Proceedings of the 10th European Conference on Artificial Intelligence (ECAI 92), Vienna, Austria 3 -7 August 1992.

Qualitative belief networks

Simon Parsons
Department of Electronic Engineering, Queen Mary and Westfield College
Mile End Road, London E1 4NS, UK

Abstract. This paper presents results concerning the qualitative behaviour of belief networks. We compare the qualitative behaviour of a simple belief network with that of the equivalent probabilistic network and analyse belief networks representing the conjunction and disjunction of distinct pieces of evidence, suggesting an abstract description of such networks. Keywords: qualitative behaviour, causal networks, belief functions.

1. Introduction

In recent years, the study of qualitative probabilistic reasoning in networks has become well established [2], [9]. This paper extends such work by considering reasoning in qualitative belief networks, that is the qualitative behaviour of reasoning in networks based upon belief functions [8]. Our interest is in the qualitative modelling of uncertainty handling systems as a means of integrating different formalisms [3]. We are thus concerned with the propagation of changes in value within networks since it is by this means that the impact of evidence may be assessed in any formalism. When we consider qualitative reasoning under uncertainty, the process of assessing the impact of new evidence becomes the determination of the qualitative change in one value given the change in another.

Belief networks were introduced [6] in the form of undirected graphs whose edges are weighted by belief functions. Smets [7] has provided the mathematical models necessary to treat directed belief graphs in a similar manner to that in which Pearl [5] treats probabilistic networks. In this paper we present some results regarding the qualitative behaviour of directed belief networks. We analyse the behaviour of the simplest possible network, and compare the results to those obtained for qualitative probabilistic networks [9]. We then analyse more complex networks representing the conjunction and disjunction of distinct pieces of evidence and suggest an abstract description of such networks.

2. A simple directed link

Consider the simplest possible belief network, a directed link between a variable A which can have values a and $\neg a$, and a variable C which can have values c and $\neg c$:

$A \in \{a, \neg a\}$ $C \in \{c, \neg c\}$

The qualitative behaviour of the link, which is referred to as $A \rightarrow C$, is determined by derivatives such as $\frac{\partial bel(c)}{\partial bel(a)}$ which specify how belief in variable C taking value c depends upon the belief in variable A taking value a. There are three possible dependencies:

Definition 2.1: The belief in a variable X taking value x is said to *follow* the belief in variable Y taking value y if $bel(x)$ increases when $bel(y)$ increases, and decreases when $bel(y)$ decreases.

The belief in a variable X taking value x is said to *vary inversely with* the belief in variable Y taking value y if $bel(x)$ decreases when $bel(y)$ increases, and increases when $bel(y)$ decreases.

The belief in a variable X taking value x is said to *be independent of* the belief in variable Y taking value y if $bel(x)$ does not change as $bel(y)$ increases and decreases.

The qualitative change in $bel(c)$ is calculated from:

$$[\Delta bel(c)] = [\Delta bel(a)] \otimes \left[\frac{\partial bel(c)}{\partial bel(a)}\right]$$

where $[x]$ represents the qualitative value of x [1], and $\otimes$ denotes qualitative multiplication. $bel(c)$ follows $bel(a)$ when $[\partial bel(c)/\partial bel(a)] > 0$, varies inversely with $bel(a)$ when $[\partial bel(c)/\partial bel(a)] < 0$ and is independent of $bel(a)$ when $[\partial bel(c)/\partial bel(a)] = 0$. Using Dempster's rule of combination to combine beliefs we have:

Theorem 2.2: For $A \rightarrow C$, $bel(c)$ follows $bel(a)$ iff $bel(c|a) > bel(c|a \cup \neg a)$, $bel(c)$ varies inversely with $bel(a)$ iff $bel(c|a) < bel(c|a \cup \neg a)$ and $bel(c)$ is independent of $bel(a)$ iff $bel(c|a) = bel(c|a \cup \neg a)$.

Proof: By Dempster's rule $bel(c) = \Sigma_{a \subseteq A} m(a)bel(c|a)$. Now, from [8] $m(a) = bel(a)$, $m(\neg a) =$

$bel(\neg a)$ and $m(a \cup \neg a) = 1 - bel(a) - bel(\neg a)$. Thus $\partial bel(c)/\partial bel(a) = bel(c|a) - bel(c|a \cup \neg a)$.■

Note that $bel(c)$ is the is the belief in hypothesis c given all the available evidence, while $bel(c|a \cup \neg a)$ is the belief induced on c by the marginalisation on $\{c \cup \neg c\}$ of the joint belief on the space $\{c \cup \neg c\} \times \{a \cup \neg a\}$.

Now $bel(c)$ follows $bel(a)$ if c is more likely to occur given a than given the whole frame. Using Dempster's rule requires conditional probabilities of the form $bel(c|a \cup \neg a)$. If such values are unavailable, we may use the disjunctive rule of combination [7]. This gives:

Theorem 2.3: For $A \rightarrow C$, $bel(c)$ either follows $bel(a)$ or is independent of it.

Proof: The disjunctive rule of combination gives $bel(c) = \sum_{a \subseteq A} m(a) \prod_{a_i \in a} bel(c|a_i)$. As before $m(a) = bel(a)$ and $m(\neg a) = bel(\neg a)$, so $\partial bel(c)/\partial bel(a) = bel(c|a)[1 - bel(c|\neg a)] \geq 0$ since $bel(x) \leq 1$ for all x, and the result follows.■

3. A comparison with qualitative probability

In qualitative probabilistic networks, simple causal links of the kind pictured above have two possible behaviours. The probability of the consequent p(c) can follow the probability of the antecedent p(a) and vary inversely with the probability of not-the-antecedent $p(\neg a)$ or vary inversely with p(a) and follow $p(\neg a)$. If p(c) follows p(a), $p(\neg c)$ varies inversely with p(a). In qualitative belief networks there is no necessary connection between the way that $bel(c)$ depends upon $bel(a)$ and $bel(\neg a)$, and the way that $bel(c)$ and $bel(\neg c)$ depend upon $bel(a)$. The full specification of the qualitative behaviour of the simple belief network above, given all the necessary conditionals, may be determined from the following generalisation of Theorem 2.2:

Theorem 3.1: The relation between $bel(x)$ and $bel(y)$ for the link $A \rightarrow C$ is such that $bel(x)$ follows $bel(y)$ iff $bel(x|y) > bel(x|y \cup \neg y)$, $bel(x)$ varies inversely with $bel(\neg y)$ iff $bel(x|y) < bel(x|y \cup \neg y)$ and $bel(x)$ is independent of $bel(y)$ iff $bel(x|y) = bel(x|y \cup \neg y)$ for all $x \in \{c, \neg c\}$, $y \in \{a, \neg a\}$.

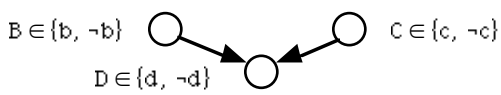
and the overall qualitative change in $bel(c)$ is:

$$[\Delta bel(c)] = \left[\frac{\partial bel(c)}{\partial bel(a)}\right] \otimes [\Delta bel(a)] \oplus \left[\frac{\partial bel(c)}{\partial bel(\neg a)}\right] \otimes [\Delta bel(\neg a)]$$

where $\oplus$ is qualitative addition [1].

4. More complex networks

There are a number of ways in which the more complex below may be treated using belief functions:



We may consider the relationship between B, C and D to be determined by one set of conditional beliefs of the form $bel(d|b,c)$, or by two sets of conditional beliefs of the form $bel(d|b)$. In either case D may be dependent upon either the conjunction or the disjunction of B and C. For a conjunctive dependence, we apply Dempster's rule of combination to the link, abbreviated to $B \wedge C \rightarrow D$, and obtain, for conditionals of the form $bel(d|b,c)$:

Theorem 4.1: The relation between $bel(d)$ and $bel(b)$ for the link $B \wedge C \rightarrow D$ is determined by:

$$\left[\frac{\partial bel(d)}{\partial bel(b)}\right] = [bel(d|b,c) - bel(d|b \cup \neg b,c)] \oplus [bel(d|b,\neg c) - bel(d|b \cup \neg b,\neg c)] \oplus [bel(d|b,c \cup \neg c) - bel(b \cup \neg b,c \cup \neg c)]$$

Proof: By Dempster's rule of combination, $bel(d) = \sum_{b \subseteq B\, c \subseteq C} m(b)m(c)bel(d|b,c)$. Now, $m(x) = bel(x)$, $m(\neg x) = bel(\neg x)$ and $m(x \cup \neg x) = 1 - bel(x) - bel(\neg x)$, so $\partial bel(d)/\partial bel(b)$ is as written.■

Thus $bel(d)$ follows $bel(b)$ iff $bel(d|b,c) > bel(d|b \cup \neg b,c)$, $bel(d|b,\neg c) > bel(d|b \cup \neg b,\neg c)$, and $bel(d|b,c \cup \neg c) > bel(b \cup \neg b,c \cup \neg c)$. For conditionals of the form $bel(d|b)$ we obtain:

Theorem 4.2: The relation between $bel(d)$ and $bel(b)$ for the link $B \wedge C \rightarrow D$ is determined by:

$$\left[\frac{\partial bel(d)}{\partial bel(b)}\right] = [bel(d|\neg b)] \oplus [bel(d|b) - bel(d|b \cup \neg b)]$$

Proof: By Dempster's rule of combination $bel(d) = \sum_{b \subseteq B,\ c \subseteq C,\ b \vee c \supset d} bel(d|b)m(b)bel(d|c)m(c)$. Thus $\partial bel(d)/\partial bel(b) = (bel(d|b) - bel(d|b \cup \neg b))(bel(d|c) + bel(d|\neg c) + bel(d|c \cup \neg c)) + (bel(d|\neg b) + bel(d|b \cup \neg b) - bel(d|b))(m(c)bel(d|c) + m(\neg c)bel(d|\neg c) + m(c \cup \neg c)bel(d|c \cup \neg c))$. Since $\Sigma_{c \subseteq C} m(c) = 1$ this reduces to $\partial bel(d)/\partial bel(b) = bel(d|\neg b).K + (bel(d|b) - bel(d|b \cup \neg b))K'$, where $K, K' > 0$, and the result follows.■

For a disjunctive relation between B, C and D, written $B \vee C \rightarrow D$, we obtain, for both conditionals such as $bel(d|b,c)$ and $bel(d|b)$, the result that:

Theorem 4.3: For the link $B \vee C \rightarrow D$, $bel(d)$ always follows $bel(b)$.

Proof: We have two separate cases. For conditionals such as $bel(d|b,c)$ the disjunctive rule gives $bel(d) = \Sigma_{b\subseteq B\ c\subseteq C} m(b)m(c)\ \Pi_{b_i\in b\ c_j\in c}\ bel(d|b_i,c_j)$ so that $\partial bel(d)/\partial bel(b) = bel(c)(bel(d|b,c)(1- bel(d|\neg b,c))) + bel(\neg c)bel(d|b,c)(1 - bel(d|\neg b,\neg c)) + bel(d|b,c)(1 - bel(c) - bel(\neg c))(1 - bel(d|\neg b,c)bel(d|\neg b,\neg c)) bel(d|b,\neg c)) > 0$. For conditionals such as $bel(d|b)$ the disjunctive rule of combination gives $bel(d) = \Sigma_{b\subseteq B\ c\subseteq C} m(b)m(c)\ \Pi_{b_i\in b\ c_j\in c}\ bel(d|b_i)\ bel(d|c_j)$. Thus $\partial bel(d)/\partial bel(b) = bel(d|b)(1 - bel(d|\neg b))((bel(d|c) + bel(d|\neg c) + bel(d|c)bel(d|\neg c)) - (m(c)\ bel(d|c) + m(\neg c)bel(d|\neg c) + bel(d|c)bel(d|\neg c)\ m(c\cup\neg c))$. Since $m(x) \leq 1$ for all x, $\partial bel(d)/\partial bel(b) > 0$, and the result follows.■

Symmetrical results hold for the relation between $bel(d)$ and $bel(c)$, $bel(\neg b)$ and $bel(\neg c)$ [3], and the overall change in $bel(d)$ is defined by:

$$[\Delta bel(d)] = \left[\frac{\partial bel(d)}{\partial bel(b)}\right]\otimes[\Delta bel(b)] \oplus \left[\frac{\partial bel(d)}{\partial bel(c)}\right]\otimes[\Delta bel(c)] \oplus \left[\frac{\partial bel(d)}{\partial bel(\neg b)}\right]\otimes[\Delta bel(\neg b)] \oplus \left[\frac{\partial bel(d)}{\partial bel(\neg c)}\right]\otimes[\Delta bel(\neg c)]$$

5. An abstract description of links

A link $A\rightarrow C$ in a qualitative belief network is described by two values; $\Delta_+^{\cdot\cdot}$ which describes its effect upon the belief in the consequent of the causal link, and $\Delta_-^{\cdot\cdot}$ which describes its effect upon the belief in not-the-consequent. The first variable in $\Delta_+^{\cdot\cdot}$ describes the effect of the belief in the antecedent , and the second the effect of the belief in not-the-antecedent. Thus if $A\rightarrow C$ is described by $\Delta_+ +0$, $\Delta_- -+$ then an increase in $bel(a)$ invokes an increase in $bel(c)$ and no change in $bel(\neg c)$, while an increase in $bel(\neg a)$ causes $bel(c)$ to decrease and $bel(\neg c)$ to increase. A chain of links $l_1, \ldots, l_n$, where the qualitative value of l_j is $\Delta_+ j_1\ j_2$, $\Delta_- j_3\ j_4$, may be replaced by a single link with the value $\Delta_+[l_1\otimes \ldots \otimes n_1]\ [l_2\otimes \ldots \otimes n_2]$, $\Delta_-[l_3\otimes \ldots \otimes n_3]\ [l_4\otimes \ldots \otimes n_4]$. A link such as $B\wedge C\rightarrow D$ is similarly described by a pair of pairs; $[\Delta_+^{ij}\ \Delta_-^{kl}]$ which determines how the belief in D depends upon that in B and $[\Delta_+^{mn}\ \Delta_-^{pq}]$ which determines how the belief in D depends upon that in C. This concept may be extended to cover an n-ary relation, $A_1\wedge\ldots\wedge A_n\rightarrow D$ which is represented by n pairs, and a tree of simple and binary links with n roots may be reduced to a single n-ary relation [4].

6. Conclusion

We have introduced the concept of qualitative belief networks as networks in which we propagate the qualitative effect of changes in belief by the rules of belief functions, and have analysed the qualitative behaviour of a number of simple networks. From our analysis it is clear that qualitative belief networks have a wider range of behaviours than qualitative probabilistic networks, and can thus be viewed as a generalisation of them.

The results presented in this paper allow us to predict the qualitative behaviour of belief networks where the parents of any one node are independent. Further work will be required to determine the behaviour of more general networks where parents may be dependent as a result of a common ancestor, in particular to determine the behaviour of networks with loops. The analysis of such networks is considerably more complex than the analysis presented here, and is discussed in detail in [3].

Acknowledgement: This work was supported by a grant from ESPRIT Basic Research Action 3085 DRUMS. Thanks to all my colleagues on the project for help and encouragement, especially Philippe Smets, Rudolph Kruse, John Fox and Mirko Dohnal.

References

[1] de Kleer, J. and Brown, J. S. (1984) A Qualitative Physics Based on Confluences, *Artificial Intelligence* 24.

[2] Henrion, M. and Druzdzel, M. J. (1990) Qualitative propagation and scenario-based approaches to explanation of probabilistic reasoning, *Proceedings of the 6th Conference on Uncertainty in AI.*

[3] Parsons, S. (1992) Qualitative approaches to reasoning under uncertainty, PhD Thesis, Department of Electronic Engineering, Queen Mary and Westfield College (in preparation).

[4] Parsons, S. (1991) Qualitative and semiqualitative methods for integrating uncertainty handling formalisms, Technical Report, Department of Electronic Engineering, Queen Mary and Westfield College.

[5] Pearl, J. (1988) *Probabilistic reasoning in intelligent systems: networks of plausible inference*. Morgan Kaufmann, San Mateo CA.

[6] Shafer, G., Shenoy, P. P. and Mellouli, K. (1987) Propagating belief functions in qualitative markov trees, International Journal of Approximate Reasoning 1.

[7] Smets (1991) Belief functions: the disjunctive rule of combination and the generalised bayesian theorem, Technical Report TR/IRIDIA/91-01-2.
[8] Smets (1988) Belief functions, in: *Non-Standard Logics for Automated Reasoning* (P.Smets, E. H. Mamdani, D. Dubois and H. Prade, eds.) Academic Press, London.
[9] Wellman, M. P. (1990) Fundamental concepts of qualitative probabilistic networks, *Artificial Intelligence*, 44.