A characterization of types of support between structured arguments and their relationship with support in abstract argumentation

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Abstract

Argumentation is an important approach in artificial intelligence and multiagent systems, providing a basis for single agents to make rational decisions, and for groups of agents to reach agreements, as well as a mechanism to underpin a wide range of agent interactions. In such work, a crucial role is played by the notion of *attack* between arguments, and the notion of attack is well-studied. There is, for example, a range of different approaches to identifying which of a set of arguments should be accepted given the attacks between them. Less well studied is the notion of *support* between arguments, yet the idea that one argument may support another is very intuitive and seems particularly relevant in the area of decision-making where decision options may have multiple arguments for and against them. In the last decade, the study of support in argumentation has regained attention among researchers, but most approaches address support in the context of abstract argumentation where the elements from which arguments are composed are ignored. In contrast, this paper studies the notion of support between arguments in the context of structured argumentation systems where the elements from which arguments are composed play a crucial role. Different forms of support are presented, each of which takes into account the structure of arguments; and the relationships between these forms of support are studied. Then, the paper investigates whether there is a correspondence between the structured and abstract forms of support, and determines whether the abstract formalisms may be instantiated using concrete forms of support in terms of structured arguments. The conclusion is that support in structured argumentation does not mesh well with support in abstract argumentation, and this suggests that more work is required to develop forms of support in abstract argumentation that model what happens in structured argumentation.

Keywords: argumentation, structured argumentation, abstract argumentation, support relation

1. Introduction

Argumentation is an important approach in artificial intelligence and multiagent systems. It provides a mechanism for single agents to make rational decisions [29], and for groups of agents to reach

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agreements [31], as well as a mechanism to underpin a wide range of agent interactions [35]. One of the reasons why argumentation is so useful is that it can handle conflicts due to inconsistent information — inconsistency naturally arises in multiagent systems since, for example, different agents represent different views of the world [7]. Such conflicts are captured with the notion of "attack" between arguments, and the argumentation literature includes a number of approaches to identifying which of a set of arguments should be considered to be "acceptable" given the attacks amongst them [14, 25]. The differences between these approaches can be explained in terms of theoretical considerations about what constitutes a good notion of acceptability. While the representational advantages of argumentation have been discussed for many years [25, 40], recent work has backed this up with strong empirical results which show that argumentation-based approaches can lead to higher quality solutions, for example: [1, 27, 30, 52].

Our interest in argumentation in this paper is a little different. Following the tradition of systems like Capsule [64] and RAGS [24], we are using argumentation to support human decision making under uncertainty. In particular, we are using argumentation to build tools for combining information from sources that are not fully trusted and to tag conclusions with an indication of the trust that can be placed in them [41, 42, 53]. In work such as [24, 64], we find that in assessing the available evidence, human subjects not only identify conflicts between arguments, but also they identify situations in which arguments *support* each other. To take account of such arguments, we need to understand support as well as attack between arguments.

In this paper, we are interested in studying the notion of support, starting with the idea that it is a positive interaction between arguments that does not depend on the existence of attacks between them. The notion of support has been present in the literature of argumentation since its foundation. In [55], Toulmin proposed a model for the structure of arguments that distinguishes between data, claim, warrant, backing, rebuttal and qualifier. Given Toulmin's scheme, we can identify two kinds of interactions among its elements. On the one hand, the backing provides support for the warrant. On the other hand, the presence of a rebuttal leads to the rejection of the claim through an attack on the argument. The influential work of Pollock, advanced in [45, 46, 47] and presented at full length in [44], which had a large impact on early work on computational argumentation, also deals with support at length. However, following the work by Dung [25], most studies on argumentation put aside the notion of support to focus on the notion of attack. Given an attack relation, a positive interaction between arguments was identified through the notion of *reinstatement*, corresponding to situations in which an argument defends another one. However, the notion of reinstatement is not, in our view, a form of support on its own, since it depends on the existence of attacks between arguments. In contrast, in the last decade, the study of a notion of support that does not rely on the existence of attacks has regained attention amongst researchers of the area. Recently, several interpretations of support have been proposed in the literature, the most widely used being the general support relation of [15], the deductive support of [6] and the necessary support of [8, 38].

Most work on support in argumentation, much of which is surveyed in [23], has been developed at the abstract level. That is, it does not consider the internal structure of arguments. However, there is other work that addresses support in a more concrete setting. In particular, DEFLOG [58] constitutes an approach to dialectical argumentation that allows for the representation of the elements in Toulmin's scheme, as well as the support links between them [59, 61]. In addition, the formalism proposed in [20] introduces a special kind of rule to represent the support relation between backings and warrants of Toulmin's scheme in the context of Defeasible Logic Programming.

It is important to note that the existing abstract argumentation formalisms addressing the notion of support do not deal with the origin of such a relation[1]. That is, they start with a given set of arguments and the corresponding attack and support relations between them and then, generally, focus on the acceptability of the given arguments by taking into account the relationships between them. As a result, they do not study the origin of the support links between the arguments. Indeed, as mentioned before, these formalisms typically just adopt an interpretation for the support relation. Then, given a particular interpretation, they characterize constraints on acceptability that are derived from it, and then take them into account by defining *complex attacks* between arguments [18, 19]. These complex attacks then make it possible to obtain sets of acceptable arguments that meet the constraints.

[1] This is in contrast to the notion of attack, which has long been given an interpretation in terms of conflicts between arguments [2, 5, 28, 49, 51].

Here, we are interested in studying how the support links between arguments originate. To do that, we need to take into account the information expressed by the arguments and/or their internal structure. As a result, we will study the notion of support in the context of a concrete argumentation formalism, and we choose to use ASPIC$^+$[36, 49], which is both widely studied and, in our opinion, steers a suitable course between concreteness (which allows us to pin down what support means) and abstraction (which allows results to be imported by any instantiation). There are several kinds of support that can be identified in a concrete setting. For instance, following the general interpretation of support by [15], where the support relation is just considered as a positive interaction between arguments, one might consider different situations in which an argument supports another because they share some positive features. For instance, we can consider that an argument A' supports another argument A, if A' provides an alternative way to derive the conclusion of A. In such a case, it is clear that there exists a positive interaction between A' and A, since they provide ways to derive the same conclusion.

To give a more concrete example of this situation, adapted from [15], A might be the argument that because we do not have any other commitments today, since the weather will be good because it is the summer, and because we like hiking, then it would be good to go for a hike today. A' might then be the argument that because we are keen to take exercise, and hiking is good exercise, then (again) it would be good to go for a hike today. Both arguments have the same conclusion, and so, in some sense, are mutually supportive. Following this line of reasoning, we can also imagine a different form of support in which the conclusion of one argument provides an alternative way to derive some step on the way to the conclusion of another. As an example, consider A'', the argument that the forecast on the radio suggests that the weather will be good, and since forecasts from this particular radio station are usually accurate, then it is likely that the weather will be good. This is another argument for the weather being good, which, being additional evidence for this fact, supports A, since deriving the conclusion of A involves first deriving this prospect (of good weather) whether from the radio report or from information about the season.

An important thing to bear in mind is that the existence of a support link between two arguments A' and A does not necessarily imply that the acceptability of A depends on the acceptability of A', nor does it imply that the acceptability of A' depends on the acceptability of A. Similarly, it does not imply that the acceptance of the conclusion of one implies the acceptance of the conclusion of the other. In other words, the existence of a support link between two arguments does not imply that the supporting argument makes the supported argument stronger, or that the supported argument makes the supporting argument stronger. Rather, support links just provide additional information that may need to be factored in when computing the acceptability status of arguments, and this can be achieved, for instance, by defining complex attacks between arguments (as, for example, in [18, 19]).

In this paper, we seek to identify different notions of support in the context of structured argumentation systems when agents have knowledge expressed in some form of rules and construct arguments using some form of rule-based inference. The work on support in abstract argumentation can represent the fact that arguments can attack or support each other (or do neither), and given sets of attacking and supporting arguments, can compute which are acceptable. However, since they deal only at this abstract level, the abstract formalisms alone are not sufficient for our work — we need to start from the formulae that make up arguments and identify which arguments support each other. (The prerequisite for identifying when structured arguments attack each other in ASPIC$^+$ has already been addressed in the literature [49].) With this aim, in this paper we enumerate a number of forms of support for ASPIC$^+$, based on the idea that support links originate from positive interactions between arguments [15]. Then, we explore some of their properties, and show how to relate the proposed forms of support back to the notions of support proposed by some abstract argumentation formalisms; namely, the support relation of [15], the deductive support of [6], and the necessary support of [8, 38]. That is, we analyze whether the notions of support for structured arguments we propose have any correspondence to those that are defined at the abstract level. Together these contributions significantly advance the study of support in argumentation.

The rest of this paper is structured as follows. Section 2 introduces ASPIC$^+$ as the framework in which we study different kinds of support. Section 3 introduces a general notion of support for structured arguments and discusses ways in which it can be realized in ASPIC$^+$. Several properties regarding the different kinds of support are proposed, relating them to one another. Section 4 presents different approaches to the notion of support in abstract argumentation. Section 5 then discusses whether there exists a correspondence between the notions of support proposed for structured and abstract arguments.

We do this in two steps. First, we analyze how the notions of support for structured arguments studied in Section 3 can be combined with different types of attack in order to infer new complex attacks. Then, starting from our structured notions of support, we investigate whether they can be fully captured at the abstract level by instantiating the formalisms presented in Section 4. Briefly, our analysis shows that while there are kinds of structured support that capture some aspects of the abstract notions of support, and some abstract notions of support that capture elements of the structured supports we identified in ASPIC$^+$, there is only one proper correspondence. Section 6 explores related work, including approaches to support in structured argumentation and abstract argumentation, as well as other argumentation systems that consider some notions that lead to support relations, but were not explicitly considered as such by the existing formalisms so far. Finally, in Section 7 we draw some conclusions and discuss some topics for future work.

2. Formal model

From the many formal argumentation systems in the literature, we take as our starting point Modgil and Prakken's ASPIC$^+$ system [36, 49], and we start by recapping the main notions from the version of ASPIC$^+$ given in [36].

2.1. Definition of Arguments

ASPIC$^+$ is deliberately defined in a rather abstract way, as a system with a minimal set of features that can capture the notion of argumentation. This is done with the intention that it can be instantiated by a number of concrete systems that then inherit all of the properties of the more abstract system. ASPIC$^+$ starts from a logical language $\mathcal{L}$ with a notion of negation. A given instantiation will then be equipped with inference rules, and ASPIC$^+$ distinguishes two kinds of inference rules: strict rules and defeasible rules. Strict rules, denoted using $\rightarrow$, are rules whose conclusions hold without exception. Defeasible rules, denoted $\Rightarrow$, are rules whose conclusions hold unless there is an exception.

The language and the set of rules define an *argumentation system*:

Definition 1 (Argumentation System [36]). *An* argumentation system *is a tuple* $\mathsf{AS} = \langle \mathcal{L}, \bar{\cdot}, \mathcal{R}, n \rangle$ *where:*

- $\mathcal{L}$ *is a logical language.*
- $\bar{\cdot}$ *is a function from* $\mathcal{L}$ *to* $2^{\mathcal{L}}$ *, such that:*
  - φ *is a* contrary *of* ψ *if* $\varphi \in \overline{\psi}$*,* $\psi \notin \overline{\varphi}$*;*
  - φ *is a* contradictory *of* ψ *if* $\varphi \in \overline{\psi}$*,* $\psi \in \overline{\varphi}$*;*
  - *each* $\varphi \in \mathcal{L}$ *has at least one contradictory.*
- $\mathcal{R} = \mathcal{R}_s \cup \mathcal{R}_d$ *is a set of strict (*$\mathcal{R}_s$*) and defeasible (*$\mathcal{R}_d$*) inference rules of the form* $\phi_1, \ldots, \phi_n \rightarrow \phi$ *and* $\phi_1, \ldots, \phi_n \Rightarrow \phi$ *respectively (where* ϕ_i, ϕ *are meta-variables ranging over wff in* $\mathcal{L}$*), and* $\mathcal{R}_s \cap \mathcal{R}_d = \emptyset$*.*
- $n : \mathcal{R}_d \mapsto \mathcal{L}$ *is a naming convention for defeasible rules.*

The function $\bar{\cdot}$ generalizes the usual symmetric notion of negation to allow non-symmetric conflict between elements of $\mathcal{L}$. The contradictory of some $\varphi \in \mathcal{L}$ is close to the usual notion of negation, and we denote that φ is a *contradictory* of ψ by "$\varphi = \neg\psi$". Note that, given the characterization of $\bar{\cdot}$, elements in $\mathcal{L}$ may have multiple contraries and contradictories. As we will see below, the naming convention for defeasible rules is necessary because there are cases in which we want to write rules that deny the applicability of certain defeasible rules. Naming the rules, and having those names be in $\mathcal{L}$ makes it possible to do this, and the denying applicability makes use of the contraries of the rule names.

An argumentation system, as defined above, is just a language and some rules which can be applied to formulae in that language. To provide a framework in which reasoning can happen, we need to add information that is known, or believed, to be true. In ASPIC$^+$, this information makes up a *knowledge base*:

Definition 2 (Knowledge Base [36]). *A* knowledge base *in an argumentation system* $\langle \mathcal{L}, \bar{\cdot}, \mathcal{R}, n \rangle$ *is a set* $\mathcal{K} \subseteq \mathcal{L}$ *consisting of two disjoint subsets* $\mathcal{K}_n$ *and* $\mathcal{K}_p$*.*

We call $\mathcal{K}_n$ the axioms and $\mathcal{K}_p$ the ordinary premises. We make this distinction between the elements of the knowledge base for the same reason that we make the distinction between strict and defeasible rules. We are distinguishing between those elements — axioms and strict rules — which are definitely true and allow truth-preserving inferences to be made, and those elements — ordinary premises and defeasible rules — which can be disputed.

Combining the notions of argumentation system and knowledge base gives us the notion of an *argumentation theory*:

Definition 3 (Argumentation Theory [36]). *An* argumentation theory AT *is a pair* $\langle \mathsf{AS}, \mathcal{K} \rangle$ *of an argumentation system* AS *and a knowledge base* $\mathcal{K}$*.*

We are now nearly ready to define an argument. But first we need to introduce some notions that can be defined just by understanding that an argument is made up of some subset of the knowledge base $\mathcal{K}$, along with a sequence of rules, that lead to a conclusion. Given this, $\mathtt{Prem}(\cdot)$ returns all the premises, $\mathtt{Conc}(\cdot)$ returns the conclusion and $\mathtt{TopRule}(\cdot)$ returns the last rule in the argument. $\mathtt{Sub}(\cdot)$ returns all the sub-arguments of a given argument, that is all the arguments that are contained in the given argument. In addition, given $A' \in \mathtt{Sub}(A)$ such that $A' \neq A$, we will say that A' is a *proper sub-argument* of A.

Definition 4 (Argument [36]). *An* argument A *from an argumentation theory* $\mathsf{AT} = \langle \langle \mathcal{L}, \bar{\cdot}, \mathcal{R}, n \rangle, \mathcal{K} \rangle$ *is:*

1. ϕ *if* $\phi \in \mathcal{K}$ *with:* $\mathtt{Prem}(A) = \{\phi\}$*;* $\mathtt{Conc}(A) = \{\phi\}$*;* $\mathtt{Sub}(A) = \{A\}$*; and* $\mathtt{TopRule}(A) =$ *undefined.*
2. $A_1, \ldots, A_n \rightarrow \phi$ *if* A_i*,* $1 \leq i \leq n$*, are arguments and there exists a strict rule of the form* $\mathtt{Conc}(A_1), \ldots, \mathtt{Conc}(A_n) \rightarrow \phi$ *in* $\mathcal{R}_s$*.* $\mathtt{Prem}(A) = \mathtt{Prem}(A_1) \cup \ldots \cup \mathtt{Prem}(A_n)$*;* $\mathtt{Conc}(A) = \phi$*;* $\mathtt{Sub}(A) = \mathtt{Sub}(A_1) \cup \ldots \cup \mathtt{Sub}(A_n) \cup \{A\}$*; and* $\mathtt{TopRule}(A) = \mathtt{Conc}(A_1), \ldots, \mathtt{Conc}(A_n) \rightarrow \phi$*.*
3. $A_1, \ldots, A_n \Rightarrow \phi$ *if* A_i*,* $1 \leq i \leq n$*, are arguments and there exists a defeasible rule of the form* $\mathtt{Conc}(A_1), \ldots, \mathtt{Conc}(A_n) \Rightarrow \phi$ *in* $\mathcal{R}_d$*.* $\mathtt{Prem}(A) = \mathtt{Prem}(A_1) \cup \ldots \cup \mathtt{Prem}(A_n)$*;* $\mathtt{Conc}(A) = \phi$*;* $\mathtt{Sub}(A) = \mathtt{Sub}(A_1) \cup \ldots \cup \mathtt{Sub}(A_n) \cup \{A\}$*; and* $\mathtt{TopRule}(A) = \mathtt{Conc}(A_1), \ldots, \mathtt{Conc}(A_n) \Rightarrow \phi$*.*

We write $\mathcal{A}(\mathsf{AT})$ *to denote the set of arguments from the theory* AT*.*

In other words, an argument is either an element of $\mathcal{K}$, or it is a rule and its conclusion such that each premise of the rule is the conclusion of an argument. Note that, as stated by the authors in [36]: "Note that all premises in ASPIC$^+$ arguments are used in deriving its conclusion, so enforcing a notion of relevance analogous to the subset minimality condition requirement on premises in classical logic approaches to argumentation".

A key concept in argumentation is the idea that even if there is an argument for some conclusion, indicating that there is a *prima facie* case for the conclusion, the conclusion may not be reasonable because there is a stronger argument that it does not hold. This notion is particularly natural in a multiagent setting, where different agents have different viewpoints, leading to conflicting arguments. However, it is perfectly possible for a single argumentation theory, representing the information held by a single individual, to be the basis of conflicting arguments. We capture this kind of interaction through the idea that one argument can attack and defeat another.

An argument can be attacked in three ways: on its ordinary premises, on its conclusion (either final or intermediate), or on its defeasible inference rules. These three kinds of attack are called *undermining*, *rebutting* and *undercutting* attacks, respectively.

Definition 5 (Attack [36]). *An argument* A attacks *an argument* B *iff* A *undermines, rebuts or undercuts* B*, where:*

- A undermines B *(on* B'*) iff* $\mathtt{Conc}(A) \in \overline{\phi}$ *for some* $B' = \phi \in \mathtt{Prem}(B)$ *and* $\phi \in \mathcal{K}_p$*.*
- A rebuts B *(on* B'*) iff* $\mathtt{Conc}(A) \in \overline{\phi}$ *for some* $B' \in \mathtt{Sub}(B)$ *of the form* $B''_1, \ldots, B''_2 \Rightarrow \phi$*.*
- A undercuts B *(on* B'*) iff* $\mathtt{Conc}(A) \in \overline{n(r)}$ *for some* $B' \in \mathtt{Sub}(B)$ *such that* $\mathtt{TopRule}(B')$ *is a defeasible rule* r *of the form* $\phi_1, \ldots, \phi_n \Rightarrow \phi$*.*

We denote "A *attacks* B*" by* (A, B)*.*

In all these cases, the idea is that an attack can be made on an element of an argument that is not known for sure to hold. An attack can thus be made on an ordinary premise — which might be an assumption or a belief — rather than an axiom, and both the other forms of attack involve defeasible rules. The difference between strict rules, using $\rightarrow$, and defeasible rules, using $\Rightarrow$, is nicely summarized by [28]. A defeasible rule captures "tentative information that may be used if nothing (can) be posed against it". The fact that "nothing can be posed against" the use of a defeasible rule is established by a proof mechanism that looks for arguments against conclusions established using defeasible rules [28]:

> (a) defeasible rule represents a weak connection between the head and the body of the rule. The effect of a defeasible rule comes from a dialectical analysis . . . which involves the consideration of arguments and counter-arguments where that rule is included.

ASPIC$^+$ allows defeasible rules to be undercut, in which case the application of the rule is attacked by an argument that states the rule does not hold[2]. Similarly, since defeasible rules are tentative, ASPIC$^+$ allows the conclusions of such rules to be rebutted. The particular notion of rebutting used in ASPIC$^+$ is said to be *restricted*, meaning that an argument with a strict $\texttt{TopRule}(\cdot)$ can rebut an argument with a defeasible $\texttt{TopRule}(\cdot)$, but not vice versa. Rebutting is thus asymmetric[3].

Typically we want to model information that is believed to different degrees, and within ASPIC$^+$ we do this using a preference order over the elements of $\mathcal{R}_\mathtt{d}$ and $\mathcal{K}_\mathtt{p}$. The question then is how these preferences combine into an ordering $\preceq$ over arguments:

Definition 6 (Preference Ordering). *A preference ordering $\preceq$ is a binary relation over arguments, i.e., $\preceq\, \subseteq \mathcal{A} \times \mathcal{A}$, where $\mathcal{A}$ is the set of all arguments from an argumentation theory. Given $\mathsf{A}, \mathsf{B} \in \mathcal{A}$, we say A's* preference level *is less than or equal to that of B iff $\mathsf{A} \preceq \mathsf{B}$.*

ASPIC$^+$ does not make any assumption about the properties of the preference ordering, but as an example of a property one might use to establish $\preceq$, consider the *weakest link* principle from [36]. This assumes two pre-orderings $\leq, \leq'$ over $\mathcal{R}_\mathtt{d}$ and $\mathcal{K}_\mathtt{p}$ respectively, and combines them into $\mathsf{A} \prec \mathsf{B}$ as follows:

- the defeasible rules in A include a rule which is weaker than (strictly less than according to $\leq$) all the defeasible rules in B, and
- the ordinary premises in A include an ordinary premise which is weaker (strictly less than according to $\leq'$) all the ordinary premises in B.

$\mathsf{A} \prec \mathsf{B}$ is then defined as usual as $\mathsf{A} \preceq \mathsf{B}$ and $\mathsf{B} \not\preceq \mathsf{A}$.

Given $\mathsf{A} \prec \mathsf{B}$, we can then use this to factor the preference over arguments into the notion of attack. Attacks can be distinguished as to whether they are preference-dependent (rebutting and undermining) or preference-independent (undercutting). The former succeed only when the attacker is preferred. The latter succeed whether or not the attacker is preferred.

By combining the definition of arguments, attack relation and preference ordering, we have the following definitions:

Definition 7 (Structured Argumentation Framework [36]). *A* Structured Argumentation Framework (SAF) *is a triple $\langle \mathcal{A}, \mathsf{Att}, \preceq \rangle$, where $\mathcal{A}$ is the set of all arguments from an argumentation theory, Att is the attack relation, and $\preceq$ is a preference ordering on $\mathcal{A}$.*

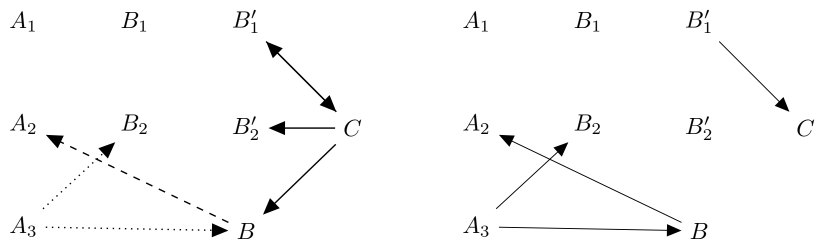
Definition 8 (Defeat [36]). *A defeats B iff A undercuts B, or if A rebuts/undermines B on B' and A's preference level is not less than that of B' ($\mathsf{A} \not\prec \mathsf{B}'$).*

Then the idea of an argumentation framework follows from Definitions 7 and 8.

Definition 9 (Argumentation Framework). *An* Argumentation Framework (AF) *corresponding to a structured argumentation framework $\mathsf{SAF} = \langle \mathcal{A}, \mathsf{Att}, \preceq \rangle$ is a pair $\langle \mathcal{A}, \mathsf{Defeats} \rangle$ such that $\mathsf{Defeats}$ is the defeat relation on $\mathcal{A}$ determined by SAF.*

[2]The canonical example here comes from [45] via [36], and is the rule that normally objects that appear red, are red. However, in the situation that everything is illuminated with red light, this rule no longer holds since under red light everything, including things that are not red, will appear to be red.

[3]This asymmetry is not uncontroversial, see [11, 33] for arguments against it.



(a) Attack relation (b) Defeat relation

Figure 1: Attack relations and defeat relations from Example 1. In (a), the solid arrows denote undermining attacks, the dashed arrow denotes a rebutting attack, and dotted arrows denote undercutting attacks.

In the general case, argumentation frameworks will include a defeat relation between arguments, and a natural question is what arguments are considered reasonable given those defeats. Now, argumentation frameworks as defined in Definition 9 correspond to the abstract argumentation frameworks of [25]. As a result, all the mechanisms that are defined in [25], and in later work such as [4, 12, 13, 60, 63], for establishing the *acceptability* of a set of arguments — that is identifying various mutually coherent subsets of arguments — can be employed. Since establishing acceptability is not our focus in this paper, we discuss these mechanisms no further, and refer the reader to [4] for an excellent overview.

Consider this example of an ASPIC$^+$ argumentation framework, adapted from [36]:

Example 1. *Consider that we have an argumentation system* $\mathsf{AS} = \langle \mathcal{L}, \bar{\cdot}, \mathcal{R}, \mathsf{n} \rangle$ *where:*

$$\mathcal{L} = \{\mathsf{a}, \mathsf{b}, \mathsf{c}, \mathsf{d}, \mathsf{e}, \mathsf{f}, \mathsf{nd}, \neg\mathsf{a}, \neg\mathsf{b}, \neg\mathsf{c}, \neg\mathsf{d}, \neg\mathsf{e}, \neg\mathsf{f}, \neg\mathsf{nd}\}$$

$\mathcal{R} = \mathcal{R}_\mathsf{s} \cup \mathcal{R}_\mathsf{d}$, *with* $\mathcal{R}_\mathsf{s} = \{\mathsf{d}, \mathsf{f} \rightarrow \neg\mathsf{b}\}$ *and* $\mathcal{R}_\mathsf{d} = \{\mathsf{a} \Rightarrow \mathsf{b}; \neg\mathsf{c} \Rightarrow \mathsf{d}; \mathsf{e} \Rightarrow \mathsf{f}; \mathsf{a} \Rightarrow \neg\mathsf{nd}\}$, *and the function* $\mathsf{n}(\cdot)$ *gives* $\mathsf{n}(\neg\mathsf{c} \Rightarrow \mathsf{d}) = \mathsf{nd}$. *We then add the knowledge base* $\mathcal{K}$ *such that* $\mathcal{K}_\mathsf{n} = \emptyset$ *and* $\mathcal{K}_\mathsf{p} = \{\mathsf{a}; \neg\mathsf{c}; \mathsf{e}; \neg\mathsf{e}\}$ *to get the argumentation theory* $\mathsf{AT} = \langle \mathsf{AS}, \mathcal{K} \rangle$. *From this we can construct the arguments:*

$$\begin{array}{l} \mathsf{A}_1 = [\mathsf{a}]; \mathsf{A}_2 = [\mathsf{A}_1 \Rightarrow \mathsf{b}]; \mathsf{A}_3 = [\mathsf{A}_1 \Rightarrow \neg\mathsf{nd}]; \\ \mathsf{B}_1 = [\neg\mathsf{c}]; \mathsf{B}_2 = [\mathsf{B}_1 \Rightarrow \mathsf{d}]; \mathsf{B}_1' = [\mathsf{e}]; \mathsf{B}_2' = [\mathsf{B}_1' \Rightarrow \mathsf{f}]; \mathsf{B} = [\mathsf{B}_2, \mathsf{B}_2' \rightarrow \neg\mathsf{b}]; \\ \mathsf{C} = [\neg\mathsf{e}]; \end{array}$$

Let us call this set of arguments $\mathcal{A}$, *so that:* $\mathcal{A} = \{\mathsf{A}_1, \mathsf{A}_2, \mathsf{A}_3, \mathsf{B}_1, \mathsf{B}_2, \mathsf{B}_1', \mathsf{B}_2', \mathsf{B}, \mathsf{C}\}$. *Note that* $\mathtt{Prem}(\mathsf{B}) = \{\neg\mathsf{c}; \mathsf{e}\}$, $\mathtt{Sub}(\mathsf{B}) = \{\mathsf{B}_1; \mathsf{B}_2; \mathsf{B}_1'; \mathsf{B}_2'; \mathsf{B}\}$, $\mathtt{Conc}(\mathsf{B}) = \neg\mathsf{b}$, *and* $\mathtt{TopRule}(\mathsf{B}) = \mathsf{d}, \mathsf{f} \rightarrow \neg\mathsf{b}$. *The attacks between these arguments are shown in Figure 1 (a). These make up the set* $\mathsf{Att} = \{(\mathsf{C}, \mathsf{B}_1'), (\mathsf{B}_1', \mathsf{C}), (\mathsf{C}, \mathsf{B}_2'), (\mathsf{C}, \mathsf{B}), (\mathsf{B}, \mathsf{A}_2), (\mathsf{A}_3, \mathsf{B}_2), (\mathsf{A}_3, \mathsf{B})\}$. *With a preference order* $\preceq$ *defined by :* $\mathsf{A}_2 \prec \mathsf{B}; \mathsf{C} \prec \mathsf{B}; \mathsf{C} \prec \mathsf{B}_1'; \mathsf{C} \prec \mathsf{B}_2'$, *we have the structured argumentation framework* $\langle \mathcal{A}, \mathsf{Att}, \preceq \rangle$. *This structured argumentation framework establishes a defeat relation* $\mathsf{Defeats} = \{(\mathsf{B}_1', \mathsf{C}), (\mathsf{B}, \mathsf{A}_2), (\mathsf{A}_3, \mathsf{B}), (\mathsf{A}_3, \mathsf{B}_2)\}$ *which is shown in Figure 1 (b). With this, we can finally write down the argumentation framework* $\langle \mathcal{A}, \mathsf{Defeats} \rangle$.

This completes a description of ASPIC$^+$ that is sufficient to understand the rest of the paper.

3. Support for structured arguments

In this section we introduce a general idea of what support for arguments in ASPIC$^+$ could be and study different ways in which it can be realized. It should be noted that, in order to characterize the different forms of support we do not extend ASPIC$^+$, but we just define new notions over the existing formalism. Also, as will be noted, some of the concrete forms of support we propose here are not entirely new since they have been present in some form in the literature of argumentation systems; however, they have not yet been explicitly addressed as forms of support and related together in the way that they are here.

We start by considering the following intuition, where an argument A_1 is considered to support another argument A_2 if there exists a positive interaction between them, leading to the construction of an argument for a formula in A_2. Formally:

Definition 10 (Support). *Let* AT *be an argumentation theory and* $\mathsf{A}_1, \mathsf{A}_2 \in \mathcal{A}(\mathsf{AT})$. *Argument* A_1 *provides* support *for* A_2 *iff* $\mathtt{Conc}(\mathsf{A}_1) = \mathtt{Conc}(\mathsf{A}')$ *for some* $\mathsf{A}' \in \mathtt{Sub}(\mathsf{A}_2)$.

First, we should note that the general notion of support proposed here does not depend on the existence of attacks between arguments. That is, A_1 can be a supporting argument for A_2 even though there is no attacking argument for A_2. However, the consideration of a supporting argument may help in reinstating the position of the supported argument.

Let us consider one of the examples given in the introduction, where it was possible to find arguments A (hike because the weather is good) and A' (hike because it is good exercise) stating that it would be good to go for a hike today. In this case, it is clear that A and A' comply with the general idea of support introduced in Definition 10, since they share the same conclusion. Let us now suppose that two people are debating going for a hike. One puts forward A. The other, who is less keen to go hiking, puts forward a new argument D stating that looking at the sky suggests that it will rain, and so the weather will not be good[4]. In this case, argument D attacks A's premise that the weather will be good (possibly, D defeats A). However, if the first person then puts forward A', which is not attacked by D, A' helps in reinstating the position of A by providing an alternative way to obtain the conclusion of A. Thus, in such a case, A' would be supporting A. Thus A' supports A whether or not there is an attack on A. However, if A is attacked, then A' can be involved in reinstating A.

As mentioned in the introduction, there may be several kinds of support that can be identified in a concrete setting. In the setting of ASPIC$^+$, we have identified four notions of support that align with our general idea of support, which exhaustively cover all cases matching Definition 10. Here we introduce these different notions of support and explore the relationships between them. The first form of support we will consider corresponds to the situation described in the hiking example. That is, it captures the situation where two arguments have the same conclusion. Since the focus is on the conclusion of arguments, we will call this form of support "conclusion support".

Definition 11 (Conclusion Support). *Let* AT *be an argumentation theory and* $\mathsf{A}_1, \mathsf{A}_2 \in \mathcal{A}(\mathsf{AT})$. *Argument* A_1 *provides* conclusion support (c-support) *for* A_2 *iff* $\mathsf{A}_1 \neq \mathsf{A}_2$ *and* $\mathtt{Conc}(\mathsf{A}_1) = \mathtt{Conc}(\mathsf{A}_2)$.

Note that this definition requires the supporting and supported arguments to be different, in order to rule out an argument supporting itself.

The following example illustrates this form of support in the context of ASPIC$^+$. From here on, we will use the symbol $\rightsquigarrow$ when we do not care about distinguishing whether an argument uses a strict rule $\rightarrow$ or a defeasible rule $\Rightarrow$. In other words, if we are making a statement about an argument $\mathsf{A} = [\mathsf{B} \rightsquigarrow \mathsf{a}]$, then we are making a statement about both arguments $\mathsf{A}' = [\mathsf{B} \rightarrow \mathsf{a}]$ and $\mathsf{A}'' = [\mathsf{B} \Rightarrow \mathsf{a}]$. Similarly, when referring to a rule $\mathsf{a} \rightsquigarrow \mathsf{b}$, we are referring to both a strict rule $\mathsf{a} \rightarrow \mathsf{b}$ and a defeasible rule $\mathsf{a} \Rightarrow \mathsf{b}$.

Example 2. *Consider that we have an argumentation system* $\mathsf{AS} = \langle \mathcal{L}, \bar{\cdot}, \mathcal{R}, \mathsf{n} \rangle$ *where* $\mathcal{R} = \{\neg \mathsf{c} \rightsquigarrow \mathsf{a}; \mathsf{a}, \mathsf{b} \rightsquigarrow \mathsf{d}; \neg \mathsf{a} \rightsquigarrow \mathsf{c}; \mathsf{c} \rightsquigarrow \mathsf{d}; \mathsf{d} \rightsquigarrow \mathsf{e}\}$. *We then add the knowledge base* $\mathcal{K}$ *such that* $\mathcal{K}_\mathsf{n} = \emptyset$ *and* $\mathcal{K}_\mathsf{p} = \{\neg \mathsf{a}; \mathsf{b}; \neg \mathsf{c}\}$ *to get the argumentation theory* $\mathsf{AT} = \langle \mathsf{AS}, \mathcal{K} \rangle$. *From this we can construct the following arguments:*

$$\mathsf{A}_1 = [\neg \mathsf{c}]; \mathsf{A}_2 = [\mathsf{A}_1 \rightsquigarrow \mathsf{a}]; \mathsf{A}_3 = [\mathsf{b}]; \mathsf{A}_4 = [\mathsf{A}_2, \mathsf{A}_3 \rightsquigarrow \mathsf{d}]; \mathsf{A} = [\mathsf{A}_4 \rightsquigarrow \mathsf{e}];$$
$$\mathsf{B}_1 = [\neg \mathsf{a}]; \mathsf{B}_2 = [\mathsf{B}_1 \rightsquigarrow \mathsf{c}]; \mathsf{B}_3 = [\mathsf{B}_2 \rightsquigarrow \mathsf{d}]; \mathsf{B} = [\mathsf{B}_3 \rightsquigarrow \mathsf{e}];$$

so that A *c-supports* B *and vice-versa.*

It should be noted that, although $\mathtt{Conc}(\mathsf{A}) = \mathtt{Conc}(\mathsf{B})$ *and* $\mathtt{TopRule}(\mathsf{A}) = \mathtt{TopRule}(\mathsf{B})$, *these two arguments are different. This becomes evident, for instance, when looking at the sets of premises from both arguments:* $\mathtt{Prem}(\mathsf{A}) = \{\neg \mathsf{c}; \mathsf{b}\}$ *whereas* $\mathtt{Prem}(\mathsf{B}) = \{\neg \mathsf{a}\}$. *Note also that conclusion support does not require that the two arguments have the same* $\mathtt{TopRule}(\cdot)$.

As Example 2 shows, conclusion support is symmetrical.

Proposition 1. *Let* AT *be an argumentation theory and* $\mathsf{A}_1, \mathsf{A}_2 \in \mathcal{A}(\mathsf{AT})$. *It holds that* A_1 *c-supports* A_2 *iff* A_2 *c-supports* A_1.

Proof: *Straightforward from Definition 11.* □

[4]Here, we are assuming that Bob does not like to hike when there is bad weather.

One natural place where support becomes of interest is when one individual proposes an argument and another individual attacks it. If argument A is proposed and D attacks A, then the proponent of A may choose to respond by putting forward the argument A' that supports A. The following example is an illustration of this kind of situation, formalized in ASPIC$^+$, and which involves c-support.

Example 3. *Recall our example from the introduction in which* A *was the argument that because we* *__d__on't have any __o__ther __c__ommitments __t__oday* (doct)*, the __w__eather will be __g__ood* (gw) *because it is the __s__ummer* (s)*, and we __l__ike __h__iking* (lh)*, then it would be good to __g__o for a __h__ike __t__oday* (ght)*. If Alice makes this argument, and then Bob responds with the attacking argument* D *that looking at the sky suggests it will __r__ain* (r) *and thus, the __w__eather will __n__ot be __g__ood* ($\neg\mathsf{gw}$)*, Alice's response* A' *— that because they are __k__een to take __e__xercise* (ke)*, and __h__iking is __g__ood __e__xercise* (hge)*, then (again) it would be good to __g__o for a __h__ike __t__oday* (ght)*. This can be formalized in* ASPIC$^+$ *as follows.*
Let $\mathsf{AS} = \langle \mathcal{L}, \bar{\cdot}, \mathcal{R}, \mathsf{n} \rangle$ *be an the argumentation system where:*

$$\mathcal{L} = \{\mathsf{doct}, \mathsf{s}, \mathsf{gw}, \mathsf{lh}, \mathsf{r}, \mathsf{ke}, \mathsf{hge}, \neg\mathsf{doct}, \neg\mathsf{s}, \neg\mathsf{gw}, \neg\mathsf{lh}, \neg\mathsf{r}, \neg\mathsf{ke}, \neg\mathsf{hge}\}$$
$$\mathcal{R} = \mathcal{R}_\mathsf{d}$$
$$\mathcal{R}_\mathsf{d} = \{\mathsf{s} \Rightarrow \mathsf{gw}; \mathsf{doct}, \mathsf{gw}, \mathsf{lh} \Rightarrow \mathsf{ght}; \mathsf{r} \Rightarrow \neg\mathsf{gw}; \mathsf{ke}, \mathsf{hge} \Rightarrow \mathsf{ght}\}$$

We then add the knowledge base $\mathcal{K}$ *such that:*

$$\mathcal{K}_\mathsf{n} = \{\mathsf{doct}; \mathsf{s}; \mathsf{hge}\}$$
$$\mathcal{K}_\mathsf{p} = \{\mathsf{lh}; \mathsf{r}; \mathsf{ke}\}$$

to get the argumentation theory $\mathsf{AT} = \langle \mathsf{AS}, \mathcal{K} \rangle$*. From this we can construct the following arguments:*

$$\mathsf{A}_1 = [\mathsf{doct}]; \mathsf{A}_2 = [\mathsf{s}]; \mathsf{A}_3 = [\mathsf{A}_2 \Rightarrow \mathsf{gw}]; \mathsf{A}_4 = [\mathsf{lh}]; \mathsf{A} = [\mathsf{A}_1, \mathsf{A}_3, \mathsf{A}_4 \Rightarrow \mathsf{ght}];$$
$$\mathsf{D}_1 = [\mathsf{r}]; \mathsf{D} = [\mathsf{D}_1 \Rightarrow \neg\mathsf{gw}];$$

such that D *rebuts* A*, and*

$$\mathsf{A}'_1 = [\mathsf{ke}]; \mathsf{A}'_2 = [\mathsf{hge}]; \mathsf{A}' = [\mathsf{A}'_1, \mathsf{A}'_2 \Rightarrow \mathsf{ght}];$$

such that A' *c-supports* A *(and vice-versa). In this case,* A' *sidesteps* D*'s attack by providing an alternative way to obtain the conclusion of* A*.*

Given the above example, suppose now that, instead of D, Bob had suggested D' — that his legs were aching from the hike they took yesterday and so it would not be good to go for a hike today. (We assume that this is a defeasible rule.) In Example 3, D rebuts A on A_3. In contrast, D' rebuts A at its final conclusion and thus, in addition to providing conclusion support for A, A' would be helping to reinstate A by rebutting D'. (A' would also be rebutted by D' of course.) On the other hand, we should note that, as will be shown in Section 5, the coexistence of conclusion support and rebutting attack leads to different conflicts between the arguments related by these attack and support relations.

We should remark that, as stated before, the existence of a support link between two arguments does not depend on the existence of attacks on them. Furthermore, as the following example shows, the existence of such a support link does not prevent the existence of attacks on the involved arguments either.

Example 4. *Given the arguments from Example 2, it was shown that arguments* A *and* B *c-support each other. Notwithstanding this, by Definition 5, several attacks involving arguments* A*,* B *and their sub-arguments occur:* A_1 *rebuts* B_2*,* B_3 *and* B (*on* B_2)*;* B_1 *rebuts* A_3*,* A_4 *and* A (*on* A_3)*;* B_1 *undermines* A_1*,* A_3*,* A_4 *and* A (*on* A_3)*; and* A_3 *undermines* B_1*,* B_2*,* B_3 *and* B (*on* B_1)*.*

Another form of support that fits with our general idea of support given in Definition 10 is rather like conclusion support for a premise:

Definition 12 (Premise Support). *Let* AT *be an argumentation theory and* $\mathsf{A}_1, \mathsf{A}_2 \in \mathcal{A}(\mathsf{AT})$*. Argument* A_1 *provides* premise support (p-support) *for* A_2 *iff* $\mathsf{A}_1 \neq \mathsf{A}_2$ *and* $\mathtt{Conc}(\mathsf{A}_1) \in \mathtt{Prem}(\mathsf{A}_2)$*.*

Premise support can be useful, for instance, when an argument A_1 is being undermined by an argument A_3 with an attack on some premise p. Thus, A_2 provides support for A_1 by giving an alternative argument for p. Notwithstanding this, as mentioned before, premise support by A_2 does not depend on A_1 being undermined. For instance, if we consider the scenario depicted in Example 3, a new argument B expressing that we do not have any other commitments today because we have cleared our schedules provides premise support for A. This form of support is also illustrated by the following example:

Example 5. *Let us consider the argumentation system* $AS = \langle \mathcal{L}, \bar{\ }, \mathcal{R}, n \rangle$ *extending the one presented in Example 2, where* $\mathcal{R} = \{a, b \rightsquigarrow d; c \rightsquigarrow d; d \rightsquigarrow e; f \rightsquigarrow \neg a; g \rightsquigarrow a\}$. *Suppose the knowledge base* $\mathcal{K}$ *now is such that* $\mathcal{K}_n = \emptyset$ *and* $\mathcal{K}_p = \{a; b; c; f; g\}$. *Then, we get the argumentation theory* $AT = \langle AS, \mathcal{K} \rangle$. *From this we can construct the following arguments:*

$$A_1 = [a]; A_2 = [b], A_3 = [A_1, A_2 \rightsquigarrow d]; A = [A_3 \rightsquigarrow e];$$
$$B_1 = [c]; B_2 = [B_1 \rightsquigarrow d]; B = [B_2 \rightsquigarrow e];$$
$$C_1 = [f]; C = [C_1 \rightsquigarrow \neg a];$$
$$D_1 = [g]; D = [D_1 \rightsquigarrow a];$$

so that C *undermines* A *because it attacks* a*, one of the premises of* A*. On the other hand, premise support for* A *is given by* D *since the latter provides an alternative way of obtaining the (challenged) premise* a *of the former.*

Note that, if we consider the rules used by arguments C and D in Example 5 to be defeasible, then these two arguments would rebut each other. However, as mentioned before, the p-support relation between D and A does not depend on the existence of attacks of any kind. Specifically, it does not depend on the existence of the undermining attack from C to A nor does it depend on the existence of the rebutting attacks between C and D.

In addition to ASPIC+ [49], other approaches to structured argumentation like Defeasible Logic Programming [28] have addressed the existence of premises by allowing for undermining attacks. Nevertheless, as stated before, aside from being able to identify attacks on premises, in this work we focus on how the existence of premises leads to a support relation between arguments.

There are forms of c-support that are not p-support (see Example 2), and there are forms of p-support that are not c-support, as shown by the preceding example. However, for a particular class of arguments, the notions of p-support and c-support coincide.

Proposition 2. *Let* AT *be an argumentation theory and* $A_1, A_2 \in \mathcal{A}(AT)$. *If* $\mathtt{Conc}(A_1) = \mathtt{Conc}(A_2) = c$ *and* $c \in \mathtt{Prem}(A_2)$*, then it holds that* A_1 *p-supports* A_2 *iff* A_1 *c-supports* A_2.

Proof: *For the first part, if* A_1 *provides p-support for* A_2 *then the conclusion* c *of* A_1 *is a premise of* A_2*. In particular, since by hypothesis* $\mathtt{Conc}(A_2) = c$*, the premise* c *of* A_2 *coincides with its conclusion; therefore,* A_1 *provides c-support for* A_2*. For the second part, if* A_1 *provides c-support for* A_2 *then the conclusions of* A_1 *and* A_2 *coincide. Then, since by hypothesis* $\mathtt{Conc}(A_2) = c$ *and* $c \in \mathtt{Prem}(A_2)$ *its conclusion is, in particular, its premise. As a result,* A_1 *p-supports* A_2. □

Note that if the definition of arguments in ASPIC$^+$ is strengthened in order to forbid an argument from having two distinct sub-arguments with the same conclusion [32], then arguments satisfying Proposition 2 (i.e., arguments that are p-supported if and only if they are c-supported) would be arguments of the form $[\mathtt{Conc}(\mathtt{Arg})]$.

The forms of support that we have considered up to now have only taken into account the conclusions and grounds of arguments. However, if we take into account the elements connecting the grounds to the conclusion of an argument (and thus, its sub-arguments), we can identify other notions of support. It can be noted that the characterization of rebutting attacks given in Definition 5 accounts for the existence of sub-arguments in the sense that arguments can be rebutted at their final conclusion or an intermediate conclusion. In contrast, the notion of c-support given in Definition 11 is restricted to the final conclusions of arguments. Thus, below we define the notion of *intermediate support*, which is akin to conclusion support and premise support, but accounts for an intermediate conclusion of the supported argument.

Definition 13 (Intermediate Support). *Let* AT *be an argumentation theory and* $A_1, A_2 \in \mathcal{A}(AT)$. *Argument* A_1 *provides* intermediate support (i-support) *for argument* A_2 *iff* $A_1 \neq A_2$ *and there exists* $A \in \mathtt{Sub}(A_2)$ *such that* $\mathtt{Conc}(A) = \mathtt{Conc}(A_1)$, $\mathtt{Conc}(A) \neq \mathtt{Conc}(A_2)$ *and* $\mathtt{Conc}(A) \notin \mathtt{Prem}(A_2)$.

In other words, if A_1 i-supports A_2, then there is some intermediate conclusion c in A_2 (neither a premise nor the final conclusion) that is the conclusion of a proper sub-argument A of A_2, and is also the conclusion of A_1. For instance, by considering the scenario depicted in Example 3, argument A'' — that the weather will be good because the forecast on the radio suggests so — provides i-support for argument A, since it is an argument for A's intermediate conclusion (the conclusion of A_3) that the weather will be good. The following is another example of i-support:

Example 6. *Let us consider the arguments built from the argumentation theory in Example 5. In this case, argument* B_2 *provides i-support for* A *since* B_2 *has the same conclusion as* A_3*, a proper sub-argument of* A*, which is not in the grounds of* A*. Similarly,* A_3 *i-supports* B *since the conclusion of* A_3 *coincides with the conclusion of* B_2*, which is an intermediate conclusion in* B*.*

Note that we require intermediate support to be focused on a proper sub-argument A of the supported argument A_2, whose conclusions differ. This is to distinguish i-support from c-support for A because, since ASPIC$^+$ arguments are not required to be minimal, it could be the case that a proper sub-argument A' of A_2 is such that $\texttt{Conc}(\mathsf{A}') = \texttt{Conc}(\mathsf{A}_2)$[5]. The definition also precludes i-support from being p-support by requiring the conclusion of A not to be a premise of A_2.

Some properties of i-support follow immediately from the definition:

Proposition 3. *Let* AT *be an argumentation theory and* $\mathsf{A}_1, \mathsf{A}_2 \in \mathcal{A}(\mathsf{AT})$*. If* A_1 *i-supports* A_2*, then there exists* $\mathsf{A} \in \texttt{Sub}(\mathsf{A}_2)$ *such that* A_1 *c-supports* A*.*

Proof: *From Definition 13, for* A_1 *to i-support* A_2 *there must exist a proper sub-argument* A *of* A_2 *such that* $\texttt{Conc}(\mathsf{A}_1) = \texttt{Conc}(\mathsf{A}_2)$*. Thus, by Definition 11,* A_1 *c-supports* A*.* □

This is exactly the case in Example 6, where the argument providing c-support is B_2 and the sub-argument being c-supported is A_3.

The following proposition relates i-support with the notions of support we have previously defined.

Proposition 4. *Let* AT *be an argumentation theory and* $\mathsf{A}_1, \mathsf{A}_2 \in \mathcal{A}(\mathsf{AT})$*. If* A_1 *i-supports* A_2*, then* A_1 *neither c-supports nor p-supports* A_2*.*

Proof: *By Definition 13, if* A_1 *i-supports* A_2 *then the conclusion of* A_1 *coincides with the conclusion of a proper sub-argument* A *of* A_2 *such that* $\texttt{Conc}(\mathsf{A}) \neq \texttt{Conc}(\mathsf{A}_2)$ *and* $\texttt{Conc}(\mathsf{A}) \notin \texttt{Prem}(\mathsf{A}_2)$*. Thus, by Definitions 11 and 12,* A_1 *does not c-support nor p-support* A_2*.* □

The notion of sub-argument has been present in the literature of argumentation, both in structured argumentation systems (e.g., [28, 49]) and in more abstract formalisms (e.g., [34]). However, the sub-argument relation had not been analyzed in the guise of a support relation until recent work [23, 50]. As one might expect given [50], the relationship between a sub-argument and its super-arguments fits the general idea of support we propose here. This is because a sub-argument A' of A constitutes an argument for a formula in A that is either a premise, an intermediate step, or the (final) conclusion of A.

Definition 14 (Sub-argument Support). *Let* AT *be an argumentation theory and* $\mathsf{A}_1, \mathsf{A}_2 \in \mathcal{A}(\mathsf{AT})$*. Argument* A_1 *provides* sub-argument support (s-support) *for argument* A_2 *iff* $\mathsf{A}_1 \in \texttt{Sub}(\mathsf{A}_2)$ *and* $\mathsf{A}_1 \neq \mathsf{A}_2$*.*

The above definition considers only proper sub-arguments, to avoid self-supporting arguments (recall that every argument is a sub-argument of itself). This is to comply with our general notion of support given in Definition 10, in which one argument supports *another* argument. The notion of s-support is illustrated in the following example:

Example 7. *Given the scenario depicted in Example 5, for instance, we have that arguments* A_1*,* A_2 *and* A_3 *s-support* A *(also,* A_1 *and* A_2 *s-support* A_3*); and argument* C_1 *s-supports argument* C*.*

[5] We return to the question of minimality of arguments in Section 5.

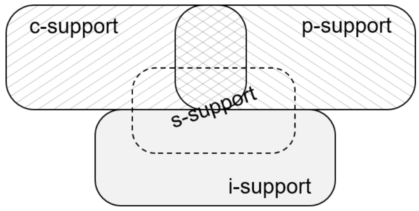


Figure 2: The relationship between types of support for structured arguments.

The notion of s-support relates arguments to their proper sub-arguments. Since, as mentioned before, arguments in ASPIC$^+$ are not required to be minimal, a proper sub-argument A' of an argument A can be such that $\mathtt{Conc}(\mathsf{A}') = \mathtt{Conc}(\mathsf{A})$ (i.e., their conclusions coincide). Moreover, it can be the case that $\mathtt{Conc}(\mathsf{A}') \in \mathtt{Prem}(\mathsf{A})$ (i.e., the conclusion of A' is a premise in A) or $\mathtt{Conc}(\mathsf{A}') \neq \mathtt{Conc}(\mathsf{A})$ and $\mathtt{Conc}(\mathsf{A}') \notin \mathtt{Prem}(\mathsf{A})$ (i.e., the conclusion of A' is an intermediate conclusion in A). Thus, as shown by the following proposition, s-support is always a form of c-support, p-support or i-support, but the reverse does not hold.

Proposition 5. *Given an argumentation theory* AT *and* $\mathsf{A}_1, \mathsf{A}_2 \in \mathcal{A}(\mathsf{AT})$*, it holds that:*

1. *If* A_1 *s-supports* A_2*, it implies that* A_1 *c-supports* A_2*,* A_1 *p-supports* A_2 *or* A_1 *i-supports* A_2*.*
2. *If* A_1 *c-supports* A_2*, it does not imply that* A_1 *s-supports* A_2*.*
3. *If* A_1 *p-supports* A_2*, it does not imply that* A_1 *s-supports* A_2*.*
4. *If* A_1 *i-supports* A_2*, it does not imply that* A_1 *s-supports* A_2*.*

Proof: *For the first part, if* A_1 *s-supports* A_2*, by Definition 14,* A_1 *is a proper sub-argument of* A_2*. Then, it can be the case that either:* $1.\mathsf{a})$ $\mathtt{Conc}(\mathsf{A}_1) = \mathtt{Conc}(\mathsf{A}_2)$ *and* $\mathtt{Conc}(\mathsf{A}_1) \notin \mathtt{Prem}(\mathsf{A}_2)$, $1.\mathsf{b})$ $\mathtt{Conc}(\mathsf{A}_1) \neq \mathtt{Conc}(\mathsf{A}_2)$ *and* $\mathtt{Conc}(\mathsf{A}_1) \in \mathtt{Prem}(\mathsf{A}_2)$, $1.\mathsf{c})$ $\mathtt{Conc}(\mathsf{A}_1) \neq \mathtt{Conc}(\mathsf{A}_2)$ *and* $\mathtt{Conc}(\mathsf{A}_1) \notin \mathtt{Prem}(\mathsf{A}_2)$*, or* $1.\mathsf{d})$ $\mathtt{Conc}(\mathsf{A}_1) = \mathtt{Conc}(\mathsf{A}_2)$ *and* $\mathtt{Conc}(\mathsf{A}_1) \in \mathtt{Prem}(\mathsf{A}_2)$*. In case* $1.\mathsf{a})$*, by Definition 11,* A_1 *c-supports* A_2*. In case* $1.\mathsf{b})$*, by Definition 12,* A_1 *p-supports* A_2*. In case* $1.\mathsf{c})$*, by Definition 13,* A_1 *i-supports* A_2*. In case* $1.\mathsf{d})$*, by Definitions 11 and 12,* A_1 *c-supports and p-supports* A_2*. For the second part, let us consider the scenario depicted in Example 2. There, argument* A *c-supports argument* B *(and vice-versa); however,* $\mathsf{A} \notin \mathtt{Sub}(\mathsf{B})$ *and thus, by Definition 14,* A *does not s-support* B*. For the third part, let us consider the scenario depicted in Example 5. There, argument* D *p-supports argument* A*; however,* $\mathsf{D} \notin \mathtt{Sub}(\mathsf{A})$ *and thus, by Definition 14,* D *does not s-support* A*. For the last part, again consider the scenario depicted in Example 2. There, argument* A_4 *i-supports argument* B*; however,* $\mathsf{A}_4 \notin \mathtt{Sub}(\mathsf{B})$ *and thus, by Definition 14,* A_4 *does not s-support* B*.* □

We are now at a point where we can summarize the relationship between the different kinds of support for structured arguments that we have identified. Proposition 2 tells us that c-support and p-support are equivalent when the conclusion of the supporting argument is also a premise of the supported one. Then, Proposition 3 shows that the existence of i-support between two arguments implies the existence of c-support between the former and a sub-argument of latter. Proposition 4 tells us that the notion of i-support does not overlap with the notions of c-support and p-support. Moreover, by Proposition 5 we know that all instances of s-support are instances of c-support, p-support or i-support; in contrast, not all instances of c-support, p-support or i-support are instances of s-support. As a result, the relationship between all the forms of support for structured arguments is depicted in Figure 2. Finally, note that the four forms of support proposed in this section (c-support, p-support, i-support and s-support) exhaustively cover all cases matching the general characterization of support given in Definition 10. Specifically, given an argument A_1 that supports an argument A_2, the sub-argument A' of A_2 that has the same conclusion as A_1 is either an argument for a premise in A_2 (in which case A_1 p-supports A_2), an argument for an intermediate conclusion in A_2 (hence, A_1 i-supports A_2), or an argument for the final conclusion of A_2 (therefore, A_1 c-supports A_2); furthermore, if A_1 is a proper sub-argument of A_2, then it would also be the case that A_1 s-supports A_2.

4. Support in abstract argumentation

The main contribution of this paper is to establish the relationships that exist between the notions of support in structured argumentation, as discussed in the previous section, and the notions of support in abstract argumentation that already exist in the literature. In order to identify these relationships, we first need to present existing notions of abstract support, and that is the topic of this section. In particular, we will introduce the support relation from the Bipolar Argumentation Framework [15], the deductive support from the meta-argumentation approach proposed in [6], and its dual interpretation, the necessary support from the Argumentation Framework with Necessities [38, 8]. These notions of support impose some acceptability constraints on the arguments they relate. Thus, the formalisms define a series of *complex attacks*[6] that can be inferred by combining supports and attacks, and reinforce the acceptability constraints of their corresponding support relations. This analysis is a prelude to Section 5 where we examine how abstract notions of support relate to the concrete notions of support in structured argumentation that we introduced in Section 3.

4.1. A general support relation

In [15] the authors provide a formalization of the Bipolar Argumentation Framework, an extension of Dung's abstract argumentation framework [25] that accounts for two independent interactions between arguments with diametrically opposed nature: the attack and support relations.

Definition 15 (Bipolar Argumentation Framework [15]). *A* Bipolar Argumentation Framework (BAF) *is a tuple $\langle \mathcal{A}, \mathcal{R}, \mathcal{S} \rangle$, where $\mathcal{A}$ is a finite non-empty set of arguments, $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ is an attack relation, and $\mathcal{S} \subseteq \mathcal{A} \times \mathcal{A}$ is a support relation.*

Here we follow the usual notation in the literature of bipolar argumentation (i.e., abstract argumentation formalisms that account for a support relation) in which $\mathcal{A}$ is used to represent the set of arguments. The attack relation $\mathcal{R}$ in a BAF is the same as in Dung's argumentation framework. On the other hand, the support relation $\mathcal{S}$ in a BAF does not impose any specific constraints on the acceptability of arguments; that is, the authors in [15] propose a *general support* relation, by just considering it as a positive interaction between arguments and refraining from saying what form that positive interaction takes.[7]

A BAF can be visualized as a directed graph called the *bipolar interaction graph*, where nodes are arguments and there are two kinds of edges, representing attack and support relations between arguments. In [15] the authors use $\not\longrightarrow$ and $\longrightarrow$ to respectively denote attack and support. Notwithstanding this, to provide a unified notation, in this paper we will follow the usual notation for Dung's argumentation frameworks in which $\longrightarrow$ denotes the attack relation. Then, the support relation between arguments will be denoted using $\Longrightarrow$[8]; furthermore, we will incorporate a label over the long double arrow to identify the interpretation given to the support relation. Thus, since the support relation of a BAF is a positive interaction between arguments, with no particular interpretation, we will use the label 'g' to denote that it is a *general* support relation. As a result, the support relation of a BAF will be denoted using $\stackrel{g}{\Longrightarrow}$.

To illustrate this, let us consider the following example.

Example 8. *Let* $\mathsf{BAF} = \langle \mathcal{A}, \mathcal{R}, \mathcal{S} \rangle$ *be a bipolar argumentation framework, where:*

$$\mathcal{A} = \{\mathsf{A}, \mathsf{B}, \mathsf{C}, \mathsf{D}, \mathsf{E}\} \qquad \mathcal{R} = \{(\mathsf{A}, \mathsf{B}), (\mathsf{D}, \mathsf{E})\} \qquad \mathcal{S} = \{(\mathsf{B}, \mathsf{C}), (\mathsf{C}, \mathsf{D})\}$$

The bipolar interaction graph associated with BAF *is depicted below.*

---

[6] This terminology is proposed by the authors of [18, 19].

[7] This is a more abstract statement than the one we made in Section 3 (Definition 10) — we see the latter as the most general instantiation of "a positive interaction between arguments" in the context of structured argumentation.

[8] Note that the single arrow ($\rightarrow$) and the double arrow ($\Rightarrow$) used for denoting strict and defeasible rules in ASPIC$^+$, correspond to short arrows. In contrast, for abstract argumentation frameworks with attack and support relations, we use the long single arrow ($\longrightarrow$) to denote attack between arguments, and the long double arrow ($\Longrightarrow$) to denote support between arguments.

$$
\begin{tikzcd}
 & & \mathsf{E} \\
\mathsf{B} \arrow[r, Rightarrow, "g"] & \mathsf{C} \arrow[r, Rightarrow, "g"] & \mathsf{D} \arrow[u] \\
\mathsf{A} \arrow[u] & &
\end{tikzcd}
$$

Here, A *attacks* B*,* B *supports* C*,* C *supports* D*, and* D *attacks* E*.*

In [15], the acceptable sets of arguments of a BAF are computed by taking the support relation into account. To achieve this, a series of *complex attacks* are obtained from the combination of the attack and support relations. These complex attacks correspond to some acceptability constraints that, although not explicitly mentioned, are imposed on the arguments related by the support relation. The complex attacks of a BAF are made up of *supported attacks* and *secondary attacks*:

Definition 16 (Supported and Secondary Attacks [15]). *Let* $\langle \mathcal{A}, \mathcal{R}, \mathcal{S} \rangle$ *be a bipolar argumentation framework and* $\mathsf{A}, \mathsf{B} \in \mathcal{A}$*.*

- *A* supported attack *from* A *to* B *is a sequence* $\mathsf{A}_1\ \mathsf{r}_1\ \mathsf{A}_2 \ldots \mathsf{A}_{n-1}\ \mathsf{r}_{n-1}\ \mathsf{A}_n$*,* $n \geq 3$*, where* $\mathsf{A}_1 = \mathsf{A}$ *and* $\mathsf{A}_n = \mathsf{B}$*, such that* $\mathsf{r}_i = \mathcal{S}$ $(1 \leq i \leq n-2)$ *and* $\mathsf{r}_{n-1} = \mathcal{R}$*.*
- *A* secondary attack *from* A *to* B *is a sequence* $\mathsf{A}_1\ \mathsf{r}_1\ \mathsf{A}_2 \ldots \mathsf{A}_{n-1}\ \mathsf{r}_{n-1}\ \mathsf{A}_n$*, with* $n \geq 3$*, where* $\mathsf{A}_1 = \mathsf{A}$ *and* $\mathsf{A}_n = \mathsf{B}$*, such that* $\mathsf{r}_1 = \mathcal{R}$ *and* $\mathsf{r}_i = \mathcal{S}$ $(2 \leq i \leq n-1)$*.*

A supported attack is thus a path through a bipolar interaction graph (such as the one from Example 8) that consists of a sequence of support arcs terminated with an attack arc. Similarly, a secondary attack is an attack followed by a sequence of supports. We will denote the supported attacks in $\langle \mathcal{A}, \mathcal{R}, \mathcal{S} \rangle$ with the relation $\mathcal{R}_{su}$, and the secondary attacks with the relation $\mathcal{R}_{se}$.

The authors in [15] state that, by extension, a sequence reduced to two arguments $\mathsf{A}\ \mathcal{R}\ \mathsf{B}$ (that is, a direct attack $\mathsf{A} \longrightarrow \mathsf{B}$) is also considered as a supported attack from A to B. Hence, given a BAF $\langle \mathcal{A}, \mathcal{R}, \mathcal{S} \rangle$ and arguments $\mathsf{A}_1, \mathsf{A}_2, \ldots, \mathsf{A}_k, \mathsf{B} \in \mathcal{A}$, a path $\mathsf{A}_1 \overset{g}{\Longrightarrow} \mathsf{A}_2 \overset{g}{\Longrightarrow} \ldots \overset{g}{\Longrightarrow} \mathsf{A}_k \longrightarrow \mathsf{B}$ in the bipolar interaction graph leads to k supported attacks from A_i to B respectively $(1 \leq i \leq k)$. Similarly, a path $\mathsf{B} \longrightarrow \mathsf{A}_1 \overset{g}{\Longrightarrow} \mathsf{A}_2 \overset{g}{\Longrightarrow} \ldots \overset{g}{\Longrightarrow} \mathsf{A}_k$ in the bipolar interaction graph leads to $k-1$ secondary attacks from B to A_j respectively $(2 \leq j \leq k)$. To illustrate these notions, let us consider the following example:

Example 9. *Given the* BAF *of Example 8, we obtain the supported and secondary attacks depicted in the graph below, where the long solid arrow* $\longrightarrow$ *represents direct attack and the dashed arrow* $\dashrightarrow$ *represents complex attack. Supports are not shown:*

$$
\begin{tikzcd}
 & & \mathsf{E} \\
\mathsf{B} \arrow[urr, dashed] & \mathsf{C} \arrow[ur, dashed] & \mathsf{D} \arrow[u] \\
\mathsf{A} \arrow[u] \arrow[ur, dashed] \arrow[urr, dashed] & &
\end{tikzcd}
$$

For instance, there is a supported attack from B *to* E *determined by the path* $\mathsf{B} \overset{g}{\Longrightarrow} \mathsf{C} \overset{g}{\Longrightarrow} \mathsf{D} \longrightarrow \mathsf{E}$ *in the bipolar interaction graph associated with the* BAF*; similarly, there is a supported attack from* C *to* E*. On the other hand, there is a secondary attack from* A *to* D *determined by the path* $\mathsf{A} \longrightarrow \mathsf{B} \overset{g}{\Longrightarrow} \mathsf{C} \overset{g}{\Longrightarrow} \mathsf{D}$ *in the bipolar interaction graph associated with the* BAF *and, analogously, a secondary attack from* A *to* C*.*

As mentioned above, the authors in [15] analyze the acceptability of arguments by taking the support relation into consideration. They extend Dung's notion of conflict-freeness [25] to take supported and secondary attacks into account when computing the acceptability of arguments, and then use the extended notion of conflict-freeness to obtain the acceptable arguments of the BAF by following Dung's extensional approach. In other words, they use a notion of attack that is the union of $\mathcal{R}$, $\mathcal{R}_{su}$, and $\mathcal{R}_{se}$ when establishing acceptability. Then the process of establishing acceptability in a BAF is defined in the same way as in [25] for Dung's argumentation systems. For example, given the preferred semantics of [25] and the BAF of Example 8, when we consider the complex attacks depicted in Example 9, the only preferred extension is $\{A, E\}$.

4.2. Deductive support

In [6], a meta-argumentation approach is introduced that allows for the consideration of support between arguments. Unlike the support relation of the bipolar argumentation frameworks, the paper introduces *deductive support*, thus giving a particular interpretation for the support relation. Briefly, the deductive interpretation of support establishes the following acceptability constraints on the arguments it relates: if argument A supports argument B, then the acceptance of A implies the acceptance of B or, equivalently, the non-acceptance of B implies the non-acceptance of A. We will use the term d-BAF to denote a bipolar argumentation framework in which the support relation has a deductive interpretation.

Instead of extending Dung's formalism by incorporating a support relation, the authors of [6] propose using meta-argumentation to instantiate an abstract argumentation framework in order to represent deductive support between arguments. Given the coexistence of supporting and attacking arguments in a d-BAF, complex attacks are considered to ensure the acceptability constraints of deductive support. First, the authors of [6] adopt supported attacks as proposed in [15]. Second, they note that secondary attacks as defined above may lead to unintended results when considering deductive support. Thus, they propose eliminating secondary attacks, and introduce *mediated attacks* instead.

Following the notation introduced in the previous section, $A \longrightarrow B$ will denote that A attacks B. As in a BAF, the attack relation in a d-BAF corresponds to the attack relation of Dung's abstract argumentation framework. Then, since this is a particular interpretation of support, we will use $A \overset{d}{\Longrightarrow} B$ to denote that A deductively supports B. Here, the label 'd' over the long double arrow indicates that the support relation between arguments is deductive. We then have:

Definition 17 (Mediated Attack [6]). *Let $\langle \mathcal{A}, \longrightarrow, \overset{d}{\Longrightarrow} \rangle$ be a d-BAF and $A, B \in \mathcal{A}$. A* mediated attack *from A to B is a sequence $A_1 \overset{d}{\Longrightarrow} \ldots \overset{d}{\Longrightarrow} A_{n-1}$ and $A_n \longrightarrow A_{n-1}$, $(n \geq 3)$, where $A_1 = B$ and $A_n = A$.*

Like a supported attack, a mediated attack is obtained from a sequence of supports followed by an attack. However, the direction of the mediated attack is from the final step in the sequence (the attacking argument) to the first step (the initial supporting argument). Mediated attack is illustrated by the following example:

Example 10. *Consider the d-BAF depicted below, where supported and mediated attacks are denoted using dashed arrows.*

$$
\begin{tikzcd}
A \arrow[r] \arrow[dr, dashed] & B \arrow[r] & C \\
 & D \arrow[u, Rightarrow, "d"] \arrow[ur, dashed] &
\end{tikzcd}
$$

Here, there is a mediated attack from A to D, and a supported attack from D to C. The justification for the mediated attack is that since D deductively supports B, the non-acceptability of B affects the acceptability of D. Then, since A's attack on B aims to make it not acceptable, A is implicitly attacking D as well.

Finally, the authors in [6] propose an approach for determining the acceptability status of arguments which involves a translation into a Dung framework using meta-arguments. The resulting framework is such that the attack relation encodes the original attacks of the d-BAF, as well as the corresponding supported and mediated attacks.

4.3. Necessary support

Next, we will present the Argumentation Framework with Necessities [8, 37, 38], an extension of Dung's argumentation framework that incorporates a specialized type of support relation between arguments: the *necessity* relation. Briefly, the necessity relation denotes that if an argument A supports an argument B, then the acceptance of A is necessary to obtain the acceptance of B. Thus, the acceptance of B implies the acceptance of A and, conversely, the non-acceptance of A implies the non-acceptance of B.

Definition 18 (Argumentation Framework with Necessities [8]). *An* Argumentation Framework with Necessities (AFN) *is a tuple $\langle \mathcal{A}, \mathcal{R}, \mathcal{N} \rangle$, where $\mathcal{A}$ is a set of arguments, $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ is an attack relation, and $\mathcal{N} \subseteq \mathcal{A} \times \mathcal{A}$ is an irreflexive and transitive necessary support relation.*

The definition of an AFN presented here corresponds to the one introduced in [8], where the necessary support relation $\mathcal{N}$ is transitive. The attack relation $\mathcal{R}$ in Definition 18 is the same as in Dung's argumentation framework. From the original attacks and the necessity relation, some complex attacks can be obtained, which enforce the acceptability constraints imposed by the support relation: if A attacks C and C is necessary for B, then A attacks B; and if C attacks B and C is necessary for A, then A attacks B. These new attacks are called *extended attacks* and are formalized in the following definition:

Definition 19 (Extended Attacks [38]). *Let $\langle \mathcal{A}, \mathcal{R}, \mathcal{N} \rangle$ be an argumentation framework with necessities and $\mathsf{A}, \mathsf{B} \in \mathcal{A}$. There is an* extended attack *from A to B, noted as $\mathsf{A}\mathcal{R}^{+}\mathsf{B}$, if there exists $\mathsf{C} \in \mathcal{A}$ such that either $\mathsf{A}\mathcal{R}\mathsf{C}$ and $\mathsf{C}\mathcal{N}\mathsf{B}$, or $\mathsf{C}\mathcal{R}\mathsf{B}$ and $\mathsf{C}\mathcal{N}\mathsf{A}$. The direct attack $\mathsf{A}\mathcal{R}\mathsf{B}$ is considered as a particular case of extended attack.*

It should be noted that the original formalization of the AFN given in [37] only considered one kind of extended attack. This was an extended attack from A to B which would only occur when A attacks C and C is necessary for B. The second kind of extended attack was added later in [38]. With both kinds of extended attack it is possible to enforce the duality between necessary and deductive support. For more details on this duality, see [19, 23].

An AFN can be graphically represented by a directed graph where nodes are arguments and there are two kinds of edges denoting the attack and support relations. In [38] the authors use $\longrightarrow$ and $\dashrightarrow$ to respectively denote attack and support between arguments. However, as mentioned before, with the aim of uniformity, we will use $\mathsf{A} \longrightarrow \mathsf{B}$ to denote $\mathsf{A}\mathcal{R}\mathsf{B}$ (attack), and $\mathsf{A} \overset{\mathfrak{n}}{\Longrightarrow} \mathsf{B}$ to denote $\mathsf{A}\mathcal{N}\mathsf{B}$ (necessary support). In this case, the label '$\mathfrak{n}$' over the long double arrow indicates that the support relation of the AFN is interpreted as necessary support. To simplify the representation, we will only include "direct" necessary support links on the graph. Therefore, the necessities obtained by transitivity on the support relation can be visualized by following the support paths on the graph. To illustrate these notions, let us consider the following example.

Example 11. *Consider the* AFN *depicted below on the left.*

$$\begin{tikzcd}
\mathsf{A} \arrow[r, Rightarrow, "\mathfrak{n}"] & \mathsf{B} \arrow[r, Rightarrow, "\mathfrak{n}"] & \mathsf{C} \arrow[d] \\
\mathsf{D} \arrow[u] \arrow[r, Rightarrow, "\mathfrak{n}"] & \mathsf{E} \arrow[r, Rightarrow, "\mathfrak{n}"] & \mathsf{F}
\end{tikzcd}
\qquad\qquad
\begin{tikzcd}
\mathsf{A} & \mathsf{B} & \mathsf{C} \arrow[d] \\
\mathsf{D} \arrow[u] \arrow[ur, dashed] \arrow[urr, dashed] & \mathsf{E} \arrow[ul, dashed] & \mathsf{F} \arrow[ull, dashed]
\end{tikzcd}$$

In addition to the direct necessities, there is a necessity from A to C, and from D to F. The graph above on the right summarizes the extended attacks obtained from the AFN *which are depicted using dashed arrows, except from those that are also direct attacks. Given that D attacks A and A supports B and C, there are extended attacks from D to B and C. In addition, since D attacks A and supports E and F, there are also extended attacks from E and F to A.*

Finally, in [38] the authors propose an alternative for computing the acceptable arguments of an AFN, which consists of transforming an AFN into an argumentation framework like that of Definition 9. This argumentation framework takes the set of arguments of the AFN and the extended attack relation $\mathcal{R}^{+}$ and computes acceptability following the approach of [25]. Since direct attacks are considered as a particular case of the extended attacks, the resulting argumentation framework contemplates both direct and extended attacks on the arguments of the original AFN. Following this approach, for instance, given the AFN of Example 11 and the preferred semantics of [25], the only preferred extension is $\{\mathsf{D}, \mathsf{E}, \mathsf{F}\}$.

4.4. Acceptability of arguments in abstract argumentation frameworks with support

The previous subsections have presented a series of abstract argumentation frameworks which characterize support relations with different interpretations (namely, the BAF, d-BAF and AFN). As mentioned before, each one of these frameworks can be associated with a Dung abstract argumentation framework (AF) that accepts the same arguments as the original framework under a given semantics. The set of arguments in such an AF coincides with the set of arguments in the BAF (respectively, the d-BAF or the AFN), and the attack relation of the AF is $\mathsf{Att}^+ = \mathsf{Att} \cup \mathsf{CAtt}$, where Att is the attack relation of the BAF (respectively, the d-BAF or the AFN) and CAtt is the set of complex attacks of the BAF (respectively, the d-BAF or the AFN). Formally:

Definition 20 (AF associated with BAF). *Let* $\mathsf{BAF} = \langle \mathcal{A}, \mathsf{Att}, \mathsf{Supp} \rangle$ *be a bipolar argumentation framework. We define the argumentation framework associated with* BAF *as* $\mathsf{AF}_{\mathsf{BAF}} = \langle \mathcal{A}, \mathsf{Att}^+ \rangle$*, where* $\mathsf{Att}^+ = \mathsf{Att} \cup \mathsf{CAtt}$ *and* $\mathsf{CAtt} = \{(\mathsf{A}, \mathsf{B}) \mid \mathsf{A}, \mathsf{B} \in \mathcal{A}$ *and there is a supported attack or a secondary attack from* A *to* B *in* $\mathsf{BAF}\}$.

Definition 21 (AF associated with d-BAF). *Let* $\mathsf{d\text{-}BAF} = \langle \mathcal{A}, \mathsf{Att}, \mathsf{Supp} \rangle$ *be a bipolar argumentation framework with deductive support. We define the argumentation framework associated with* $\mathsf{d\text{-}BAF}$ *as* $\mathsf{AF}_{\mathsf{dBAF}} = \langle \mathcal{A}, \mathsf{Att}^+ \rangle$*, where* $\mathsf{Att}^+ = \mathsf{Att} \cup \mathsf{CAtt}$ *and* $\mathsf{CAtt} = \{(\mathsf{A}, \mathsf{B}) \mid \mathsf{A}, \mathsf{B} \in \mathcal{A}$ *and there is a supported attack or a mediated attack from* A *to* B *in* $\mathsf{d\text{-}BAF}\}$.

Definition 22 (AF associated with AFN). *Let* $\mathsf{AFN} = \langle \mathcal{A}, \mathsf{Att}, \mathsf{Supp} \rangle$ *be an argumentation framework with necessities. We define the argumentation framework associated with* AFN *as* $\mathsf{AF}_{\mathsf{AFN}} = \langle \mathcal{A}, \mathsf{Att}^+ \rangle$*, where* $\mathsf{Att}^+ = \mathsf{Att} \cup \mathsf{CAtt}$ *and* $\mathsf{CAtt} = \{(\mathsf{A}, \mathsf{B}) \mid \mathsf{A}, \mathsf{B} \in \mathcal{A}$ *and there is an extended attack from* A *to* B *in* $\mathsf{AFN}\}$.

We come back to these definitions in Section 5.3.

5. Relating support at the abstract and structured level

Having established a set of notions of support for structured arguments (Section 3) and discussed various notions of support for abstract arguments that can be found in the literature (Section 4), we now consider how to relate the two. We will analyze whether the notions of support for structured arguments characterized in Section 3 can be used to instantiate the abstract forms of support presented in Section 4.

Given their abstract nature, the formalisms presented in Section 4 do not elaborate on the origin of their support relations. Rather, they just define the support relation as an interaction between pairs of arguments. Furthermore, in the case of BAFs, no acceptability constraints associated with the adopted interpretation of support are given beforehand. In contrast, d-BAFs and AFNs explicitly mention the acceptability constraints associated with their deductive and necessary interpretations of support. Notwithstanding this, in all three cases, the nature of the support relation is determined by the way in which the complex attacks are characterized, so that they comply with the corresponding acceptability constraints[9]. As a result, in order to try to establish a correspondence between the structured and abstract forms of support, we need to be able to determine whether c-support, p-support, i-support or s-support are able to comply with the acceptability constraints derived from the nature of the abstract forms of support. In other words, we need to determine whether the complex attacks for BAFs, d-BAFs and AFNs make sense when considering the structured forms of support from Section 3.

To this end, we will analyze how c-support, p-support, i-support and s-support can be combined with the different types of attack from Section 2 in order to infer new attacks which, following the naming convention adopted in abstract argumentation, will be referred to as *complex attacks*. We will consider four scenarios in which, given three arguments A, B and C, two of them are related by the support relation while the remaining argument attacks, or is attacked by, one of the arguments linked by support. These are:

1. A supports B and A is attacked by C;

[9] In the case of BAFs, since no acceptability constraints are given beforehand, these constraints are just inferred from the characterization of the complex attacks.

2. A supports B and B is attacked by C;
3. A supports B and A attacks C;
4. A supports B and B attacks C;

These four scenarios match those scenarios in which complex attacks take place in BAFs, d-BAFs and AFNs, as presented in Section 4. Hence, along with all their symmetric variants, these four scenarios cover all the possible ways in which three arguments can interact when there is one attack and one support between them (we ignore the case where one pair of arguments both attack and support each other).

It should be noted that ASPIC$^+$ arguments are minimal in the sense that given an argument A, removing any sub-argument of A will mean that A is no longer an argument. However, it is possible for A to contain sub-arguments with the same conclusion or repeated premises, and such an argument is non-minimal in the sense that it is possible to obtain an argument A′ with the same conclusion as A, but using a smaller set of premises or rules. As a result, an argument satisfying this stronger form of minimality cannot have two distinct sub-arguments with the same conclusion [32]. Since arguments satisfying the stronger form of minimality are the kinds of argument that one encounters most often in the literature (see e.g., [5, 28]), we will only consider such arguments in our analysis.

For each scenario, in Section 5.1 we consider all forms of structured attack — rebut[10], undermine and undercut — and support — c-support, p-support, i-support, s-support — relations, and identify the complex attacks that can be inferred from their combination. These complex attacks are illustrated in Tables 2–5, specifying the corresponding scenario along with the conditions under which they can be inferred. For example, in the case where A supports B and A is attacked by C, the relevant table (Table 2) shows what complex attacks can be inferred between C and B. Each table also depicts the complex attacks that are obtained by the abstract formalisms from Section 4 in those same scenarios. It is important to remark that we will only consider complex attacks arising from the combination of both attack and support relations (and possibly some additional constraints). That is, for instance, attacks that may arise by looking at the attack relation alone will not be considered as complex attacks.

The similarities and differences between the complex attacks for structured and abstract forms of support, thus the possibility of instantiating the abstract supports with concrete ones, is discussed in Section 5.2. Finally, the results from our analysis are formalized in Section 5.3.

5.1. Complex attacks for minimal structured arguments

Following the notation introduced in Section 4, we will use the long single arrow $\longrightarrow$ to denote the attacks, including a label to distinguish between rebutting, undermining, and undercutting attacks. Similarly, we will use a labeled long double arrow $\Longrightarrow$ to denote support, distinguishing between c-support, p-support, i-support, s-support, and the abstract forms of support: g-support (general support in BAFs), d-support (deductive support in d-BAFs) and n-support (necessary support in AFNs). In all cases, the complex attacks will be denoted using a dashed arrow $\dashrightarrow$ which, when required, will be labeled with the corresponding type of attack. The notation is summarized in Table 1.

As can be observed in Tables 2–5, the complex attacks in the abstract formalisms can be inferred by looking at the attack and support relations alone, without considering additional constraints. In contrast, none of the forms of support between structured arguments is such that it enables the inference of complex attacks in all scenarios by just looking at the attack and support relations. That is, most rows in every table that correspond to a form of support between structured arguments are such that at least one of the inferred complex attacks comes with attached conditions (the only exceptions are s-support in Table 2 and c-support in Tables 4 and 5). Put another way, at the abstract level, complex attacks may be inferred, naturally enough, without needing to consider the structure of arguments. However, at the structured level, the details of the structure heavily influence what can be inferred.

To illustrate the need for side-conditions when identifying the complex attacks for structured arguments, let us consider the following examples. Let A, B and C be such that C undermines A and A p-supports B ($\mathsf{C} \xrightarrow{um} \mathsf{A} \xRightarrow{p} \mathsf{B}$ in Table 2). Then, from the existence of the undermining attack, we know that $\mathtt{Conc}(\mathsf{C})$ is a contrary of a premise in A and that $\mathtt{Conc}(\mathsf{A}) \in \mathtt{Prem}(\mathsf{B})$. Furthermore, if it is the case

[10] Recall that, by Definition 5, rebutting attacks in ASPIC$^+$ are *restricted*, meaning that an argument with a strict $\mathtt{TopRule}(\cdot)$ can rebut an argument with a defeasible $\mathtt{TopRule}(\cdot)$, but not vice versa.

Attack	**Support**
$\xrightarrow{\mathtt{r}}$: rebutting attack	$\xRightarrow{\mathtt{c}}$: conclusion support (c-support)
$\xrightarrow{\mathtt{um}}$: undermining attack	$\xRightarrow{\mathtt{p}}$: premise support (p-support)
$\xrightarrow{\mathtt{uc}}$: undercutting attack	$\xRightarrow{\mathtt{i}}$: intermediate support (i-support)
$\dashrightarrow$: complex attack	$\xRightarrow{\mathtt{s}}$: sub-argument support (s-support)
	$\xRightarrow{\mathtt{g}}$: general support in a BAF (g-support)
	$\xRightarrow{\mathtt{d}}$: deductive support in a d-BAF (d-support)
	$\xRightarrow{\mathtt{n}}$: necessary support in an AFN (n-support)

Table 1: The notation we use to denote different forms of attack and the structured and abstract forms of support.

that $\mathtt{C}$ undermines $\mathtt{A}$ on $\mathtt{A}$, then we know that $\mathtt{A}$ is an argument of the form [$\mathtt{Conc}(\mathtt{A})$] (i.e., $\mathtt{Conc}(\mathtt{A})$ is the only premise in $\mathtt{A}$). As a result, if this side-condition is met, we can infer a complex undermining attack from $\mathtt{C}$ to $\mathtt{B}$. To give another example, let us consider the case where $\mathtt{A} \xRightarrow{\mathtt{c}} \mathtt{B}$ and $\mathtt{C} \xrightarrow{\mathtt{r}} \mathtt{B}$ in Table 3. There, the side condition establishes that if $\mathtt{C}$ rebuts $\mathtt{B}$ on $\mathtt{B}$ and $\mathtt{TopRule}(\mathtt{A})$ is defeasible, then a complex rebutting attack from $\mathtt{C}$ to $\mathtt{A}$ can be inferred. It is important to note that in order to infer that complex attack, we need to explicitly account for the c-support relationship between $\mathtt{A}$ and $\mathtt{B}$; otherwise, the information expressed by the side-condition alone would not be sufficient to infer that complex attack.

As a third and final example, let us consider the combination of s-support and the different forms of attack in the scenario depicted in Table 3. For instance, if $\mathtt{A} \xRightarrow{\mathtt{s}} \mathtt{B}$ and $\mathtt{C} \xrightarrow{\mathtt{r}} \mathtt{B}$, then without further information we cannot infer a complex attack from $\mathtt{C}$ to $\mathtt{A}$. Suppose we know that $\mathtt{C}$ rebuts $\mathtt{B}$ on $\mathtt{A}$. Then, $\mathtt{C}$ would also rebut $\mathtt{A}$ (on $\mathtt{A}$). However, in such a case, the rebutting attack from $\mathtt{C}$ to $\mathtt{A}$ would be inferred by just considering information regarding the attack from $\mathtt{C}$ to $\mathtt{B}$ (that is, without explicitly considering the s-support link between $\mathtt{A}$ and $\mathtt{B}$). As a result, attacks like that one are not considered in our analysis (hence the blank cells for s-support in Table 3) since they are not regarded as *complex attacks*[11].

By looking at the complex attacks for the structured forms of support in Tables 2–5, we can identify some relevant features which, in some cases, are shared across different tables. First, we can note a difference in terms of side conditions between the complex attacks inferred in Tables 2 and 3, and those inferred in Tables 4 and 5. Almost every complex attack in the first two tables (all but those related to s-support), which correspond to scenarios where the original and the inferred complex attacks target the two arguments related by support, come with attached side-conditions. Moreover, in many cases, the associated side-conditions determine very specific situations. In contrast, the complex attacks in Tables 4 and 5, which correspond to scenarios where the original and inferred complex attacks originate from the arguments related by support, only have side-conditions when considering p-support. Furthermore, the side-conditions for p-support in these two tables coincide and are quite simple. Furthermore, the side-conditions in some tables are symmetric (such as those for c-support in Tables 4 and 5).

Regarding the specification of side-conditions in Tables 2–5, it can be noted that many of those that consider rebutting and undermining attacks, rely on the nature of such attacks. Specifically, for rebutting attacks, most of them consider that rebut occurs at the final conclusion of the attacked argument. This would suggest that intermediate rebut (i.e., rebut at an intermediate conclusion of an argument) is not very useful for inferring complex attacks. Even in the case where intermediate rebut is considered (e.g., combination of i-support and rebutting attack in Table 3, where $\mathtt{A} \xRightarrow{\mathtt{i}} \mathtt{B}$ and $\mathtt{C} \xrightarrow{\mathtt{r}} \mathtt{B}$), the inferred complex attack is in turn a rebutting attack on the final conclusion of $\mathtt{A}$. On the other hand, regarding the side-conditions in cases where undermining attacks are involved, we can note that most of them rely on the fact that the attacked premise is also the final conclusion of the attacked argument (thus, that the attacked argument is of the form [$\mathtt{Conc}(\mathtt{Arg})$]). As a result, one could say that undermining attacks on other kinds of arguments are not that useful for inferring complex attacks. A similar thing occurs for

[11]As mentioned before, following the terminology introduced in Section 4, we only consider as complex attacks those attacks that are inferred from the combination of the attack and support relations.

$\Rightarrow$ \ $\rightarrow$	$\xrightarrow{r}$	$\xrightarrow{um}$	$\xrightarrow{uc}$
$\overset{c}{\Longrightarrow}$	$C \xrightarrow{r} A \overset{c}{\Longrightarrow} B$, $C \overset{r}{\dashrightarrow} B$ If C rebuts A on A; and $\mathtt{TopRule}(B)$ is defeasible. $C \xrightarrow{r} A \overset{c}{\Longrightarrow} B$, $C \overset{um}{\dashrightarrow} B$ If C rebuts A on A; and $\mathtt{Conc}(B) \in \mathtt{Prem}(B)$.	$C \xrightarrow{um} A \overset{c}{\Longrightarrow} B$, $C \overset{r}{\dashrightarrow} B$ If C undermines A on A; and $\mathtt{TopRule}(B)$ is defeasible.	
$\overset{p}{\Longrightarrow}$	$C \xrightarrow{r} A \overset{p}{\Longrightarrow} B$, $C \overset{um}{\dashrightarrow} B$ If C rebuts A on A.	$C \xrightarrow{um} A \overset{p}{\Longrightarrow} B$, $C \overset{um}{\dashrightarrow} B$ If C undermines A on A.	
$\overset{i}{\Longrightarrow}$	$C \xrightarrow{r} A \overset{i}{\Longrightarrow} B$, $C \overset{r}{\dashrightarrow} B$ If C rebuts A on A; and $\mathtt{TopRule}(B')$ is defeasible, where B' is the sub-argument of B originating the support from A.	$C \xrightarrow{um} A \overset{i}{\Longrightarrow} B$, $C \overset{r}{\dashrightarrow} B$ If C undermines A on A; and $\mathtt{TopRule}(B')$ is defeasible, where B' is the sub-argument of B originating the support from A.	
$\overset{s}{\Longrightarrow}$	$C \xrightarrow{r} A \overset{s}{\Longrightarrow} B$, $C \overset{r}{\dashrightarrow} B$	$C \xrightarrow{um} A \overset{s}{\Longrightarrow} B$, $C \overset{um}{\dashrightarrow} B$	$C \xrightarrow{uc} A \overset{s}{\Longrightarrow} B$, $C \overset{uc}{\dashrightarrow} B$
$\overset{g}{\Longrightarrow}$	$C \rightarrow A \overset{g}{\Longrightarrow} B$, $C \dashrightarrow B$		
$\overset{d}{\Longrightarrow}$			
$\overset{n}{\Longrightarrow}$	$C \rightarrow A \overset{n}{\Longrightarrow} B$, $C \dashrightarrow B$		

Table 2: Complex attacks for $C \longrightarrow A \Longrightarrow B$, C attacks A and A supports B, when arguments are minimal. The three columns correspond to the three possible kinds of attack: rebutting ($\xrightarrow{r}$), undermining ($\xrightarrow{um}$), and undercutting ($\xrightarrow{uc}$). The rows each correspond to one of the forms of support: conclusion support ($\overset{c}{\Longrightarrow}$), premise support ($\overset{p}{\Longrightarrow}$), intermediate support ($\overset{i}{\Longrightarrow}$), sub-argument support ($\overset{s}{\Longrightarrow}$), general support ($\overset{g}{\Longrightarrow}$), deductive support ($\overset{d}{\Longrightarrow}$) and necessary support ($\overset{n}{\Longrightarrow}$). Blank cells indicate that no complex attacks can be inferred.

$\Rightarrow$ \ $\rightarrow$	$\xrightarrow{r}$		$\xrightarrow{um}$		$\xrightarrow{uc}$
$\xRightarrow{c}$	$A \xRightarrow{c} B$, $C \xrightarrow{r} B$, $C \overset{r}{\dashrightarrow} A$ If C rebuts B on B; and TopRule(A) is defeasible.	$A \xRightarrow{c} B$, $C \xrightarrow{r} B$, $C \overset{um}{\dashrightarrow} A$ If C rebuts B on B; and Conc(A) $\in$ Prem(A).	$A \xRightarrow{c} B$, $C \xrightarrow{um} B$, $C \overset{r}{\dashrightarrow} A$ If C undermines B on B; and TopRule(A) is defeasible.		
$\xRightarrow{p}$			$A \xRightarrow{p} B$, $C \xrightarrow{um} B$, $C \overset{um}{\dashrightarrow} A$ If $\lvert$Prem$(B)\rvert = 1$; and Conc(A) $\in$ Prem(A).	$A \xRightarrow{p} B$, $C \xrightarrow{um} B$, $C \overset{r}{\dashrightarrow} A$ If $\lvert$Prem$(B)\rvert = 1$; and TopRule(A) is defeasible.	
$\xRightarrow{i}$	$A \xRightarrow{i} B$, $C \xrightarrow{r} B$, $C \overset{r}{\dashrightarrow} A$ If C rebuts B on B', where B' is the sub-argument of B originating the support from A; and TopRule(A) is defeasible.	$A \xRightarrow{i} B$, $C \xrightarrow{r} B$, $C \overset{um}{\dashrightarrow} A$ If C rebuts B on B', where B' is the sub-argument of B originating the support from A; and Conc(A) $\in$ Prem(A).			
$\xRightarrow{s}$					
$\xRightarrow{g}$					
$\xRightarrow{d}$	$A \xRightarrow{d} B$, $C \longrightarrow B$, $C \dashrightarrow A$				
$\xRightarrow{n}$					

Table 3: Complex attacks for $A \Longrightarrow B \longleftarrow C$, C attacks B and A supports B, when arguments are minimal. The three columns correspond to the three possible kinds of attack: rebutting ($\xrightarrow{r}$), undermining ($\xrightarrow{um}$), and undercutting ($\xrightarrow{uc}$). The rows each correspond to one of the forms of support: conclusion support ($\xRightarrow{c}$), premise support ($\xRightarrow{p}$), intermediate support ($\xRightarrow{i}$), sub-argument support ($\xRightarrow{s}$), general support ($\xRightarrow{g}$), deductive support ($\xRightarrow{d}$) and necessary support ($\xRightarrow{n}$). Blank cells indicate that no complex attacks can be inferred.

$\overset{\longrightarrow}{\Rightarrow}$	$\xrightarrow{r}$	$\xrightarrow{um}$	$\xrightarrow{uc}$
$\overset{c}{\Longrightarrow}$	$\begin{tikzcd} A \arrow[r, Rightarrow, "c"] \arrow[d, "r"'] & B \arrow[dl, dashed, "r"] \\ C \end{tikzcd}$	$\begin{tikzcd} A \arrow[r, Rightarrow, "c"] \arrow[d, "um"'] & B \arrow[dl, dashed, "um"] \\ C \end{tikzcd}$	$\begin{tikzcd} A \arrow[r, Rightarrow, "c"] \arrow[d, "uc"'] & B \arrow[dl, dashed, "uc"] \\ C \end{tikzcd}$
$\overset{p}{\Longrightarrow}$	$\begin{tikzcd} A \arrow[r, Rightarrow, "p"] \arrow[d, "r"'] & B \arrow[dl, dashed, "r"] \\ C \end{tikzcd}$ If $\texttt{Conc}(B) \in \texttt{Prem}(B)$.	$\begin{tikzcd} A \arrow[r, Rightarrow, "p"] \arrow[d, "um"'] & B \arrow[dl, dashed, "um"] \\ C \end{tikzcd}$ If $\texttt{Conc}(B) \in \texttt{Prem}(B)$.	$\begin{tikzcd} A \arrow[r, Rightarrow, "p"] \arrow[d, "uc"'] & B \arrow[dl, dashed, "uc"] \\ C \end{tikzcd}$ If $\texttt{Conc}(B) \in \texttt{Prem}(B)$.
$\overset{i}{\Longrightarrow}$			
$\overset{s}{\Longrightarrow}$			
$\overset{g}{\Longrightarrow}$			
$\overset{d}{\Longrightarrow}$			
$\overset{n}{\Longrightarrow}$	$\begin{tikzcd} A \arrow[r, Rightarrow, "n"] \arrow[d] & B \arrow[dl, dashed] \\ C \end{tikzcd}$		

Table 4: Complex attacks for $C \longleftarrow A \Longrightarrow B$, A supports B and A attacks C, when arguments are minimal. The three columns correspond to the three possible kinds of attack: rebutting ($\xrightarrow{r}$), undermining ($\xrightarrow{um}$), and undercutting ($\xrightarrow{uc}$). The rows each correspond to one of the forms of support: conclusion support ($\overset{c}{\Longrightarrow}$), premise support ($\overset{p}{\Longrightarrow}$), intermediate support ($\overset{i}{\Longrightarrow}$), sub-argument support ($\overset{s}{\Longrightarrow}$), general support ($\overset{g}{\Longrightarrow}$), deductive support ($\overset{d}{\Longrightarrow}$) and necessary support ($\overset{n}{\Longrightarrow}$). Blank cells indicate that no complex attacks can be inferred.

$\Rightarrow$ \ $\longrightarrow$	$\xrightarrow{r}$	$\xrightarrow{um}$	$\xrightarrow{uc}$
$\xRightarrow{c}$	$\begin{tikzcd} A \arrow[r, Rightarrow, "c"] \arrow[dr, dashed, "r"'] & B \arrow[d, "r"] \\ & C \end{tikzcd}$	$\begin{tikzcd} A \arrow[r, Rightarrow, "c"] \arrow[dr, dashed, "um"'] & B \arrow[d, "um"] \\ & C \end{tikzcd}$	$\begin{tikzcd} A \arrow[r, Rightarrow, "c"] \arrow[dr, dashed, "uc"'] & B \arrow[d, "uc"] \\ & C \end{tikzcd}$
$\xRightarrow{p}$	$\begin{tikzcd} A \arrow[r, Rightarrow, "p"] \arrow[dr, dashed, "r"'] & B \arrow[d, "r"] \\ & C \end{tikzcd}$ If `Conc`(B) $\in$ `Prem`(B).	$\begin{tikzcd} A \arrow[r, Rightarrow, "p"] \arrow[dr, dashed, "um"'] & B \arrow[d, "um"] \\ & C \end{tikzcd}$ If `Conc`(B) $\in$ `Prem`(B).	$\begin{tikzcd} A \arrow[r, Rightarrow, "p"] \arrow[dr, dashed, "uc"'] & B \arrow[d, "uc"] \\ & C \end{tikzcd}$ If `Conc`(B) $\in$ `Prem`(B).
$\xRightarrow{i}$			
$\xRightarrow{s}$			
$\xRightarrow{g}$	$\begin{tikzcd} A \arrow[r, Rightarrow, "g"] \arrow[dr, dashed] & B \arrow[d] \\ & C \end{tikzcd}$		
$\xRightarrow{d}$	$\begin{tikzcd} A \arrow[r, Rightarrow, "d"] \arrow[dr, dashed] & B \arrow[d] \\ & C \end{tikzcd}$		
$\xRightarrow{n}$			

Table 5: Complex attacks for $A \Longrightarrow B \longrightarrow C$, A supports B and B attacks C, when arguments are minimal. The three columns correspond to the three possible kinds of attack: rebutting ($\xrightarrow{r}$), undermining ($\xrightarrow{um}$), and undercutting ($\xrightarrow{uc}$). The rows each correspond to one of the forms of support: conclusion support ($\xRightarrow{c}$), premise support ($\xRightarrow{p}$), intermediate support ($\xRightarrow{i}$), sub-argument support ($\xRightarrow{s}$), general support ($\xRightarrow{g}$), deductive support ($\xRightarrow{d}$) and necessary support ($\xRightarrow{n}$). Blank cells indicate that no complex attacks can be inferred.

the inferred undermining complex attacks, where the side-conditions rely on the fact that the supported argument is of the form [$\texttt{Conc}(\mathsf{Arg})$]. Finally, this is also the case for complex attacks inferred from the consideration of p-support in Tables 4 and 5, where the supported argument is also deemed to be of the form [$\texttt{Conc}(\mathsf{Arg})$].

5.2. Relating abstract support to structured support for minimal arguments

As expressed at the beginning of this section, the complex attacks defined for BAFs, d-BAFs and AFNs can be considered as determining the nature of the corresponding support relations. This is because, even though d-BAFs and AFNs specify acceptability constraints on the arguments related by deductive and necessary support (respectively, d-support and n-support), those constraints are not explicitly taken into account when establishing what arguments are ultimately accepted. Instead, the constraints are expressed via the characterization of the corresponding complex attacks.

Our search for a correspondence between the structured and abstract forms of support is then based on matching the results summarized in Tables 2–5. First, we look to see if a structured support/attack pair (for example, c-support and rebutting attack) permits the inference of a complex attack in the same scenarios in which an abstract formalism permits such a complex attack to be inferred. Second, we look to see if a structured support/attack pair does not permit the inference of a complex attack in the same scenarios in which an abstract formalism does not permit an attack to be inferred. Only when this match holds across all scenarios — structured and abstract support allow exactly the same inferences in each of the four scenarios in Tables 2–5 — do we have a correspondence.[12] It should be noted that, if we were only to consider a partial correspondence, for instance, check only if a structured attack/support pair allows to infer complex attacks in just one scenario where an abstract formalism does, undesired (also, incorrect) results could be obtained. This is because, when instantiating the abstract formalisms with the structured attack/support pair, the abstract formalism would infer complex attacks that do not make sense in the context of the structured framework. Such is the case of the combination of undercutting attack and c-support, when considering the instantiation of an AFN. Whereas a complex undercutting attack is inferred in Table 4 from the combination of undercutting attack and c-support, as well as in the AFN, the AFN would also infer an undercutting complex attack in a scenario matching the one in Table 2; however, the cell corresponding to the undercutting attack/c-support combination in that table is empty, meaning that no complex attack is inferred in the context of the structured framework.

There are some key features of the abstract formalisms from Section 4 that should be taken into account during our analysis. First, the attack relations of BAFs, d-BAFs and AFNs do not distinguish between different kinds of attack. That is, every attack (either an attack specified in the attack relation or an inferred complex attack) in a BAF (respectively, in a d-BAF or an AFN) is considered to be of the same kind. As a result, when looking for correspondences between abstract and structured complex attacks, we insist that any attack inferred at the structured level is of the same type as the original attack. The reason for doing this is that, if we were to instantiate an abstract formalism with a structured one allowing for different kinds of attack, and we do not distinguish between them at the abstract level, the corresponding abstract formalism may lead us to infer complex attacks that do not make sense when we go back to the structured level. For instance, consider the combination of rebutting attack and p-support in Table 2 with rebut occurring at the final conclusion of A, where a complex undermining attack from C to B is inferred. If we analyze this scenario in the context of a bipolar abstract framework (BAF), a complex attack from C to B would be inferred, but no reference to its nature will be made (because the BAF makes no distinction between different kinds of attack). Hence, when going back to the structured framework where the original attacks and supports occurred, it could be the case that the inferred complex attack is interpreted as a rebutting attack from C to B; however, as illustrated in Table 2, the inferred complex attack should be catalogued as undermining. Another feature that should be taken into account during our analysis is that all attacks in the abstract formalisms are considered to be successful, in contrast with rebutting and undermining attacks in ASPIC$^+$, whose success depends on the preference ordering over arguments. To match this, for the moment we will ignore preferences

[12]Since each table relates to one scenario, the first four rows of a table relate to structured support, and the last three rows of a table relate to abstract support. The search for a correspondence reduces to checking whether there are two rows, one from the first four and the other from the last three, such that there is an entry in one if and only if there is an entry in the other. If this is true for all tables, then a correspondence exists.

between structured arguments, and assume that all attacks succeed. Below, in Section 5.3, when we formalize the results of the analysis performed here, we will consider how to ensure that all attacks succeed.

It can be noted that almost every structured form of support (all but s-support) is such that its combination with rebutting, undermining attack leads to inferring at least one complex attack that comes with side-conditions. Furthermore, many of them are such that, when combined with a particular kind of attack, the inferred complex attack is of a different kind (e.g., the combination of p-support and rebutting attack leads to inferring a complex undermining attack in Table 2). Together, these disqualify c-support, p-support and i-support from being in correspondence with g-support, d-support or n-support.

Let us now turn to analyze s-support. Even though this structured form of support does not suffer from the above mentioned problems, there is still the issue of matching the scenarios in which the abstract formalisms infer complex attacks. In the case of BAFs, complex attacks provide the only available information behind the nature of the g-support relation. Thus, in order to have a correspondence between s-support and g-support, s-support would need to infer complex attacks in the scenarios of both Tables 2 and 5. As a result, since complex attacks for s-support are only inferred in Table 2, such a correspondence would not hold.

Let us now consider d-support and n-support. Recall that the acceptability constraints on the arguments related by them are: given $\mathsf{A} \stackrel{\mathrm{d}}{\Longrightarrow} \mathsf{B}$: "if A is accepted, then B is accepted (conversely, if B is not accepted, then A is not accepted)"; and, given $\mathsf{A} \stackrel{\mathrm{n}}{\Longrightarrow} \mathsf{B}$: "if B is accepted, then A is accepted (conversely, if A is not accepted, then B is not accepted)". By looking at how those acceptability constraints are modeled through the characterization of complex attacks in d-BAFs and AFNs, we note the following. On the one hand, the complex attack for d-support in Table 3 makes it possible to propagate attacks on B to A (thus capturing the fact that if B is not accepted, then A should not be accepted either). Similarly, the complex attack for n-support in Table 2 propagates attacks on A to B (capturing the fact that if A is not accepted, then B should not be accepted either). In contrast, the characterization of complex attacks for d-support in Table 5 and for n-support in Table 4 can be considered to be redundant. This is because, if $\mathsf{A} \stackrel{\mathrm{d}}{\Longrightarrow} \mathsf{B} \longrightarrow \mathsf{C}$ (Table 5), then if A is accepted we also have that B is accepted and thus, given the attack from B, C would not be accepted; therefore, the addition (or not) of a complex attack from A to C does not modify the outcome. Similarly, if $\mathsf{C} \longleftarrow \mathsf{A} \stackrel{\mathrm{n}}{\Longrightarrow} \mathsf{B}$ (Table 4), by having $\mathsf{A} \longrightarrow \mathsf{C}$ in a scenario where B is accepted, in which case A is also accepted, C would not be accepted. Thus, the non-acceptance of C is a consequence of the acceptance of A, and the addition (or not) of a complex attack from B to C also does not modify the outcome.

As a result, since no complex attacks for s-support are inferred in the scenario corresponding to Table 3, we cannot establish a correspondence between s-support and d-support. On the other hand, since s-support infers complex attacks only in Table 2, and this scenario coincides with the one where non-redundant complex attacks for n-support are inferred, we can have a match between s-support and n-support. In fact, as mentioned in Section 4, the original formalization of AFNs (the one in [37]) only considered one kind of complex attack — the kind of attack covered in Table 2. Finally, to fully establish the correspondence between s-support and n-support, we need to account for the fact that the n-support relation of an AFN is irreflexive and transitive (see Definition 18). Then, since s-support relates proper sub-arguments to their super-arguments, it is easy to see that the s-support relation is both irreflexive and transitive.

5.3. Formalizing the relationship between abstract and structured support for minimal arguments

Earlier in this section we discussed the relationship between the forms of support we proposed in Section 3 for structured arguments and those adopted by the abstract formalisms from Section 4. In this subsection, we will formalize that informal analysis.

Given a particular kind of Structured Argumentation Framework (SAF) (see Definition 7), we can define an associated abstract Argumentation Framework with Support (AFS), which corresponds to a BAF, a d-BAF or an AFN having the same arguments and attacks as the SAF, where the support relation of the AFS corresponds to one of the forms of support presented in Section 3. Recall that every attack in the abstract formalisms from Section 4 is a successful attack, and so in our informal discussion of correspondence, we assumed that all attacks between structured arguments were successful. Now, in order to be able to establish a formal correspondence, we again assume that the preference ordering $\preceq$ of the SAF is such that every preference-dependent attack (i.e., every rebutting and undermining attack) succeeds[13]; moreover, we also assume that every argument in the SAF is *minimal*. We capture these assumptions in the notion of a Simple Structured Argumentation Framework:

Definition 23 (Simple Structured Argumentation Framework). *A* Simple Structured Argumentation Framework (SSAF) *is a structured argumentation framework* $\langle \mathcal{A}, \mathsf{Att}, \preceq \rangle$ *such that every argument* $\mathsf{A} \in \mathcal{A}$ *is minimal* ($\nexists \mathsf{A}', \mathsf{A}'' \in \mathtt{Sub}(\mathsf{A})$ *s.t.* $\mathsf{A}' \neq \mathsf{A}''$ *and* $\mathtt{Conc}(\mathsf{A}') = \mathtt{Conc}(\mathsf{A}'')$) *and* $\preceq$ *is such that* $\forall (\mathsf{A}, \mathsf{B}) \in \mathsf{Att} : \mathsf{A}$ *defeats* B.

Then, we can define the *Argumentation Framework with Support (AFS)* associated with a SSAF as follows.

Definition 24 (AFS associated with SSAF for σ-support). *Let* $\mathsf{SSAF} = \langle \mathcal{A}, \mathsf{Att}, \preceq \rangle$ *be a simple structured argumentation framework. We define the* AFS associated with SSAF for σ-support *as* $\langle \mathcal{A}, \mathsf{Att}, \sigma\text{-}\mathsf{Supp} \rangle$*, where* $\sigma\text{-}\mathsf{Supp} = \{(\mathsf{A}, \mathsf{B}) \mid \mathsf{A}, \mathsf{B} \in \mathcal{A}$ *and* A *σ-supports* B *in* $\mathsf{SSAF}\}$ *and* $\sigma \in \{\mathsf{c}, \mathsf{p}, \mathsf{i}, \mathsf{s}\}$.

Note that the support relation of the associated AFS is such that it corresponds to only one of the forms of support between arguments in the SSAF, hence the mention of the AFS being associated with the SSAF for a particular form of support. That is, $\sigma\text{-}\mathsf{Supp}$ will be instantiated with either c-support, p-support, i-support or s-support.

Following the discussion in the previous subsection about the kind of AFN that we can capture with s-support, we modify Definition 19 to give:

Definition 25 (Original Extended Attacks). *Let* $\langle \mathcal{A}, \mathcal{R}, \mathcal{N} \rangle$ *be an argumentation framework with necessities and* $\mathsf{A}, \mathsf{B} \in \mathcal{A}$*. There is an* extended attack *from* A *to* B*, noted as* $\mathsf{A}\mathcal{R}^+\mathsf{B}$*, if there exists* $\mathsf{C} \in \mathcal{A}$ *such that* $\mathsf{A}\mathcal{R}\mathsf{C}$ *and* $\mathsf{C}\mathcal{N}\mathsf{B}$*. The direct attack* $\mathsf{A}\mathcal{R}\mathsf{B}$ *is considered as a particular case of extended attack.*

From now on, we use the term "AFN" to mean an AFN with extended attacks as defined in Definition 19, and "AFN with original extended attacks" to mean an AFN with extended attacks as defined in Definition 25. In the following, whenever we want to refer to an abstract argumentation framework with support that can either be a BAF, a d-BAF or an AFN (with or without the original extended attacks), we will simply refer to it as an AFS. Then, given the AFS associated with a SSAF for σ-support, we characterize the notion of *correct abstraction*, which reflects the fact that the sets of accepted arguments of the SSAF under a given semantics coincide with the sets of accepted arguments of the associated AFS under the same semantics.

Definition 26 (Correct abstraction for σ-support). *Let* $\mathsf{SSAF} = \langle \mathcal{A}, \mathsf{Att}, \preceq \rangle$ *be a simple structured argumentation framework and* $\mathsf{AFS} = \langle \mathcal{A}, \mathsf{Att}, \sigma\text{-}\mathsf{Supp} \rangle$ *its associated abstract argumentation framework for σ-support. We say that* AFS *is a* correct abstraction of SSAF for σ-support *if* $\mathsf{E}_\theta(\mathsf{AF}_{\mathsf{SSAF}}) = \mathsf{E}_\theta(\mathsf{AF}_{\mathsf{AFS}})$ *for any given semantics* θ*, where* $\mathsf{AF}_{\mathsf{SSAF}}$ *is the argumentation framework corresponding to* SSAF *by Definition 9,* $\mathsf{AF}_{\mathsf{AFS}}$ *is the argumentation framework corresponding to* AFS *by Definitions 20–22, and* $\mathsf{E}_\theta(\cdot)$ *is a function returning the set of extensions of an argumentation framework under the semantics* θ.

Then, the following lemma identifies a condition under which an AFS is a correct abstraction of a SSAF for a given form of support.

Lemma 1. *Let* $\mathsf{SSAF} = \langle \mathcal{A}, \mathsf{Att}, \preceq \rangle$ *be a simple structured argumentation framework,* $\Gamma = \langle \mathcal{A}, \mathsf{Att}, \sigma\text{-}\mathsf{Supp} \rangle$ *its associated* AFS *for σ-support, and* $\mathsf{A}, \mathsf{B} \in \mathcal{A}$*. If every complex attack from* A *to* B *in* Γ *is such that* $(\mathsf{A}, \mathsf{B}) \in \mathsf{Att}$*, then* Γ *is a correct abstraction of* SSAF *for σ-support.*

Proof: *By Definition 23, the preference ordering* $\preceq$ *of* SSAF *is such that all attacks in* Att *succeed. Then, by Definition 9,* $\mathsf{AF}_{\mathsf{SSAF}} = \langle \mathcal{A}, \mathsf{Att} \rangle$*. Furthermore, if every complex attack from* A *to* B *in* Γ *is such that* $(\mathsf{A}, \mathsf{B}) \in \mathsf{Att}$*, then by Definitions 20–22,* $\mathsf{AF}_\Gamma = \langle \mathcal{A}, \mathsf{Att} \rangle$*. Therefore, since* $\mathsf{AF}_{\mathsf{SSAF}} = \mathsf{AF}_\Gamma$*, it holds that* $\mathsf{E}_\theta(\mathsf{AF}_{\mathsf{SSAF}}) = \mathsf{E}_\theta(\mathsf{AF}_\Gamma)$ *for any semantics* θ *and, as a result,* Γ *is a correct abstraction of* SSAF *for σ-support.* □

[13] A simple example of such a preference ordering is the preference ordering that assigns every argument the same preference level (in which case all arguments are equally preferred).

As discussed in Section 5.2, the combination of s-support and rebutting, undermining and/or undercutting attack would be suitable for instantiating an AFN with original extended attacks like the one in [37]. This result is formalized by the following lemma and proposition.

Lemma 2. *Let* $\mathsf{SSAF} = \langle \mathcal{A}, \mathsf{Att}, \preceq \rangle$ *be a simple structured argumentation framework and* $\Gamma = \langle \mathcal{A}, \mathsf{Att}, \text{s-}\mathsf{Supp} \rangle$ *its associated* AFS *for s-support. If* Γ *is an* AFN *with original extended attacks, then it holds that every complex attack from* A *to* B *in* Γ *is such that* $(\mathsf{A}, \mathsf{B}) \in \mathsf{Att}$.

Proof: *Let us suppose that there exists a complex attack from* A *to* B *in* Γ. *Since* Γ *is an* AFN *with original extended attacks, by Definition 25 there is only one kind of extended attack in* Γ. *Hence, it should be the case that there exists* $\mathsf{C} \in \mathcal{A}$ *such that* $(\mathsf{A}, \mathsf{C}) \in \mathsf{Att}$ *and* $(\mathsf{C}, \mathsf{B}) \in \text{s-}\mathsf{Supp}$. *Given that* Γ *is the* AFS *associated with* SSAF *for s-support, this would imply that* C *is a proper sub-argument of* B *and that* A *rebuts, undermines or undercuts* C *in* SSAF. *Therefore, by Definition 5, this implies that* A *rebuts, undermines or undercuts* B *in* SSAF, *hence* $(\mathsf{A}, \mathsf{B}) \in \mathsf{Att}$. □

Proposition 6. *Let* $\mathsf{SSAF} = \langle \mathcal{A}, \mathsf{Att}, \preceq \rangle$ *be a simple structured argumentation framework and* $\Gamma = \langle \mathcal{A}, \mathsf{Att}, \text{s-}\mathsf{Supp} \rangle$ *its associated* AFS *for s-support. If* Γ *is an* AFN *with original extended attacks, then it holds that* Γ *is a correct abstraction of* SSAF *for s-support.*

Proof: *Direct from Lemmas 1 and 2.* □

Given a SSAF and its associated AFS for a given form of support, it can be easily shown that if the AFS is an AFN that allows for two kinds of extended attack (as in Definition 19), then it can be the case that a complex attack inferred by the AFS is not included in the original attack relation. A situation like that would occur, for instance, when instantiating the AFS with s-support in a scenario matching the one depicted in Table 4. This is illustrated by the following example.

Example 12. *Let* $\mathsf{AS} = \langle \mathcal{L}, \bar{\cdot}, \mathcal{R}, n \rangle$ *be an argumentation system where:*

$$\mathcal{L} = \{a, b, c, d, \neg a, \neg b, \neg c, \neg d\}$$

$\mathcal{R} = \mathcal{R}_s \cup \mathcal{R}_d$, *with* $\mathcal{R}_s = \{a \to c\}$ *and* $\mathcal{R}_d = \{c \Rightarrow b; d \Rightarrow \neg c\}$. *We then add the knowledge base* $\mathcal{K}$ *such that* $\mathcal{K}_n = \emptyset$ *and* $\mathcal{K}_p = \{a; d\}$ *to get the argumentation theory* $\mathsf{AT} = \langle \mathsf{AS}, \mathcal{K} \rangle$. *From this we can construct the arguments:*

$$\begin{array}{l} \mathsf{A}_1 = [a]; \mathsf{A}_2 = [\mathsf{A}_1 \to c]; \mathsf{A}_3 = [\mathsf{A}_2 \Rightarrow b]; \\ \mathsf{B}_1 = [d]; \mathsf{B}_2 = [\mathsf{B}_1 \Rightarrow \neg c]; \end{array}$$

Let us call this set of arguments $\mathcal{A}$, *so that:* $\mathcal{A} = \{\mathsf{A}_1, \mathsf{A}_2, \mathsf{A}_3, \mathsf{B}_1, \mathsf{B}_2\}$. *From this set of arguments, we have that* A_2 *rebuts* B_2 *(on* B_2*) to make up the set* $\mathsf{Att} = \{(\mathsf{A}_2, \mathsf{B}_2)\}$. *With a preference order* $\preceq$ *defined by* $\mathsf{B}_2 \prec \mathsf{A}_2$, *we have the simple structured argumentation framework* $\mathsf{SSAF} = \langle \mathcal{A}, \mathsf{Att}, \preceq \rangle$. *Then, the AFS associated with* SSAF *for s-support is* $\Gamma = \langle \mathcal{A}, \mathsf{Att}, \text{s-}\mathsf{Supp} \rangle$, *where* $\text{s-}\mathsf{Supp} = \{(\mathsf{A}_1, \mathsf{A}_2), (\mathsf{A}_2, \mathsf{A}_3), (\mathsf{A}_1, \mathsf{A}_3), (\mathsf{B}_1, \mathsf{B}_2)\}$. *Then, if we consider* Γ *to be an* AFN, *by Definition 19 there exists an extended attack from* A_3 *to* B_2 *in* AFN*; however,* $(\mathsf{A}_3, \mathsf{B}_2) \notin \mathsf{Att}$.

Recall that Lemma 1 identifies one condition under which an AFS (in particular, an AFN) would be a correct abstraction of a SSAF for a given form of support. However, it can be the case that an AFN not satisfying that requirement (namely, an AFN considering both kinds of extended attack) is still a correct abstraction of the SSAF. That is, although the condition established in Lemma 1 is a sufficient requirement for identifying a correct abstraction, it is not a necessary condition. Specifically, the following proposition shows that an AFN with two kinds of extended attack is a correct abstraction of a SSAF for s-support.

Proposition 7. *Let* $\mathsf{SSAF} = \langle \mathcal{A}, \mathsf{Att}, \preceq \rangle$ *be a simple structured argumentation framework and* $\Gamma = \langle \mathcal{A}, \mathsf{Att}, \text{s-}\mathsf{Supp} \rangle$ *its associated AFS for s-support. If* Γ *is an* AFN, *then it holds that* Γ *is a correct abstraction of* SSAF *for s-support.*

Proof: *By Definition 23, the preference ordering* $\preceq$ *of* SSAF *is such that all attacks in* Att *succeed. Then, by Definition 9,* $\mathsf{AF}_{\mathsf{SSAF}} = \langle \mathcal{A}, \mathsf{Att} \rangle$ *is the abstract argumentation framework associated with* SSAF. *By Definition 22, since* Γ *is an* AFN, $\mathsf{AF}_{\Gamma} = \langle \mathcal{A}, \mathsf{Att}^{+} \rangle$ *is the abstract argumentation framework* *associated with* Γ, *where* $\mathsf{Att}^+ = \mathsf{Att} \cup \{(\mathsf{A},\mathsf{B}) \mid$ *there exists an extended attack from* A *to* B *in* $\Gamma\}$. *For every extended attack from* A *to* B *in* Γ, *by Definition 19,* $\exists \mathsf{C} \in \mathcal{A}$ *such that either* $(\mathsf{A},\mathsf{C}) \in \mathsf{Att}$ *and* $(\mathsf{C},\mathsf{B}) \in \mathsf{s\text{-}Supp}$; *or* $(\mathsf{C},\mathsf{B}) \in \mathsf{Att}$ *and* $(\mathsf{C},\mathsf{A}) \in \mathsf{s\text{-}Supp}$. *By Lemma 2, every extended attack from* A *to* B *where* $(\mathsf{A},\mathsf{C}) \in \mathsf{Att}$ *and* $(\mathsf{C},\mathsf{B}) \in \mathsf{s\text{-}Supp}$ *is such that* $(\mathsf{A},\mathsf{B}) \in \mathsf{Att}$. *Then, concerning the extended attacks of the second kind we have two cases:* (a) *they are all included in* Att; *or* (b) *there exists an extended attack from* A *to* B *of the second kind such that* $(\mathsf{A},\mathsf{B}) \notin \mathsf{Att}$.

(a) *Suppose that all extended attacks of second kind are included in* Att, *in which case* $\mathsf{Att} = \mathsf{Att}^+$. *Then, since* $\mathsf{AF}_{\mathsf{SSAF}} = \mathsf{AF}_{\Gamma}$, *it holds that* $\mathsf{E}_\theta(\mathsf{AF}_{\mathsf{SSAF}}) = \mathsf{E}_\theta(\mathsf{AF}_{\Gamma})$ *for any semantics* θ *and, as a result,* Γ *is a correct abstraction of* SSAF *for s-support.*
(b) *Suppose that there exists an extended attack from* A *to* B *in* Γ *such that* $(\mathsf{C},\mathsf{B}) \in \mathsf{Att}$, $(\mathsf{C},\mathsf{A}) \in \mathsf{s\text{-}Supp}$ *and* $(\mathsf{A},\mathsf{B}) \notin \mathsf{Att}$. *The existence of that extended attack implies that if* A *is accepted, then* B *cannot be accepted. Then, given an extension* E *of* SSAF *under a semantics* θ, *it holds that if* $\mathsf{A} \in \mathsf{E}$, *then* $\mathsf{C} \in \mathsf{E}$. *This is because* C *is a proper sub-argument of* A *and, by Definition 5, all arguments attacking* C *will also attack* A; *moreover, by Definition 23, these attacks will result in defeats. Similarly, given an extension* E' *of* Γ *under the semantics* θ, *it holds that if* $\mathsf{A} \in \mathsf{E}'$, *then* $\mathsf{C} \in \mathsf{E}'$. *This is because, since* Γ *is an* AFN, C *necessary supports* A *and, by Definition 19, all arguments attacking* C *will also attack* A. *Therefore, since* $(\mathsf{C},\mathsf{B}) \in \mathsf{Att}$, *in either case it will hold that* B *is not accepted (respectively,* $\mathsf{B} \notin \mathsf{E}$ *and* $\mathsf{B} \notin \mathsf{E}'$*). That is, extended attacks of the second kind in* Γ *do not affect the outcome of the framework. As a result,* $\mathsf{E}_\theta(\mathsf{AF}_{\mathsf{SSAF}}) = \mathsf{E}_\theta(\mathsf{AF}_{\Gamma})$ *and* Γ *is a correct abstraction of* SSAF *for s-support.*

□

Finally, the following example shows that the preceding result does not hold if the AFS associated with a SSAF for s-support is a BAF or a d-BAF.

Example 13. *Let* $\mathsf{AS} = \langle \mathcal{L}, \bar{\cdot}, \mathcal{R}, n \rangle$ *be an argumentation system where:*

$$\mathcal{L} = \{a, b, c, d, e\neg a, \neg b, \neg c, \neg d, \neg e\}$$

$\mathcal{R} = \mathcal{R}_s \cup \mathcal{R}_d$, *with* $\mathcal{R}_s = \{d \to \neg b; b \to c\}$ *and* $\mathcal{R}_d = \{a \Rightarrow b; e \Rightarrow \neg c\}$. *We then add the knowledge base* $\mathcal{K}$ *such that* $\mathcal{K}_n = \emptyset$ *and* $\mathcal{K}_p = \{a; d; e\}$ *to get the argumentation theory* $\mathsf{AT} = \langle \mathsf{AS}, \mathcal{K} \rangle$. *From this we can construct the arguments:*

$$\begin{aligned}&\mathsf{A}_1 = [a]; \mathsf{A}_2 = [\mathsf{A}_1 \Rightarrow b]; \mathsf{A}_3 = [\mathsf{A}_2 \to c];\\ &\mathsf{A}_4 = [d]; \mathsf{A}_5 = [\mathsf{A}_4 \to \neg b];\\ &\mathsf{A}_6 = [e]; \mathsf{A}_7 = [\mathsf{A}_6 \Rightarrow \neg c]\end{aligned}$$

Let us call this set of arguments $\mathcal{A}$, *so that:* $\mathcal{A} = \{\mathsf{A}_1, \mathsf{A}_2, \mathsf{A}_3, \mathsf{A}_4, \mathsf{A}_5, \mathsf{A}_6, \mathsf{A}_7\}$. *From this set of arguments, we have that* A_3 *rebuts* A_7 *(on* A_7*),* A_5 *rebuts* A_2 *(on* A_2*), and* A_5 *rebuts* A_3 *(on* A_2*), to make up the set* $\mathsf{Att} = \{(\mathsf{A}_3,\mathsf{A}_7), (\mathsf{A}_5,\mathsf{A}_2), (\mathsf{A}_5,\mathsf{A}_3)\}$. *With a preference order* $\preceq$ *defined by:* $\mathsf{A}_7 \prec \mathsf{A}_3$ *and* $\mathsf{A}_2 \prec \mathsf{A}_5$, *we have the simple structured argumentation framework* $\mathsf{SSAF} = \langle \mathcal{A}, \mathsf{Att}, \preceq \rangle$. *This structured argumentation framework establishes a defeat relation* $\mathsf{Defeats} = \mathsf{Att} = \{(\mathsf{A}_3,\mathsf{A}_7), (\mathsf{A}_5,\mathsf{A}_2), (\mathsf{A}_5,\mathsf{A}_3)\}$. *With this, we can finally obtain the argumentation framework* $\mathsf{AF}_{\mathsf{SSAF}} = \langle \mathcal{A}, \mathsf{Defeats} \rangle$ *associated with* SSAF.

Let $\Gamma = \langle \mathcal{A}, \mathsf{Att}, \mathsf{s\text{-}Supp} \rangle$ *be the* AFS *associated with* SSAF *for s-support, where* $\mathsf{s\text{-}Supp} = \{(\mathsf{A}_1,\mathsf{A}_2), (\mathsf{A}_2,\mathsf{A}_3), (\mathsf{A}_1,\mathsf{A}_3), (\mathsf{A}_4,\mathsf{A}_5), (\mathsf{A}_6,\mathsf{A}_7)\}$:

- *If* Γ *is a* BAF, *then we can infer the following complex attacks: the supported attacks* $\mathsf{Att}_{su} = \{(\mathsf{A}_4,\mathsf{A}_2), (\mathsf{A}_4,\mathsf{A}_3), (\mathsf{A}_1,\mathsf{A}_7), (\mathsf{A}_2,\mathsf{A}_7)\}$ *and the secondary attacks* $\mathsf{Att}_{se} = \{(\mathsf{A}_5,\mathsf{A}_3)\}$.
- *If* Γ *is a* $\mathsf{d\text{-}BAF}$, *then we can infer the following complex attacks: the supported attacks* $\mathsf{Att}_{su} = \{(\mathsf{A}_4,\mathsf{A}_2), (\mathsf{A}_4,\mathsf{A}_3), (\mathsf{A}_1,\mathsf{A}_7), (\mathsf{A}_2,\mathsf{A}_7)\}$ *and the mediated attacks* $\mathsf{Att}_{me} = \{(\mathsf{A}_5,\mathsf{A}_1), (\mathsf{A}_5,\mathsf{A}_2), (\mathsf{A}_3,\mathsf{A}_6)\}$.

Given the set of arguments $\mathcal{A}$ *and all the attacks in* Γ *(those in* Att *plus the inferred complex attacks, depending on whether* Γ *is a* BAF *or a* $\mathsf{d\text{-}BAF}$*), we can obtain its associated AF. Let* $\mathsf{AF}_{\Gamma_{\mathsf{BAF}}} = \langle \mathcal{A}, \mathsf{Att} \cup \mathsf{Att}_{su} \cup \mathsf{Att}_{se} \rangle$ *be the AF associated with* Γ *in the case where it is a* BAF. *Similarly, let* $\mathsf{AF}_{\Gamma_{\mathsf{dBAF}}} \langle \mathcal{A}, \mathsf{Att} \cup \mathsf{Att}_{su} \cup \mathsf{Att}_{me} \rangle$ *be the AF associated with* Γ *in the case where it is a* $\mathsf{d\text{-}BAF}$.

Finally, for instance, if we consider the preferred semantics we obtain the following sets of preferred extensions for $\mathsf{AF}_{\mathsf{SSAF}}$, $\mathsf{AF}_{\Gamma_{\mathsf{BAF}}}$ *and* $\mathsf{AF}_{\Gamma_{\mathsf{dBAF}}}$: $\mathsf{E}_{pr}(\mathsf{AF}_{\mathsf{SSAF}}) = \{\{\mathsf{A}_1, \mathsf{A}_4, \mathsf{A}_5, \mathsf{A}_6, \mathsf{A}_7\}\}$, $\mathsf{E}_{pr}(\mathsf{AF}_{\Gamma_{\mathsf{BAF}}}) =$

$\{\{A_1, A_4, A_5, A_6\}\}$ *and* $E_{pr}(AF_{\Gamma_{dBAF}}) = \{\{A_4, A_5, A_6, A_7\}\}$. *Note that, even though each one of these sets contains a unique preferred extension,* Γ *is not a correct abstraction of* SSAF *for s-support in the cases where it is a* BAF *or a* d-BAF. *This is because* $AF_{\Gamma_{BAF}}$ *does not accept* A_7 *under the preferred semantics (which belongs to the preferred extension of* AF_{SSAF}*), whereas* $AF_{\Gamma_{dBAF}}$ *does not accept* A_1 *under the preferred semantics (which belongs to the preferred extension of* AF_{SSAF}*).*

Finally, note that the results in this section are based on a correspondence between a SSAF and its associated AFS for a given form of support. Thus, they apply to a wide range of semantics, including those based on the notion of acceptability such as complete, preferred, ideal and grounded semantics, as well as to other semantics such as stable and semi-stable, among others.

5.4. Summary

In this section, we looked for relationships between the versions of support for ASPIC$^+$ that we identified in Section 3 and different abstract notions of support that can be found in the literature, which we summarized in Section 4. We did this by considering which complex attacks identified by the abstract notions of support (which, in turn, determine the nature of the corresponding support relations) are captured by the different notions of support in ASPIC$^+$. Since complex attacks involve both support and attack, we thus considered all possible combinations of support and attack in ASPIC$^+$ and looked for combinations in which the resulting complex attacks corresponded exactly with the complex attacks from the different abstract supports. Section 5.2 provided this analysis for ASPIC$^+$ arguments that are minimal, and this analysis was then formalized in Section 5.3.

The overall conclusion is that while there are kinds of support in ASPIC$^+$ that capture some aspects of the abstract notions of support, and some abstract notions of support that capture elements of the supports we identified in ASPIC$^+$ — that is, there are some partial matches in Tables 2–5 — there is only one proper correspondence: that is between s-support (and any form of attack) and the n-support relation of AFNs with or without original extended attacks. Furthermore, we note that our results are highly dependent on the structure of arguments and the way in which attacks are defined in ASPIC$^+$. Hence, if we were to consider a different structured argumentation system, we might obtain other results regarding the correspondence between structured and abstract forms of support[14].

6. Related work

In this paper, we have studied different notions of support between arguments in the context of structured argumentation systems. Even though the notion of support was present in the literature of argumentation since its foundations [55], later studies on argumentation systems, especially those which followed Dung [25] in studying abstract argumentation, put aside the notion of support to focus on the notion of attack. Notwithstanding this, in the last decade or so, the study of support between arguments has regained attention among the researchers of the area. Indeed, as [26] points out, support and attack are both important elements in argumentation, and can be either modelled as complementary notions, as in bipolar argumentation frameworks, or can be handled separately, with support implicit in the construction of structured arguments (as we have demonstrated, notions of support emerge from the way that arguments are built) and attack between arguments handled explicitly.

The complementarity between support and attack is considered in depth in [3], where the authors discuss the existence of bipolarity in the different steps of the argumentation process. In particular, when considering the construction of arguments, they argue that bipolarity appears in this step of the argumentation process in the form of arguments in favor of a conclusion and arguments against that conclusion. Then, they state that the role of arguments in favor of a conclusion is to *support* that conclusion, whereas the role of arguments against a conclusion is to *attack* it. It is important to note that the mention of *support* in the building of arguments in [3] is used in a different way than we have used it throughout this paper. That is, the authors of [3] take support as a relation linking arguments to conclusions, whereas we take support as a relation *between arguments*. In contrast, when considering bipolarity at the argument interaction level, [3] refers to Bipolar Argumentation Frameworks, where

[14]Indeed, an earlier version of this paper studied a model of argumentation that was closer to that of [43] and did obtain different results in which a version of c-support came closest to mapping to abstract support.

support and attack relations between arguments exist, treating support in the same way as we do in this paper (i.e., as relating pairs of arguments).

In [57, 58], Verheij proposed DefLog, a theory of dialectical argumentation. Using DefLog as a starting point, a reconstruction of Toulmin's ideas was provided in [59, 61], allowing for the representation of the elements of Toulmin's scheme, as well as the support and attack links between them. Following the spirit of [59, 61], in [20, 21], the authors addressed the notion of support based on Toulmin's model for the layout of arguments. Specifically, they considered the support that Toulmin's backings provide for their associated warrants, both in the context of abstract argumentation and Defeasible Logic Programming. It is important to note that the notion of support used in [20] and [21] is closely related to the notion of undercutting defeater proposed by Pollock [45]. In particular, in [20, 21], the authors regard a backing argument as providing support for another argument's inference step, aiming to defend it against possible undercutting defeaters. Then, in [22], the authors showed that it is possible to instantiate the abstract argumentation framework proposed in [21] with the structured argumentation system presented in [20]. Therefore, [22] shares one of the goals of our work, which consists of trying to bridge the gap between the notions of support at the abstract and structured levels.

In the last decade, several interpretations for the notion of support have been proposed (see [19, 23]). The first of these proposals was developed in the context of the Bipolar Argumentation Framework (BAF) [15, 16, 17, 18, 19], which provides a general approach to support in abstract argumentation where the support relation between arguments is left abstract, just like the notion of attack in Dung-style frameworks. The line of research on BAFs clearly motivated later work on the study of the notion of support (while leaving aside the previous work on support in argumentation that was outlined above). The introduction of BAFs was followed by several approaches to abstract argumentation with different interpretations of support such as deductive support [6, 62] and necessary support [8, 38].

The different approaches to support in abstract argumentation define a series of *complex attacks* that enforce the acceptability constraints they impose on the arguments related by the support relation. In Section 5, we showed that some complex attacks defined for the general support relation of [15], the deductive support of [6] and the necessary support of [8, 38] also occur when considering the different notions of support for structured arguments defined in this paper. Furthermore, we showed that by instantiating the Abstract Argumentation Framework with Necessities (AFN) with rebutting, undermining or undercutting attacks and the sub-argument support (s-support) relation of a Simple Structured Argumentation Framework (SSAF), the resulting AFN is a correct abstraction of the SSAF for s-support. In other words, we showed that both frameworks lead to obtaining the same outcome when considering the acceptability of arguments.

The most obvious difference between the work on support in abstract argumentation and the work in this paper is that we are concerned with the internal structure of arguments. This concern is shared with Prakken's work in [50], where he considered the possibility of instantiating abstract forms of support using ASPIC$^+$. It can be noted that Prakken's paper is complementary to ours. This is because he started by considering abstract forms of support and then tried to match those using ASPIC$^+$; in contrast, we started by considering different forms of support between structured arguments in ASPIC$^+$, and then exploring the relationship between the structured forms of support and those existing in the literature for abstract argumentation.

On the one hand, in [50], Prakken defines a simple abstract argumentation framework with support called SuppAF that incorporates a binary support relation with the constraint that if B supports C and A attacks B, then A also attacks C. Then, he shows that SuppAF can be instantiated with ASPIC$^+$ by considering ASPIC$^+$ defeats between arguments and its sub-argument relation. This result aligns with the correspondence we identified in Section 5, where we formally showed that an AFN (with or without original extended attacks) is a correct abstraction of an ASPIC$^+$ Simple Structured Argumentation Framework (SSAF) for s-support.

On the other hand, Prakken [50] analyzes whether the BAF and the Evidential Argumentation System (EAS) [39] can be instantiated using ASPIC$^+$. In the case of the BAF, [50] claims that such an instantiation is not possible since the BAF accounts for the existence of supported attacks (see Definition 16), which lead to counterintuitive results when considering ASPIC$^+$ sub-argument relation. Again, this non-correspondence between s-support and g-support was also determined by our analysis in Section 5. Then, in [50], it is shown that an EAS can be instantiated with ASPIC$^+$. In particular, this was proven for preferred semantics only since, as remarked in [50], that is the semantics on which [39] concentrates.

Although our work is similar to [50] in that they both consider the instantiation of abstract forms of support with more concrete ones, there are several aspects that distinguish them. As mentioned before, Prakken's work starts by considering three forms of abstract support (namely, the support relations of the SuppAF framework, the BAF and the EAS) and analyzes whether they can be instantiated with ASPIC^{+} sub-argument relation. In contrast, our approach begins with the study of various notions of support in structured argumentation and the relationships between them, and then moves to analyze whether there is any correspondence between the notions of structured support we propose here and several abstract notions of support. In other words, our work goes further in identifying forms of support in structured argumentation and in relating those forms of support to one another. Specifically, whereas Prakken only considers the s-support relation between sub-arguments and their super arguments, our analysis also accounts for the existence of c-support, p-support and i-support. In addition, there is also a difference between the abstract forms of support we considered. Similarly to [50], we consider the g-support relation of the BAF; however, our analysis again goes further [50] because it also takes into account the deductive support (d-support) of the d-BAF and the necessary support (n-support) of the AFN.

As the previous paragraph highlights, we have not studied the relationship between our notions of support for structured arguments and the support relation of the EAS [39]. This is because, unlike the other abstract formalisms we considered and introduced in Section 4, the EAS considers that arguments should be backed up by evidence in order to be considered as "active". Hence, the support relation of the EAS is intended to capture the intuition that an argument cannot be accepted unless it is supported by evidence from the environment. Thus, an argument in an EAS will be acceptable if it is defended against attacks coming from arguments supported by evidence, and the argument itself is backed up by evidence.

Another area of research that is connected to support in argumentation is work on Abstract Dialectical Frameworks (ADFs). ADFs were first proposed in [10]. Briefly, an ADF is a directed graph, whose nodes represent arguments which can be accepted or not, and the links between the nodes represent dependencies. Each argument X in the graph is associated with an acceptance condition $\mathsf{C}(\mathsf{X})$, which is some propositional function whose truth status is determined by the corresponding values of the acceptance conditions for those arguments Y such that (Y, X) is a link in the ADF (i.e., Y is a parent of X). As a result, ADFs allow for the representation of different dependencies between arguments. For instance, there can be nodes which are rejected unless they are supported by some accepted nodes (leading to supporting links); also, there can be links of different strength, and even links which support or attack a node depending on the context.

Regarding the notion of support, the authors in [10] state that the acceptance conditions in ADFs are more flexible than the constraints described by specific interpretations of support such as deductive, necessary or evidential support. However, in spite of the flexibility provided by the acceptance conditions, since the status of an argument in an ADF (a node in a graph) depends only on the statuses of its parents, ADFs might not be able to capture the acceptability constraints imposed by specific interpretations of support. For example, given the deductive support of [6], if A supports B and C attacks B, then there exists a mediated attack from C to A. An ADF that depicts this situation would have the nodes A, B and C, and the links (A, B) and (C, B). However, no acceptance condition in the ADF would be capable of ensuring that A is not accepted whenever C is accepted, since there would be no link between A and C in the ADF. This is analogous for any of the complex attacks depicted in Tables 2–5, which result from the combination of different attack and support relations.

A revisited approach to ADFs was proposed in [9], where the authors state that the acceptance conditions specify the parents a node depends on implicitly. Then, they do not require to give the links in the graph explicitly and thus, an ADF can be simply specified by determining the nodes and their associated acceptance conditions. Hence, the links between nodes are inferred from the corresponding acceptance conditions. By applying the revisited approach to the scenario described above, the ADF would be able to model that A is not accepted whenever C is accepted (as expressed by the mediated attack from C to A) by formulating the acceptance condition accordingly. However, this would mean that there was a link between A and C in the ADF, corresponding to a different situation from the one in which the only links between nodes were (A, B) (support) and (C, B) (attack).

From existing work in the literature that addresses support in argumentation systems, we can note that most, and, especially, most of the most recent are abstract argumentation approaches. The study of different kinds of support in structured argumentation, as proposed here and in [22, 50], provides an important counter-point which helps us to understand the meaning behind the existing abstract notions of support.

Finally, although some of the notions of support we have addressed are not entirely new (since they have been present in the literature of argumentation), they have not been explicitly addressed as support relations so far. Such is the case of the sub-argument relation present in structured and abstract formalisms like [28, 49, 34] which, prior to [23], had not been accounted for by the literature of bipolar argumentation systems. Similarly, [28] and [49] also address the existence of premises and consider the existence of attacks on those premises; the former by allowing for undermining attacks on premises that are not axioms, and the by latter allowing for undermining attacks on presumptions. However, none of them explicitly accounted for the existence of a support relation that involves the premises of arguments.

7. Conclusion

We began this paper with the aim of identifying what support, as addressed in abstract argumentation approaches like [6, 15, 38], could mean for structured argumentation. Given our interest in constructing practical systems that reason using argumentation, as discussed in [54], this kind of understanding of support is essential — without knowing what support is in concrete logical terms, we cannot identify support relations between such arguments.

We identified four different ideas of support for structured arguments. The first two of them (namely, c-support and p-support) make reference only to the premises and conclusion of arguments. The remaining two forms of support (i-support and s-support), on the other hand, account for elements that connect premises to conclusion of an argument. Specifically, they account for the existence of sub-arguments that may lead to obtaining intermediate conclusions. (The elements distinguished by the different forms of argument provide points at which attacks may be made, and, in our formulation, identify propositions that can be the basis of support.) Characterizing these four forms of support, discussing some of their properties, and showing how they relate to one another is the first contribution of this paper.

We should note that the notions of attack and support for structured arguments that we are proposing here, are independent in the following sense. Given arguments A and B, it can be the case that A attacks (respectively, supports) B even though there is no supporting (respectively, attacking) argument for B. However, the existence of an attack link (respectively, a support link) from A to B does not prevent the existence of a support link (respectively, an attack link) between them. Notwithstanding this, we should also note that some forms of attack used in this paper have their counterpart in the support relation. That is, where a rebutting argument attacks the final conclusion of another argument, a c-supporting argument provides an alternative way for deriving that conclusion. The same occurs if a rebutting argument attacks an intermediate conclusion of another argument, where an i-supporting argument would provide an alternative way for deriving that intermediate conclusion. Similarly, where an undermining attack provides a reason against a premise of an argument, a p-supporting argument provides support for it. Thus, we can establish a duality between rebutting attack and c-support or i-support, and between undermining attack and p-support.

In Section 5, we analyzed how the notions of support we propose here relate to different notions of support present in the literature of abstract argumentation: the general support relation (g-support) of [15], the deductive support (d-support) of [6], and necessary support (n-support) of [8, 38]. There are two main results from our analysis.

The first is that, of the kinds of support that we established in Section 3, only s-support, where one argument supports another by being a proper sub-argument of it, has a corresponding notion of support at the abstract level; namely, the n-support relation of the AFN. Even this match is somewhat unsatisfactory because some complex attacks inferred by the AFN (the extended attacks of the second kind, where C attacks B and C supports A, meaning that A attacks B) are not related to s-support. Nevertheless, it was shown that such extended attacks do not affect the outcome of the framework, allowing for the AFN to be a correct abstraction of the SSAF for s-support. Furthermore, the s-support relation is irreflexive and transitive like the necessary support relation of AFNs. As a result, given the correspondence between s-support and n-support, it is possible to instantiate an AFN with a rebutting, undermining or undercutting attack relation and the s-support relation between ASPIC$^+$arguments. The second main result of our analysis of the correspondence between abstract and structured notions of support is that s-support and n-support is the *only* pair for which a correspondence exists. This is a strong argument that the existing literature on support in abstract argumentation fails to capture most kinds of support that seem very natural when considering structured arguments. Together, these two contributions go further than any existing work in analyzing support in structured argumentation and its relationship with abstract notions of support.

The results from our analysis suggest that more work is required in two directions. First, we need to further investigate if it is possible to map existing notions of support at the abstract level into support between structured arguments, and if so, how this may be achieved. The abstract view of argumentation is useful for studying different properties about concrete instantiations of arguments — Dung's semantics [25] are helpful because they provide a way to extract justified conclusions from an inconsistent propositional knowledge base.[15] Hence, ideas which work at the abstract level but do not have a sensible instantiation at the concrete (structured) level are less useful. This seems to be the case with some forms of support. We believe that looking for concrete instantiations of existing abstract notions of support will help to determine which abstract forms of support are useful at the structured level and which are not. Second, we need to identify if there are abstract notions of support which capture the intuitions behind the structured notions of support we have presented here. This is the dual of the study proposed above, and if we can characterize our concrete notions of support at the abstract level, then we can make use of existing work to characterize their properties in comparison to those existing for abstract systems. In particular, since our analysis only accounted for the general support from BAFs, the necessary support from AFNs and the deductive support of d-BAFs, future studies could consider (among other things) the evidential support of [39] or the backing interpretation of [21], which have shown to be somehow related to other abstract notions of support.

Finally, it would be interesting to study the relationship between the notions of support we proposed here and other relationships proposed in the literature of argumentation. Specifically, the notion of argument accrual [5, 48, 56] could be related to the notion of support between arguments. Briefly, accrual is based on the intuitive idea that having more reasons for a given conclusion makes such a conclusion more credible, so one might imagine that having more support for an argument makes its conclusion more credible. However, it is important to note that the kind of support we have considered in this paper is different from the support expressed by argument accrual [5, 48, 56], since in the underlying formalism, conclusions do not become more credible when they have more support. Notwithstanding this, accrual is an important topic for argumentation, and our account of support in structured argumentation will remain incomplete until we can relate our results to existing work on accrual. This is another topic for future work.

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[15] The procedure is: 1) construct arguments; 2) detect attack relations between arguments; 3) select a semantics (grounded, preferred, etc); and 4) compute which arguments are acceptable given the semantics. The conclusions of the acceptable arguments are then the justified conclusions.

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