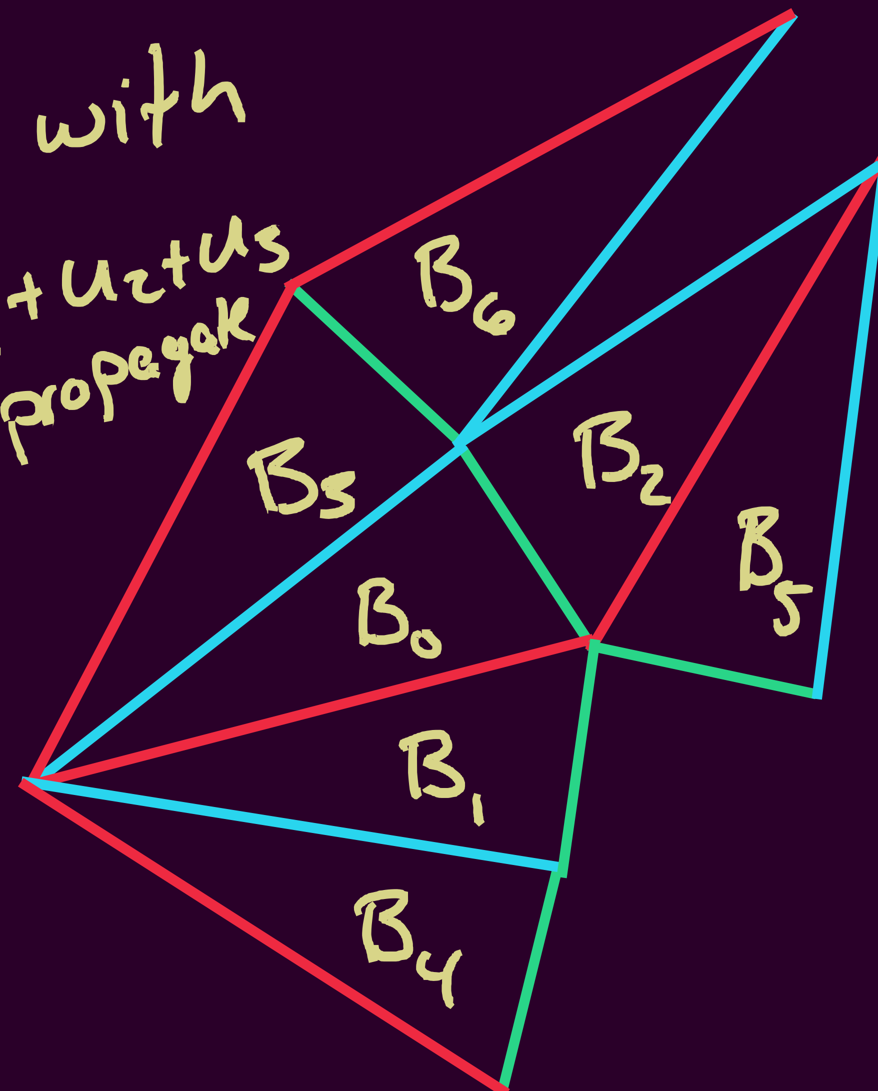
- On outside boundary, $u = 0$
- On inner boundary, $u_i = u_j$ and
$$\partial_n u_i = \partial_n u_j$$




start with

$v_0 = u_1 + u_2 + u_3$

and propegate




$$u_j = u\big|_{A_j}$$

$$v_j = v\big|_{B_j}$$

# CAN WE HEAR ANYTHING?

- ISOPERIMETRIC RESULTS: