Two forms of minimality in ASPIC^{+}

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Abstract. Many systems of structured argumentation explicitly require that the facts and rules that make up the argument for a conclusion be the minimal set required to derive the conclusion. ASPIC^{+} does not place such a requirement on arguments, instead requiring that every rule and fact that are part of an argument be used in its construction. Thus ASPIC^{+} arguments are minimal in the sense that removing any element of the argument would lead to a structure that is not an argument. In this paper we discuss these two types of minimality and show how the first kind of minimality can, if desired, be recovered in ASPIC^{+}.

1 Introduction

A large part of the work on computational argumentation is concerned with *structured*, or *logic-based* argumentation. In this work, much of the focus is on the way that arguments are constructed from some set of components, expressed in some logic. At this point, perhaps the most widely studied system of structured argumentation is ASPIC^{+}, which builds on what is now quite a lengthy tradition, a tradition which goes back at least as far as [10]. In addition to Pollock's work on OSCAR [9, 11], we can count the work of Loui [7], Krause *et al.* [6], Prakken and Sartor [13], Besnard and Hunter [2], Amgoud and Cayrol [1], García and Simari [5] and Dung *et al.* [4] as being in the same lineage. ASPIC^{+} [8, 12] is more recent, but very influential, providing a very general notion of argumentation that captures many of the structured systems which precede it. In all these systems, there is, often explicitly, a notion of an argument as a pair $\langle \Delta, c \rangle$ which relates the conclusion of the argument, c, and the set of statements Δ from which that conclusion is derived. The form of derivation, and what these "statements" consist of, are two of the aspects of these systems which vary widely.

One difference between ASPIC^{+} and other systems of structured argumentation is that many of the latter require that arguments be minimal in the sense that the set Δ in any argument $\langle \Delta, c \rangle$ has to be minimal. That is, Δ has to be the smallest set from which c can be derived. We can find this explicitly expressed, for example, in [1, 2, 5]. In contrast, like the assumption-based system from [4], ASPIC$^+$ does not explicitly require arguments to be minimal in this sense. Instead ASPIC$^+$ arguments satisfy a different form of minimality in which arguments cannot include premises or rules that are not used in the derivation of their conclusion. In recent work using ASPIC$^+$[3], we discovered some cases in which the difference between these two forms of minimality was important, and so needed to investigate those differences in the context of ASPIC$^+$. In this paper we report our findings.

Note that while the first form of minimality is stronger than the native minimality of ASPIC$^+$, because there are ASPIC$^+$ arguments that are not minimal in this sense, this form of minimality is completely compatible with ASPIC$^+$, and indeed with assumption-based argumentation (which shares the same mechanism for defining an argument). As we show, when the stronger form of minimality is required, we can simply invoke a definition for arguments in ASPIC$^+$ which does require this form of minimality.

The rest of the paper is organized as follows. In Section 2 we introduce background notions from ASPIC$^+$. Then, in Section 3 we discuss the native form of minimality of arguments in ASPIC$^+$, and propose two equivalent ways of providing a stronger notion of minimality, which prevents redundancy and circularity in arguments. Section 3 also includes formal results regarding the characterization of arguments in ASPIC$^+$, as well as relating the forms of minimality we proposed. Later, in Section 4, we analyze related work, and finally, in Section 5, we draw some conclusions and comment on future lines of work.

2 Background

ASPIC$^+$ is deliberately defined in a rather abstract way, as a system with a minimal set of features that can capture the notion of argumentation. This is done with the intention that it can be instantiated by a number of concrete systems that then inherit all of the properties of the more abstract system. ASPIC$^+$ starts from a logical language $\mathcal{L}$ with a notion of negation. A given instantiation will then be equipped with inference rules, and ASPIC$^+$ distinguishes two kinds of inference rules: strict rules and defeasible rules. Strict rules, denoted using $\rightarrow$, are rules whose conclusions hold without exception. Defeasible rules, denoted $\Rightarrow$, are rules whose conclusions hold unless there is an exception.

The language and the set of rules define an *argumentation system*:

Definition 1 (Argumentation System [8]). *An* argumentation system *is a tuple* $AS = \langle \mathcal{L}, \bar{\cdot}, \mathcal{R}, n \rangle$ *where:*

- *$\mathcal{L}$ is a logical language.*
- *$\bar{\cdot}$ is a function from $\mathcal{L}$ to $2^{\mathcal{L}}$, such that:*
  - *φ is a* contrary *of ψ if $\varphi \in \overline{\psi}$, $\psi \notin \overline{\varphi}$;*
  - *φ is a* contradictory *of ψ if $\varphi \in \overline{\psi}$, $\psi \in \overline{\varphi}$;*
  - *each $\varphi \in \mathcal{L}$ has at least one contradictory.*
- *$\mathcal{R} = \mathcal{R}_s \cup \mathcal{R}_d$ is a set of strict ($\mathcal{R}_s$) and defeasible ($\mathcal{R}_d$) inference rules of the form $\phi_1, \ldots, \phi_n \rightarrow \phi$ and $\phi_1, \ldots, \phi_n \Rightarrow \phi$ respectively (where ϕ_i, ϕ are meta-variables ranging over wff in $\mathcal{L}$), and $\mathcal{R}_s \cap \mathcal{R}_d = \emptyset$.*

– $n : \mathcal{R}_d \mapsto \mathcal{L}$ *is a naming convention for defeasible rules.*

The function $\bar{\cdot}$ generalizes the usual symmetric notion of negation to allow non-symmetric conflict between elements of $\mathcal{L}$. The contradictory of some $\varphi \in \mathcal{L}$ is close to the usual notion of negation, and we denote that φ is a *contradictory* of ψ by "$\varphi = \neg\psi$". Note that, given the characterization of $\bar{\cdot}$, elements in $\mathcal{L}$ may have multiple contraries and contradictories. The naming convention for defeasible rules is necessary because there are cases in which we want to write rules that deny the applicability of certain defeasible rules. Naming the rules, and having those names be in $\mathcal{L}$ makes it possible to do this, and the denying applicability makes use of the contraries of the rule names.

An argumentation system, as defined above, is just a language and some rules which can be applied to formulae in that language. To provide a framework in which reasoning can happen, we need to add information that is known, or believed, to be true. In ASPIC$^+$, this information makes up a *knowledge base*:

Definition 2 (Knowledge Base [8]). *A* knowledge base *in an argumentation system* $\langle \mathcal{L}, \bar{\cdot}, \mathcal{R}, n \rangle$ *is a set* $\mathcal{K} \subseteq \mathcal{L}$ *consisting of two disjoint subsets* $\mathcal{K}_n$ *and* $\mathcal{K}_p$*.*

We call $\mathcal{K}_n$ the axioms and $\mathcal{K}_p$ the ordinary premises. We make this distinction between the elements of the knowledge base for the same reason that we make the distinction between strict and defeasible rules. We are distinguishing between those elements — axioms and strict rules — which are definitely true and allow truth-preserving inferences to be made, and those elements — ordinary premises and defeasible rules — which can be disputed.

Combining the notions of argumentation system and knowledge base gives us the notion of an *argumentation theory*:

Definition 3 (Argumentation Theory [8]). *An* argumentation theory *AT is a pair* $\langle AS, \mathcal{K} \rangle$ *of an argumentation system AS and a knowledge base* $\mathcal{K}$*.*

We are now nearly ready to define an argument. But first we need to introduce some notions which can be defined just understanding that an argument is made up of some subset of the knowledge base $\mathcal{K}$, along with a sequence of rules, that lead to a conclusion. Given this, $\texttt{Prem}(\cdot)$ returns all the premises, $\texttt{Conc}(\cdot)$ returns the conclusion and $\texttt{TopRule}(\cdot)$ returns the last rule in the argument. $\texttt{Sub}(\cdot)$ returns all the sub-arguments of a given argument, that is all the arguments that are contained in the given argument.

Definition 4 (Argument [8]). *An* argument *A from an argumentation theory* $AT = \langle\langle \mathcal{L}, \bar{\cdot}, \mathcal{R}, n \rangle, \mathcal{K}\rangle$ *is:*

1. ϕ *if* $\phi \in \mathcal{K}$ *with:* $\texttt{Prem}(A) = \{\phi\}$; $\texttt{Conc}(A) = \{\phi\}$; $\texttt{Sub}(A) = \{A\}$; *and* $\texttt{TopRule}(A) =$ *undefined.*
2. $A_1, \ldots, A_n \rightarrow \phi$ *if* A_i, $1 \leqslant i \leqslant n$, *are arguments and there exists a strict rule of the form* $\texttt{Conc}(A_1), \ldots, \texttt{Conc}(A_n) \rightarrow \phi$ *in* $\mathcal{R}_s$. $\texttt{Prem}(A) = \texttt{Prem}(A_1) \cup \ldots \cup \texttt{Prem}(A_n)$; $\texttt{Conc}(A) = \phi$; $\texttt{Sub}(A) = \texttt{Sub}(A_1) \cup \ldots \cup \texttt{Sub}(A_n) \cup \{A\}$; *and* $\texttt{TopRule}(A) = \texttt{Conc}(A_1), \ldots, \texttt{Conc}(A_n) \rightarrow \phi$.

3. $A_1, \ldots, A_n \Rightarrow \phi$ *if* A_i, $1 \leqslant i \leqslant n$, *are arguments and there exists a defeasible rule of the form* $\texttt{Conc}(A_1), \ldots, \texttt{Conc}(A_n) \Rightarrow \phi$ *in* $\mathcal{R}_d$. $\texttt{Prem}(A) = \texttt{Prem}(A_1) \cup \ldots \cup \texttt{Prem}(A_n)$*;* $\texttt{Conc}(A) = \phi$*;* $\texttt{Sub}(A) = \texttt{Sub}(A_1) \cup \ldots \cup \texttt{Sub}(A_n) \cup \{A\}$*; and* $\texttt{TopRule}(A) = \texttt{Conc}(A_1), \ldots, \texttt{Conc}(A_n) \Rightarrow \phi$*.*

We write $\mathcal{A}(AT)$ *to denote the set of arguments from the theory* AT*.*

In other words, an argument is either an element of $\mathcal{K}$, or it is a rule and its conclusion such that each premise of the rule is the conclusion of an argument. From here on, we will use the symbol $\rightsquigarrow$ when we do not care about distinguishing whether an argument uses a strict rule $\rightarrow$ or a defeasible rule $\Rightarrow$. Thus, if we are making a statement about an argument $A = [B \rightsquigarrow a]$, then we are making a statement about both arguments $A' = [B \rightarrow a]$ and $A'' = [B \Rightarrow a]$. Similarly, when referring to a rule $a \rightsquigarrow b$, we are referring to both a strict rule $a \rightarrow b$ and a defeasible rule $a \Rightarrow b$.

The above is a standard presentation of an argument in ASPIC$^+$. In this paper we wish to refer to an additional element of an argument, and to describe an argument in a somewhat different way. In particular, we wish to refer to $\texttt{Rules}(A)$, which identifies the set of all the strict and defeasible rules used in the argument A.

Definition 5 (Argument Rules). *Let* $AT = \langle AS, \mathcal{K} \rangle$ *be an argumentation theory and* $A \in \mathcal{A}(AT)$*. We define the set of rules of* A *as follows:*

$$\texttt{Rules}(A) = \begin{cases} \varnothing & A \in \mathcal{K} \\ \{\texttt{TopRule}(A)\} \cup \bigcup_{i=1}^{n} \texttt{Rules}(A_i) & A = [A_1, \ldots, A_n \rightsquigarrow \texttt{Conc}(A)] \end{cases}$$

We can then describe an argument A as a triple (G, R, c), where $G = \texttt{Prem}(A)$ are the grounds on which A is based, $R = \texttt{Rules}(A)$ is the set of rules that are used to construct A from G, and $c = \texttt{Conc}(A)$ is the conclusion of A.

Example 1. Consider that we have an argumentation system $AS_1 = \langle \mathcal{L}_1, \bar{\cdot}, \mathcal{R}_1, n \rangle$, where $\mathcal{L}_1 = \{p, q, r, s, t, u, v, \neg p, \neg q, \neg r, \neg s, \neg t, \neg u, \neg v\}$, $\mathcal{R}_1 = \{p, q \rightsquigarrow r; t, u \rightsquigarrow r; r \rightsquigarrow s; u \rightsquigarrow v\}$. By adding the knowledge base $\mathcal{K}_1 = \{p, q, t, u\}$ we obtain the argumentation theory $AT_1 = \langle AS_1, \mathcal{K}_1 \rangle$, from which we can construct the following arguments:

$$A_1 = [p]; A_2 = [q]; A_3 = [A_1, A_2 \rightsquigarrow r]; A = [A_3 \rightsquigarrow s];$$
$$B_1 = [t]; B_2 = [u]; B_3 = [B_1, B_2 \rightsquigarrow r]; B = [B_3 \rightsquigarrow s]$$

such that $A_1 = (\{p\}, \varnothing, p)$, $A_2 = (\{q\}, \varnothing, q)$, $A_3 = (\{p, q\}, \{p, q \rightsquigarrow r\}, r)$, $A = (\{p, q\}, \{p, q \rightsquigarrow r; r \rightsquigarrow s\}, s)$, $B_1 = (\{t\}, \varnothing, t)$, $B_2 = (\{u\}, \varnothing, u)$, $B_3 = (\{t, u\}, \{t, u \rightsquigarrow r\}, r)$ and $B = (\{t, u\}, \{t, u \rightsquigarrow r; r \rightsquigarrow s\}, s)$.

3 Minimality

Now, as mentioned above, unlike some definitions of arguments in the literature — for example [1, 5] — Definition 4 does not impose any minimality requirement on the grounds or the set of rules. However, this does not mean that ASPIC$^+$ arguments are not, in some sense, minimal, as we will now show.

The following example illustrates the fact that any element (proposition or rule) in the grounds and rules of an argument needs to be used in the derivation of the conclusion of that argument:

Example 2. Given the argumentation theory from Example 1, the structure

$$C = (\{p, q, t, u\}, \{p, q \rightsquigarrow r; t, u \rightsquigarrow r; r \rightsquigarrow s\}, s)$$

is not an argument. In particular, C is not an argument because the third clause of Definition 4 only justifies adding the rules and grounds of one argument for each premise of the rule that is the subject of the clause. Thus, it allows $p, q \rightsquigarrow r$ to be added to an argument with conclusion r, or it allows $t, u \rightsquigarrow r$ to be added, but it does not permit both to be added. Similarly,

$$D = (\{p, q\}, \{p, q \rightsquigarrow r; r \rightsquigarrow s; u \rightsquigarrow v\}, s)$$
$$E = (\{t, u\}, \{t, u \rightsquigarrow r; r \rightsquigarrow s; u \rightsquigarrow v\}, s)$$

are not arguments because Definition 4 does not allow rules that are not used in the derivation of the conclusion of an argument to be part of the set of rules of that argument. Finally, neither of

$$F = (\{p, q, t, u\}, \{p, q \rightsquigarrow r; r \rightsquigarrow s\}, s)$$
$$G = (\{p, q, t, u\}, \{t, u \rightsquigarrow r; r \rightsquigarrow s\}, s)$$

are arguments, because Definition 4 does not allow the addition of propositions to the grounds of an argument if they do not correspond to premises of a rule in the argument.

Thus, as the preceding example shows, an argument A, described by the triple $A = (G, R, c)$, cannot contain any elements in G or R that are not used in the derivation of c. Therefore, Definition 4 implies that arguments are minimal in the sense that they do not contain any extraneous propositions or rules. This intuition is also pointed out by the authors in [8], and we formalize it in the following proposition:

Proposition 1. *Let $AT = \langle AS, \mathcal{K} \rangle$ be an argumentation theory and $A \in \mathcal{A}(AT)$. It holds that either:*

(a) $A = (\{c\}, \varnothing, c)$*; or*
(b) $A = (G, R, c)$ *and*
  - *i. for every $g \in G$: there exists $A' \in \texttt{Sub}(A)$ such that $A' = (\{g\}, \varnothing, g)$ and there exists $r \in R$ such that $r = p_1, \ldots, g, \ldots, p_n \rightsquigarrow p'$; and*
  - *ii. for every $r' \in R$ such that $r' = p_1, \ldots, p_m \rightsquigarrow p''$: there exists $A'' \in \texttt{Sub}(A)$ such that $A'' = (G'', R'' \cup \{r'\}, p'')$, with $G'' \subseteq G$ and $R'' \subseteq R$.*

Proof. *Definition 4 includes three clauses that define when $A = (G, R, c)$ is an argument. In the first clause, the base case of the recursive definition, $c \in \mathcal{K}$, R is the empty set and $G = \{c\}$, satisfying case (a).*

The rest of this proof concerns case (b). Now, the second and third clauses of Definition 4, which define the recursive step of the definition, tells us that (G, R, c) *is an argument if there exists a rule in* R *of the form* $c_1, \ldots, c_n \rightsquigarrow c$ *and for each* c_i $(1 \leqslant i \leqslant n)$ *there exists an argument* $A_i \in \mathcal{A}(AT)$ *such that* $\mathtt{Conc}(A_i) = c_i$. *In other words, for every premise* c_i *of the rule there is a sub-argument* A_i *of* A *whose conclusion is that premise. Unwinding each of those sub-arguments in turn, they are either of the form* $(\{c_i\}, \varnothing, c_i)$*, or can be deconstructed into a rule with sub-arguments for each premise, where that rule is in* R. *In the first of these cases, the first clause of Definition 4 tells us that* $c_i \in G$*, and so case (b.i) holds. From the second of these cases we can infer that for every rule* $p_1, \ldots, p_m \rightsquigarrow p'' \in R$*, there is a sub-argument* $(G'', R'' \cup \{p_1, \ldots, p_m \rightsquigarrow p''\}, p'')$ *of* A*, and case (b.ii) is proved.* □

Given an argument $A = (G, R, c)$, Proposition 1 states that every element in G is the conclusion of a sub-argument A' of A and is the premise of a rule in R, and that every rule in R is the $\mathtt{TopRule}(\cdot)$ of a sub-argument A'' of A. In other words, it states that every element of the grounds G and the rules R is part of the derivation of c. However, as the following example shows, Definition 4 does not imply that for any argument (G, R, c) there is no argument (G', R', c) such that $G' \subset G$ and $R' \subset R$:

Example 3. Consider the argumentation system $AS_3 = \langle \mathcal{L}_3, \bar{\cdot}, \mathcal{R}_3, n \rangle$, where $\mathcal{L}_3 = \{p, q, r, s, t, \neg p, \neg q, \neg r, \neg s, \neg t\}$ and $\mathcal{R}_3 = \{p, q \rightsquigarrow s; s \rightsquigarrow q; q, r \rightsquigarrow t\}$. By adding the knowledge base $\mathcal{K}_3 = \{p, q, r\}$ we obtain the argumentation theory $AT_3 = \langle AS_3, \mathcal{K}_3 \rangle$, from which we can construct the following arguments:

$$A_1 = [p]; A_2 = [q]; A_3 = [A_1, A_2 \rightsquigarrow s]; A_4 = [A_3 \rightsquigarrow q]; A_5 = [r];$$
$$A = [A_4, A_5 \rightsquigarrow t]; B = [A_2, A_5 \rightsquigarrow t]$$

such that $A = (\{p, q, r\}, \{p, q \rightsquigarrow s; s \rightsquigarrow q; q, r \rightsquigarrow t\}, t)$ and $B = (\{q, r\}, \{q, r \rightsquigarrow t\}, t)$. Here, it is clear that the grounds and rules of argument B are proper subsets of those of argument A.

Consider now the set of rules $\mathcal{R}_{3'} = \{p, q \rightsquigarrow r; r \rightsquigarrow s; s \rightsquigarrow t; t \rightsquigarrow r\}$. We can obtain a new argumentation system $AS_{3'} = \langle \mathcal{L}_3, \bar{\cdot}, \mathcal{R}_{3'}, n \rangle$ and combine it with the knowledge base $\mathcal{K}_3$ to obtain the argumentation theory $AT_{3'} = \langle AS_{3'}, \mathcal{K}_3 \rangle$, from which we can construct the arguments:

$$C_1 = [p]; C_2 = [q]; C_3 = [C_1, C_2 \rightsquigarrow r]; C_4 = [C_3 \rightsquigarrow s]; C_5 = [C_4 \rightsquigarrow t];$$
$$C = [C_5 \rightsquigarrow r]; D = C_3 = [C_1, C_2 \rightsquigarrow r]$$

Here, $C = (\{p, q\}, \{p, q \rightsquigarrow r; r \rightsquigarrow s; s \rightsquigarrow t; t \rightsquigarrow r\}, r)$ and $D = (\{p, q\}, \{p, q \rightsquigarrow r\}, r)$; hence, $\mathtt{Rules}(D) \subset \mathtt{Rules}(C)$.

Finally, if we consider a set of rules $\mathcal{R}_{3''} = \{p \rightsquigarrow r; r \rightsquigarrow s; q \rightsquigarrow r; r, s \rightsquigarrow t\}$ and a knowledge base $\mathcal{K}_{3'} = \{p, q\}$ we can define an argumentation system $AS_{3''} = \langle \mathcal{L}_3, \bar{\cdot}, \mathcal{R}_{3''}, n \rangle$ and an argumentation theory $AT_{3''} = \langle AS_{3''}, \mathcal{K}_{3'} \rangle$, from which we obtain:

$$E_1 = [p]; E_2 = [E_1 \rightsquigarrow r]; E_3 = [E_2 \rightsquigarrow s]; E_4 = [q]; E_5 = [E_4 \rightsquigarrow r];$$
$$E = [E_5, E_3 \rightsquigarrow t]; F = [E_2, E_3 \rightsquigarrow t]$$

In this case, $E = (\{p, q\}, \{p \rightsquigarrow r; r \rightsquigarrow s; q \rightsquigarrow r; r, s \rightsquigarrow t\}, t)$ and $F = (\{p\}, \{p \rightsquigarrow r; r \rightsquigarrow s; r, s \rightsquigarrow t\}, t)$. As a result, the grounds and rules of F are proper subsets of those of E.

At first sight, this seems a bit contradictory. Example 2 and Proposition 1 show that arguments only contain elements that are used in the derivation of their conclusion, yet Example 3 shows that elements can be removed from the grounds or the rules of an argument, and what remains is still an argument. There is, however, no contradiction. Rather, there are two ways in which this phenomenon might arise. The first is illustrated by the first two cases in Example 3. There we have arguments that are *circular*[4] — if you follow the chain of reasoning from premises to conclusion in A in Example 3, we start with q, then derive q, then use q to derive the final conclusion; similarly, when considering C, we start with p and q to derive r, then derive s and t to derive (again) r. In B and D, these loops are removed to give us more compact arguments with the same conclusions. The second way in which this phenomenon might arise is illustrated by the third case in Example 3, where we have arguments that are *redundant*. There, the cause is that the set of rules provides two ways to derive r, one that relies on p and another that relies on q, and r appears twice in the derivation of t: once to produce s, and once when the rule $r, s \rightsquigarrow t$ is applied. Then E, the redundant argument, uses both of the rules for deriving r while F uses just one of them, again providing a more compact derivation.

Furthermore, as shown by the following example, circularity in arguments may lead to having two distinct arguments A and B such that their descriptions as a triple (G, R, c) coincide. Hence, while we can extract a unique description (G, R, c) from a given ASPIC$^+$ argument A, the reverse is not true.[5]

Example 4. Consider that we have an argumentation system $AS_4 = \langle \mathcal{L}_4, \bar{\cdot}, \mathcal{R}_4, n \rangle$, where $\mathcal{L}_4 = \{a, b, c, \neg a, \neg b, \neg c\}$ and $\mathcal{R}_4 = \{a \rightsquigarrow c; c \rightsquigarrow b; b \rightsquigarrow a\}$. We then add the knowledge base $\mathcal{K}_4 = \{a\}$ to get the argumentation theory $AT_4 = \langle AS_4, \mathcal{K}_4 \rangle$. From this we can construct the following arguments:

$$A_1 = [a]; A_2 = [A_1 \rightsquigarrow c]; A_3 = [A_2 \rightsquigarrow b]; A = [A_3 \rightsquigarrow a];$$
$$B_1 = [A \rightsquigarrow c]; B_2 = [B_1 \rightsquigarrow b]; B = [B_2 \rightsquigarrow a]$$

[4] We use the term *circular* to reflect the idea of circular reasoning [15] and "begging the question" [14].

[5] This is a version of the issue pointed out by [4, p119], that any inference-based description of an argument allows multiple arguments to be described in the same way. In fact what we have here is a stronger version of the problem, because [4] pointed out the problem for arguments which, in our terms, were described just by their grounds and conclusion. What we have here is the problem arising even when we state the inference rules as well. This issue the is converse of the problem that describing arguments by their entire structure, as ASPIC$^+$ and the assumption-based argumentation of [4] do, allows for redundant elements in the arguments, as we have just shown.

Here, both arguments A and B are described by the triple (G, R, a), where $G = \texttt{Prem}(A) = \texttt{Prem}(B) = \{a\}$, $R = \texttt{Rules}(A) = \texttt{Rules}(B) = \mathcal{R}_4$ and $a = \texttt{Conc}(A) = \texttt{Conc}(B)$.

Given the preceding analysis we can note that, even though the characterization of ASPIC$^+$arguments accounts for some form of minimality (see [8]), it allows for circular and redundant arguments. These notions of circularity and redundancy are formalized next.

Definition 6 (Circular Argument). *Let AT be an argumentation theory and $A \in \mathcal{A}(AT)$. We say that A is a* circular argument *if $\exists A_1, A_2 \in \texttt{Sub}(A)$ such that $A_1 \neq A_2$, $\texttt{Conc}(A_1) = \texttt{Conc}(A_2)$ and $A_1 \in \texttt{Sub}(A_2)$.*

Note that the usual definition of a circular argument in the literature [14, 15] involves starting with some premise and then inferring that premise — a typical pattern is "Assume a, then a is true". What we define here as circular is more general.

Example 5. Considering Example 3 in the light of Definition 6 and looking at A, the two sub-arguments that define its circularity are $A_1 = (\{q\}, \emptyset, q)$ and $A_4 = (\{p, q\}, \{p, q \rightsquigarrow s, s \rightsquigarrow q\}, q)$. Then, if we consider argument C, the two sub-arguments that define its circularity are $C_3 = (\{p, q\}, \{p, q \rightsquigarrow r\}, r)$ and $C = (\{p, q\}, \{p, q \rightsquigarrow r, r \rightsquigarrow s, s \rightsquigarrow t, t \rightsquigarrow r\}, r)$. Here, A follows the classic form of a circular argument. In contrast, C illustrates the more general form of circularity, not related to the premises of the argument.

Next, we formalize the notion of redundancy:

Definition 7 (Redundant Argument). *Let AT be an argumentation theory and $A \in \mathcal{A}(AT)$. We say that A is a* redundant argument *if $\exists A_1, A_2 \in \texttt{Sub}(A)$ such that $A_1 \neq A_2$, $\texttt{Conc}(A_1) = \texttt{Conc}(A_2)$, $A_1 \notin \texttt{Sub}(A_2)$ and $A_2 \notin \texttt{Sub}(A_1)$.*

Example 6. Considering Example 3 in the light of Definition 7, the two sub-arguments that define the redundancy of E are $E_2 = (\{p\}, \{p \rightsquigarrow r\}, r)$ and $E_5 = (\{q\}, \{q \rightsquigarrow r\}, r)$.

We say that arguments that are non-circular and non-redundant are *regular* arguments since they are the kinds of argument that one encounters most often in the literature. Clearly this is the same as saying:

Definition 8 (Regular Argument). *Let AT be an argumentation theory and $A \in \mathcal{A}(AT)$. We say that A is* regular *if $\nexists A_1, A_2 \in \texttt{Sub}(A)$ such that $A_1 \neq A_2$ and $\texttt{Conc}(A_1) = \texttt{Conc}(A_2)$.*

Now, to tie this back to the notion of minimality frequently used in the literature (e.g., [1, 2, 5]), that of a minimal set of information from which a conclusion is derived, we need a notion of inference that works for ASPIC$^+$. We start with a notion of *closure*. Given an argumentation theory, we can define the closure of a set of propositions in the knowledge base under a set of rules of the theory.

Definition 9 (Closure). *Let $AT = \langle AS, \mathcal{K} \rangle$ be an argumentation theory, where AS is the argumentation system $AS = \langle \mathcal{L}, \bar{\cdot}, \mathcal{R}, n \rangle$. We define the* closure *of a set of propositions $P \subseteq \mathcal{K}$ under a set of rules $R \subseteq \mathcal{R}$ as $Cl(P)_R$, where:*

1. *$P \subseteq Cl(P)_R$;*
2. *if $p_1, \ldots, p_n \in Cl(P)_R$ and $p_1, \ldots, p_n \rightsquigarrow p \in R$, then $p \in Cl(P)_R$; and*
3. *$\nexists S \subset Cl(P)_R$ such that S satisfies the previous conditions.*

Based on the notion of closure, we can define a notion of inference from a set of propositions and rules of an argumentation theory.

Definition 10 (Inference). *Let $AT = \langle AS, \mathcal{K} \rangle$ be an argumentation theory, where AS is the argumentation system $AS = \langle \mathcal{L}, \bar{\cdot}, \mathcal{R}, n \rangle$. Given a set of propositions $P \subseteq \mathcal{K}$, a set of rules $R \subseteq \mathcal{R}$ and a proposition $p \in \mathcal{K}$, we say that p* is inferred from *P* and *R, noted as $P \vdash_R p$, if $p \in Cl(P)_R$.*

Now, with this notion of inference, we can characterize *minimal arguments*. These arguments are such that they have minimal (with respect to $\subseteq$) sets of grounds and rules that allow to infer their conclusion.

Definition 11 (Minimal Argument). *Let $AT = \langle AS, \mathcal{K} \rangle$ be an argumentation theory and $A \in \mathcal{A}(AT)$. We say that $A = (G, R, c)$ is a* minimal argument *if $\nexists G' \subset G$ such that $G' \vdash_R c$ and $\nexists R' \subset R$ such that $G \vdash_{R'} c$.*

The following example illustrates the first condition in Definition 11.

Example 7. Let $AT_7 = \langle AS_7, \mathcal{K}_7 \rangle$ be an argumentation theory, where $AS_7 = \langle \mathcal{L}_7, \bar{\cdot}, \mathcal{R}_7, n \rangle$, $\mathcal{R}_7 = \{d \rightsquigarrow b; b \rightsquigarrow c; b, c \rightsquigarrow a\}$ and $\mathcal{K}_7 = \{b, d\}$. From AT we can construct the following arguments:

$$A_1 = [d]; A_2 = [A_1 \rightsquigarrow b]; A_3 = [A_2 \rightsquigarrow c]; A_4 = [b]; A = [A_4, A_3 \rightsquigarrow a];$$
$$B = [A_2, A_3 \rightsquigarrow a]; A_5 = [A_4 \rightsquigarrow c]; C = [A_4, A_5 \rightsquigarrow a]$$

Here, $A = (G, R, a)$, with $G = \{b, d\}$ and $R = \mathcal{R}_7$. In this case, A is not minimal since $\exists G' \subset G$, with $G' = \{d\}$, such that $G' \vdash_R a$; moreover, $B = (G', R, a)$. On the other hand, argument C is represented by the triple (G'', R', a), with $G'' = \{b\}$ and $R' = \{b \rightsquigarrow c; b, c \rightsquigarrow a\}$. In particular, argument C is minimal. Furthermore, B is also minimal since, even though $R' \subset R$, it is not the case that $G' \vdash_{R'} a$.

It should be noted that, since the notion of minimality characterized in Definition 11 explicitly accounts for the set of grounds of the arguments, this notion of minimality is different from those used in other structured argumentation systems such as DeLP [5]. Arguments in DeLP do not include the grounds: they are specified by a pair $\langle \Delta, c \rangle$, where Δ is the set of defeasible rules used to derive the conclusion c. Thus, the notion minimality in DeLP considers only the defeasible rules used in an argument. As a result, if we consider the arguments given in Example 7, argument B would not be minimal in DeLP.

To illustrate the second condition of Definition 11, let us consider the situation depicted in Example 4. There, we have arguments A and B, which are both described by the triple (G, R, a), with $G = \{a\}$ and $R = \{a \rightsquigarrow c; c \rightsquigarrow b; b \rightsquigarrow a\}$. Also, there is argument $A_1 = (G', \varnothing, a)$, with $G' = \{a\}$. As a result, $\exists G' \subset G$ such that $G' \vdash_R a$ and therefore, arguments A and B are not minimal, in contrast with A_1.

Given the characterization of regular and minimal arguments, the following proposition shows that these notions are equivalent.

Proposition 2. *Let $AT = \langle AS, \mathcal{K} \rangle$ be an argumentation theory and $A \in \mathcal{A}(AT)$, with $A = (G, R, c)$. A is a regular argument iff A is a minimal argument.*

Proof. The proof follows the same form as that of Proposition 1, being based around the three clauses of Definition 4.

*Let us start with the if part. In the first clause of Definition 4, c is a proposition in $\mathcal{K}$, R is empty, and G contains just c. Clearly, in this case there is no $R' \subset R$, nor $G' \subset G$ such that $G' \vdash_R c$ or $G \vdash_{R'} c$, so A is minimal. It is also regular. The second and third clauses in Definition 4 define the recursive case. Here, $A = (G, R, c)$ is an argument if c is the conclusion of a rule, let us call it r, and there is an argument in $\mathcal{A}(AT)$ for each of the premises of r. G is then the union of the grounds of all the arguments with conclusions that are premises of r; we will call this set of arguments **Args**, and R is the union of all the rules for **Args**, call them **Rs**, plus r. If all the arguments in **Args** are minimal, then A will be minimal, so long as (i) adding r does not introduce any non-minimality, and (ii) the union of the grounds and the rules of the arguments in **Args** do not introduce any non-minimality. Let us consider case (i). For the addition of r to introduce non-minimality, it must be the case that $(G, \boldsymbol{Rs}, c)$ is an argument. In that case, $(G, \boldsymbol{Rs}, c)$ will be a sub-argument of A and thus, by Definition 6, A is circular, contradicting the hypothesis that it is a regular argument. Let us now consider case (ii). Here, in order for A not to be minimal, there have to be minimal arguments $(G_1, R_1, p_1), \ldots, (G_n, R_n, p_n)$ in **Args** such that $p_1, \ldots, p_n$ are the premises in rule r and $A = (\bigcup_{i=1}^n G_i, \bigcup_{i=1}^n R_i \cup \{r\}, c)$ is not minimal. Because we are taking the unions, no duplication can be introduced. Since $G_1, \ldots, G_n$ are just sets of propositions, their union cannot be the cause of any non-minimality, and we know from case (i) that any non-minimality is not due to r. So if any non-minimality is introduced, it is in $\bigcup_{i=1}^n R_i$. Since by Proposition 1 every rule in R_i must be used in deriving p_i, the only way that $\bigcup_{i=1}^n R_i$ can make A non-minimal is if there is some rule in R_j which allows the derivation of the same conclusion as a rule in R_k (with $1 \leqslant j, k \leqslant n$, and $j \neq k$). In such a case, A would have two distinct sub-arguments with the same conclusion, where one is not a sub-argument of the other; hence, by Definition 7, A would be redundant, contradicting the hypothesis that A is regular.*

Let us now address the only if part. In the first clause of Definition 4, c is a proposition in $\mathcal{K}$, R is empty, and G contains just c. Clearly, in this case A is regular since A is the only sub-argument of A; thus, there exist no distinct sub-arguments of A with the same conclusion. A is also minimal. The second and third clauses in Definition 4 define the recursive case. Here, $A = (G, R, c)$ is an argument if c is the conclusion of a rule, let us call it r, and there is an argument in $\mathcal{A}(AT)$ for each of the premises in rule r. G is then the union of the grounds *of all the arguments with conclusions that are premises in r, and R is the union of all the rules for those arguments, plus r. Since by hypothesis $A = (G, R, c)$ is minimal, it must be the case that $\nexists G' \subset G$ such that $G' \vdash_R c$ and $\nexists R' \subset R$ such that $G \vdash_{R'} c$. Suppose by contradiction that A is not regular. Hence, there should exist two distinct sub-arguments $A_1 = (G_1, R_1, p')$ and $A_2 = (G_2, R_2, p')$ of A such that $G_1 \neq G_2$, $R_1 \neq R_2$, or both. However, this would imply that $\exists G' \subset G$ (with $G' = (G \backslash G_1) \cup G_2$, or $G' = (G \backslash G_2) \cup G_1$) or $\exists R' \subset R$ (with $R' = (R \backslash R_1) \cup R_2$, or $R' = (R \backslash R_2) \cup R_1$) such that $G' \vdash_R c$ and $\nexists R' \subset R$ such that $G \vdash_{R'} c$, contradicting the hypothesis that A is minimal.* □

Next, we illustrate the relationship between regular and minimal arguments.

Example 8. Let us consider the arguments from Example 7, where it was shown that B and C are minimal arguments, whereas A is not. Then, we have that B is also regular, since it has no pair of sub-arguments with the same conclusion. Specifically, $\texttt{Sub}(B) = \{B, A_2, A_3, A_1\}$, and $\texttt{Conc}(B) = a$, $\texttt{Conc}(A_2) = b$, $\texttt{Conc}(A_3) = c$, $\texttt{Conc}(A_1) = d$. Similarly, C is also regular since $\texttt{Sub}(C) = \{C, A_4, A_5\}$, where $\texttt{Conc}(C) = a$, $\texttt{Conc}(A_4) = b$ and $\texttt{Conc}(A_5) = c$. In contrast, if we consider argument A, which was shown to be non-minimal in Example 7, we have $\texttt{Sub}(A) = \{A, A_4, A_3, A_2, A_1\}$ where, in particular, $\texttt{Conc}(A_4) = b$ and $\texttt{Conc}(A_2) = b$; therefore, A is not a regular argument.

On the other hand, if we consider the arguments from Example 3, it was shown in Examples 5 and 6 that A, C and E are not regular arguments (the first two by being circular and the last one by being redundant). Then, if we look at the minimality of these arguments, we have that $A = (G_a, R_a, t)$, with $G_a = \{p, q, r\}$, $R_a = \{p, q \rightsquigarrow s; s \rightsquigarrow q; q, r \rightsquigarrow t\}$, and $\exists G'_a = \{r\}$, $\exists R'_a = \{r \rightsquigarrow t\}$ such that $G'_a \vdash_{R_a} t$ and $G_a \vdash_{R'_a} t$; hence, A is not a minimal argument. In the case of $C = (G_c, R_c, r)$, with $G_c = \{p, q\}$ and $R_c = \{p, q \rightsquigarrow r; r \rightsquigarrow s; s \rightsquigarrow t; t \rightsquigarrow r\}$, we have that $\exists R'_c = \{p, q \rightsquigarrow r\}$ such that $G_c \vdash_{R'_c} r$ and therefore, C is not minimal. Finally, given $E = (G_e, R_e, t)$, with $G_e = \{p, q\}$ and $R_e = \{p \rightsquigarrow r; r \rightsquigarrow s; q \rightsquigarrow r; r, s \rightsquigarrow t\}$, it is the case that $\exists G'_e = \{p\}$, $\exists G''_e = \{q\}$, $\exists R'_e = \{p \rightsquigarrow r; r \rightsquigarrow s; r, s \rightsquigarrow t\}$, $\exists R''_e = \{r \rightsquigarrow s; q \rightsquigarrow r; r, s \rightsquigarrow t\}$ such that $G'_e \vdash_{R_e} t$, $G''_e \vdash_{R_e} t$, $G_e \vdash_{R'_e} t$ and $G_e \vdash_{R''_e} t$; thus, E is not a minimal argument.

Let us consider another example regarding minimal and non-minimal arguments.

Example 9. Consider that we have an argumentation system $AS_9 = \langle \mathcal{L}_9, \bar{\cdot}, \mathcal{R}_9, n \rangle$, where $\mathcal{L}_9 = \{p, q, r, \neg p, \neg q, \neg r\}$ and $\mathcal{R}_9 = \{p \rightsquigarrow q; q \rightsquigarrow r\}$. By adding the knowledge base $\mathcal{K}_9 = \{p, q\}$ we obtain the argumentation theory $AT_9 = \langle AS_9, \mathcal{K}_9 \rangle$, from which we can build the following arguments:

$$H_1 = [p];\ H_2 = [H_1 \rightsquigarrow q];\ H = [H_2 \rightsquigarrow r];$$
$$I_1 = [q];\ \ I = [I_1 \rightsquigarrow r]$$

such that $H = (\{p\}, \{p \rightsquigarrow q; q \rightsquigarrow r\}, r)$ and $I = (\{q\}, \{q \rightsquigarrow r\}, r)$.

Even though arguments H and I in Example 9 have the same conclusion and use the rule $q \rightsquigarrow r$ to draw that conclusion, they are both minimal. This is because, according to Definition 11, an argument is minimal if there is no argument for the same conclusion built from a smaller set of grounds (respectively, rules) combined with the set of rules (respectively, grounds) of the former. Thus it is possible to have two minimal arguments for the same conclusion, where the latter uses a subset of the rules of the former, so long as the grounds of the latter are not included in the grounds of the former. Similarly, we could have two minimal arguments for the same conclusion, where the latter uses a subset of the grounds of the former, so long as the rules of the latter are not included in the set of rules of the former.

On the other hand, the situation depicted in Example 9 relates to the one involving arguments $C_3 = (\{p,q\},\{p,q \rightsquigarrow r\},r)$ and $C = (\{p,q\},\{p,q \rightsquigarrow r; r \rightsquigarrow s; s \rightsquigarrow t; t \rightsquigarrow r\},r)$ in Example 3. However, even though C_3 and C have the same conclusion, differently from H and I, they are such that one is a sub-argument of the other (specifically, C_3 is a sub-argument of C, with the sets of grounds and rules of C_3 being contained in those of C). As a result, C is not regular nor minimal.

Finally, it should be noted that, since ASPIC$^+$ arguments are not required to be minimal in the sense of Definition 11, it can be the case that two different arguments A and B have the same description as a triple (G,R,c), as occurred in Example 4. However, as shown by the following proposition, that cannot be the case when considering minimal arguments.

Proposition 3. *Let $AT = \langle AS, \mathcal{K}\rangle$ be an argumentation theory and $A \in \mathcal{A}(AT)$, with $A = (G,R,c)$. If A is a minimal argument, then $\nexists B \in \mathcal{A}(T)$ such that $B \neq A$ and $B = (G,R,c)$.*

Proof. *Suppose that $A = (G,R,c)$ is a minimal argument and $\exists B \in \mathcal{A}(T)$ such that $B \neq A$ and $B = (G,R,c)$. By Proposition 1, every element in the grounds G and every rule in R is used in the derivation of A's and B's conclusion c. Furthermore, since by hypothesis A is minimal, by Definition 11 it is the case that $\nexists G' \subset G$, $\nexists R' \subset R$ such that $G' \vdash_R c$ or $G \vdash_{R'} c$. If $B \neq A$, then it must be the case that the difference between them is on the number of times they use the rules in R. Since by hypothesis A is minimal, there must be a rule $r = p_1,\ldots,p_n \rightsquigarrow p \in R$ that is used more times in B than in A. Now consider the derivation of A and B. From what we have said so far, these must be largely the same, so we can think of them starting from the same set of grounds and applying rules, one by one. Thinking of the two arguments like this, side by side, so to speak, since B uses some rule r more times than A does, then at some stage B uses the rule r to derive p, whereas r is not used in A at that point. Hence, since p is needed at that point as part of the derivation for A's conclusion c, there must be an alternative derivation for p in A, which does not require the use of the rule r. However, this would imply that there exists a rule $r' \in R$ such that $r' = p'_1,\ldots,p'_m \rightsquigarrow p$ or $p \in G$, contradicting the hypothesis that A is minimal. As a result, if A is a minimal argument, then $\nexists B \in \mathcal{A}(T)$ such that $B \neq A$ and $B = (G,R,c)$.* □

We finish by noting that even though arguments H and I in Example 9, are both minimal in the sense of Definition 11, argument I could be considered to be, in some sense, "more minimal" than H since the sets of grounds of both arguments are the same size, while I has a smaller set of rules. This suggests that further forms of minimality may be worth investigating.

4 Related Work

In this section we will discuss how the notion of minimality is handled by other approaches to structured argumentation.

As we have mentioned before, the formalism of *Assumption-Based Argumentation (ABA)* proposed in [4] shares some characteristics with ASPIC$^+$. Arguments in ABA are deductions of claims using rules based on a set of assumptions. Deductions are defined as trees, where leaves correspond to assumptions and non-leave nodes correspond to sentences that are the heads of rules, whose children correspond to the sentences in the body of those rules. That is, arguments in ABA are built following the same strategy as ASPIC$^+$, where some form of minimality is implicit. Specifically, like in ASPIC$^+$, irrelevant pieces of information cannot be introduced in a deduction in ABA. Thus, ABA arguments have ASPIC$^+$ native form of minimality, in which minimality relates to relevance.

In [2] the authors propose a framework for structured argumentation based on classical logic. In their approach, an argument A is a pair $\langle \Phi, \alpha \rangle$, where Φ is a minimal (w.r.t. $\subseteq$) set of formulae that is consistent and allows to prove α. Relating their proposal to ASPIC$^+$, if we consider an argument $A = (G, R, \alpha)$, the set Φ would be the combination of the sets of grounds and rules of A (i.e., $\Phi = G \cup R$). Then, since Definition 11 establishes that A is minimal if there exists no $G' \subset G$ and no $R' \subset R$ such that $G' \vdash_R \alpha$ or $G \vdash_{R'} \alpha$, this is the same as saying that there is no $\Phi' \subset \Phi$ such that $\Phi \vdash \alpha$ in [2]. As a result, the characterization of minimality for ASPIC$^+$ arguments that we proposed in this paper could be considered to be equivalent to the one given in [2]. Thus we might claim to have extended the notion of minimality from [2] to fit ASPIC$^+$.

Another work in which the notion of minimality becomes present when defining the structure of arguments is [1], where a framework for dealing with preferences between arguments is proposed. There, arguments are assumed to be built from a propositional knowledge base, by means of classical inference. Then, an argument is defined as a pair (H, h), where H is a consistent and minimal (w.r.t. $\subseteq$) set of formulae from the knowledge base that allows to infer h. That is, the notion of minimality considered in [1] coincides with that of [2]. Therefore, as discussed above, it could be considered to be equivalent to the notion of minimality we proposed in this paper for ASPIC$^+$.

Let us now consider *Defeasible Logic Programming (DeLP)*, the structured argumentation system proposed in [5]. An argument in DeLP is defined as a pair $\langle \Delta, c \rangle$, where Δ is a set of rules used to derive the conclusion c. The first difference between the characterization of arguments in DeLP and in ASPIC$^+$(as well as in the formaslisms of [2, 1, 4]) relies on the fact that DeLP does not include in Δ the set of grounds used for building the argument. Furthermore, the set Δ does not include *every* rule used in the derivation process, but only includes the *defeasible* rules. In other words, the set Δ only includes the defeasible knowledge of the argument. This is because arguments in [5] are required to be consistent with the strict knowledge of a DeLP program, which is determined by the facts and strict rules of the program. Then, the minimality requirement on DeLP arguments accounts only for the defeasible part of the arguments (i.e., Δ has to be a minimal set — w.r.t. $\subseteq$ — that is consistent with the strict knowledge of the program and allows to derive the conclusion c).

The characterization of arguments in DeLP, leaving the strict knowledge aside, results in that minimal arguments cannot be uniquely mapped into a single derivation. This is because there might be alternative derivations for a given argument, which make use of different sets of facts and strict rules that allow to derive the same conclusions. Furthermore, the derivation for the conclusion of a given argument may not be minimal, in the sense that it may include irrelevant facts or strict rules. In addition, since minimality only accounts for the set of defeasible rules, it could be the case that ASPIC$^+$ arguments satisfying the notion of minimality from Definition 11 are not minimal under DeLP's notion of minimality, as discussed after Example 7. Finally, it should be noted that, since there exist scenarios (like the one in Example 7) where arguments are not 'valid' (thus, they are not arguments at all) in DeLP but they are 'valid' (furthermore, minimal) arguments in ASPIC$^+$, the outcome of the two argumentation systems in such scenarios may differ, because different sets of arguments are considered. This difference opens up a space that we are interested in exploring in the future.

5 Conclusion

In this paper we have studied the notion of minimality of arguments in the context of ASPIC$^+$. We have considered two forms of minimality. The first of these corresponds to the native minimality of ASPIC$^+$, which implies that arguments do not include irrelevant grounds or rules. We have noted that, under the native form of minimality, redundant and circular arguments may be obtained. Although there is nothing inherently wrong with circular and redundant arguments, in some cases it may be helpful to work with arguments that satisfy a stronger form of minimality. The second, stronger form of minimality that we considered, is satisfied by what we have identified as *regular* arguments, since these are the arguments that one encounters most often in the literature of argumentation. Specifically, regular arguments do not have two (or more) distinct sub-arguments with the same conclusion. It should be noted that an argument A satisfying the stronger form of minimality uses the same grounds and rules for deriving a proposition p at every step in which p is required in the derivation of A's conclusion. Furthermore, we have shown that regular arguments, satisfying the stronger form of minimality, can be unequivocally described by a triple (G, R, c), distinguishing their grounds, rules and conclusion. In contrast, that is not the case for arguments complying only with ASPIC$^+$ native form of minimality. Finally, as discussed in Section 4, the stronger form of minimality we proposed in this paper is related to the notion of minimality considered in other approaches for structured argumentation like [2] and [1], but not to others, such as [5]. As a result, we can say that the way in which arguments are characterized, and the way in which the minimality restrictions are imposed on arguments, heavily influence the outcome of an argumentation system.

In the future we are interested in further studying the notion of minimality in the context of ASPIC$^+$, and investigate whether alternative forms of minimality could provide results that align with the behavior of structured systems like [5]. In addition, we are interested in studying the impact the notion of minimality could have in determining the existence of interactions between arguments, including attack and support relations.

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