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# Universality in glassy low-temperature physics

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**Abstract.** – We propose a microscopic translationally invariant glass model which exhibits two-level tunneling systems, and shows the salient low-temperature anomalies of glassy systems. Results so far obtained are in good accord with experiment. Qualitative universality is due to the collective origin of the glassy potential energy landscape. However, we obtain a simple explanation also for the mysterious so-called *quantitative* universality that manifests itself, *e.g.*, in the unusually weak dependence of values for the internal friction plateau on substance or system parameters.

*Introduction.* – The physics of glassy systems at low temperatures differs strikingly from that of their crystalline counterparts [1]. Differences are observed in thermal properties, in transport phenomena, and in dielectric and acoustic response, and are believed to be due to the existence of tunneling excitations with a broad range of energy splittings and relaxation times, which are absent in ideal crystals. The standard tunneling model (STM) [2], and the soft potential model (SPM) [3], which paraphrase this idea quantitatively, were for many years considered to provide a satisfactory rationalization of glassy low-temperature physics.

Questions on the *origin of the universality* of these phenomena [4] have not yet found clearcut answers; see, however, [5]. In particular, the remarkable degree of *quantitative* universality of sound attenuation in the 1 K regime and (related to it) of the thermal conductivity have so far remained a mystery [6]. Also, relations between low- and high-temperature phenomena could not possibly emerge from phenomenological descriptions. Moreover, a number of recent experiments [7–9] are difficult to reconcile with predictions of prevalent models.

*The model.* – In order to address these issues, we have recently proposed *microscopic* models for glassy low-temperature physics [10,11] using heuristics taken from spin-glass theory. The purpose of the present letter is to improve upon [10, 11] with respect to both modeling and scope of analysis. We investigate a glass model described by a Hamiltonian with a *translationally invariant* interaction energy of the form

$$U_{\rm int} = \frac{1}{4}\sum_{ij} J_{ij}(u_i - u_j)^2 + \frac{g}{2N}\sum_{ij}(u_i - u_j)^4. \tag{1}$$

The $u_i$ designate deviations of particle coordinates from some reference positions, and (1) may be thought of as arising from a Born von Karman expansion of the full interaction energy

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about these positions —taken to be stationary but not necessarily stable. As before [10, 11], glassy properties are modeled by assuming the expansion coefficients at the harmonic level to be Gaussian random couplings with mean $\overline{J_{ij}} = J_0/N$ and variance $\overline{(\delta J_{ij})^2} = J^2/N$. The non-random quartic potential is for stabilization, and so requires $g > 0$. Non-translationally invariant versions [10] or models with partial translational invariance [11] have been introduced before. We emphasize the following: i) We do not claim our model to be microscopic in the sense of being *microscopically realistic*. Rather, the line of reasoning is as follows. For the very reason that glassy low-temperature physics *is* universal, one should be justified in describing it within virtually *any* model of an interacting many-particle system which produces a glassy low-temperature phase, *and* which might with some reason claim to capture the essence of glassy physics, *without* necessarily adhering to all detail. As to what this essence might be, we believe —in view of our results so far obtained— that our choice of a translationally invariant interaction (reflecting the *structure* of a Born von Karman expansion), with some randomness generating frustration, could well be close sufficient. ii) The glassy low-temperature phase in such a model may then justly be expected to exhibit the universal low-temperature anomalies in the way virtually *any* (real) glass does, and —if it does— to reveal some of the mechanisms responsible for these phenomena —once more, without claiming to be faithful in all details.

Main results which go beyond our earlier analysis [10, 11] are: i) By honoring translational invariance, the description of glassy low-temperature physics is considerably improved. In particular, in accord with experiment, we find that the low-temperature specific heat is a *super-linear* (rather than linear) function of temperature, and that the ultrasound attenuation plateau shows a weak frequency dependence. ii) We demonstrate that these features are not independent but *quantitatively related*. iii) We find that the distribution of parameters characterizing the effective single-site potentials of the glassy potential energy landscape differs qualitatively from what is assumed in the phenomenological models. iv) Most importantly, however, within our approach we have a simple explanation for the mysterious quantitative universality of glassy low-temperature anomalies.

*Analysis.* – We now turn to the analysis. Due to the translationally invariant interaction, the system contains global translations as zero modes, and the partition function must be evaluated orthogonal to these modes, *i.e.* with the constraint $\sum_i u_i = 0$. Apart from this detail, the analysis follows standard reasoning. The free energy is obtained by averaging an $n$-fold replicated partition sum, $-\beta f(\beta) = \lim_{N\to\infty, n\to 0}(Nn)^{-1}\ln[Z_N^n]_{\rm av}$, and can be expressed in terms of an Edwards-Anderson matrix of two-replica overlaps $q_{ab}$. Although four-replica overlaps must also be introduced during the calculation, the final result does not depend on them. The replica free energy reads

$$n\beta f(\beta) = \frac{(\beta J)^2}{4}\sum_{ab} q_{ab}^2 - 3\beta g \sum_a q_{aa}^2 - \ln \tilde{Z}_n \tag{2}$$

with $\tilde{Z}_n$ a single-site partition function corresponding to the $n$-replica potential $U_n = \frac{1}{2}\sum_a (J_0 + 12 g q_{aa}) u_a^2 - \frac{\beta J^2}{2}\sum_{ab} q_{ab} u_a u_b - \frac{\beta J^2}{8}\big(\sum_a u_a^2\big)^2 + g\sum_a u_a^4$, that is, $\tilde{Z}_n = \int \prod_a \mathrm{d}u_a \exp[-\beta U_n]$. The order parameters $q_{ab}$ must satisfy the fixed-point equations $q_{ab} = \langle u_a u_b\rangle$ with angle brackets denoting a Gibbs average with respect to the potential $U_n$. As usual, to perform the $n \to 0$ limit, one starts from parameterizations of the $q_{ab}$ matrix based on assumptions concerning transformation properties of solutions under permutation of the replica. We have looked at the replica symmetric (RS) solution and a solution with one step of replica symmetry breaking (1RSB). Both in RS and 1RSB (and at all higher levels of Parisi's RSB scheme) the system is described by an ensemble of effective single-site potentials of the form

$$U_{\rm eff}(u) = d_1 u + d_2 u^2 + d_4 u^4, \tag{3}$$

in which $d_4 = g$ is the coupling constant of the quartic stabilizing interaction, and $d_1$ and $d_2$ are randomly varying parameters. We shall occasionally refer to $d_1$ and $d_2$ as to an effective local field $h_{\rm eff}$ and an effective harmonic coupling $k_{\rm eff}$ via $d_1 = -h_{\rm eff}$ and $d_2 = \frac{1}{2}k_{\rm eff}$. Their joint distribution is expressed in terms of the order parameters of the system which are in turn obtained in terms of Gibbs averages self-consistently evaluated over the $U_{\rm eff}$ ensemble (3). This ensemble of effective single-site potentials is a representation of the glassy potential energy landscape within a mean-field description. Glassy low-temperature anomalies follow from considering the effects of quantized excitations within this ensemble of local potentials.

In RS one assumes $q_{aa} = q_d$ and $q_{ab} = q$ for $a \neq b$. These must solve the equations

$$q_d = \big\langle \langle u^2 \rangle \big\rangle_{\overline{z},z}\,, \qquad q = \big\langle \langle u \rangle^2 \big\rangle_{\overline{z},z} \tag{4}$$

in which inner brackets denote a Gibbs average with respect to the single-site potential (3) with

$$d_1 = d_1^{\rm RS} = -J\sqrt{q}z, \qquad d_2 = d_2^{\rm RS} = \frac{1}{2}\big(J_0 + 12gq_d - J^2C + J\overline{z}\big), \tag{5}$$

and $C = \beta(q_d - q)$, and outer brackets an average over the zero-mean, unit-variance Gaussians $\overline{z}$ and $z$ in $d_1$ and $d_2$. The ensemble of single-site potentials thus comprises both SWPs and DWPs, the latter with a broad spectrum of barrier heights. A glassy state is signaled by $q \neq 0$, thus a non-degenerate distribution of asymmetries of single-site potentials. In the present model the transition is continuous and the critical condition is given by $1 = (\beta_{\rm c}J)^2\big\langle\langle u^2\rangle^2\big\rangle_{\overline{z},z}$, which is just the de Almeida-Thouless condition for the occurrence of a RSB instability. For models with symmetric coupling distributions the transition temperature satisfies $T_{\rm c}(J,g) = \frac{J^2}{g}T_{\rm c}(1,1)$ with $k_{\rm B}T_{\rm c}(1,1) \simeq 0.133E_*$ [12]. In RS, $d_1$ and $d_2$ are Gaussian and uncorrelated. This would be qualitatively in line with assumptions of the SPM, although there are quantitative differences. In contrast to the STM, correlations are predicted between asymmetries and tunneling matrix elements of tunneling systems in DWPs.

The RS solution is unstable and thus strictly not acceptable throughout the glassy phase. For a non-translationally invariant model [10], we have shown RSB effects to be small for the specific heat. Here, we look in greater detail also at the distribution of $h_{\rm eff}$ and $k_{\rm eff}$. We shall find it to be *qualitatively* different from what has been obtained in the RS approximation, and thus also from what is assumed in the phenomenological models.

In 1RSB, expected to exhibit the salient RSB effects, one assumes $q_{aa} = q_d$, $q_{ab} = q_1$ for $1 < |a-b| \leq m$ and $q_{ab} = q_0$ for $|a-b| > m$. The fixed-point equations are now much more complicated, and we shall not reproduce them here. Whereas $d_2$ has the same form as in RS, $d_2^{\rm 1RSB} = \frac{1}{2}(J_0 + 12gq_d - J^2C + J\overline{z})$, except that now $C = \beta(q_d - q_1)$, the local field is more complicated. It is formulated in terms of *two* variables $z_1$ and $z_0$,

$$d_1 = d_1^{\rm 1RSB} = -J\sqrt{q_1 - q_0}z_1 - J\sqrt{q_0}z_0, \tag{6}$$

of which $z_0$ is a standard Gaussian, whereas $z_1$, while deriving from a Gaussian, becomes correlated with $z_0$ and $\overline{z}$ (thus $d_2$) in an intricate way through $U_{\rm eff}$. In the low-temperature limit, to which we restrict our attention here, the result is

$$p(z_1 \mid z_0, \overline{z}) = \frac{\exp\big[-z_1^2/2 - DU_{\rm eff}(\hat{u})\big]}{\sqrt{2\pi}\int \mathcal{D}z_1 \exp\big[-DU_{\rm eff}(\hat{u})\big]}\,. \tag{7}$$

Here $D = \beta m$, which has a finite $T \to 0$ limit, $\hat{u} = \hat{u}(z_1, z_0, \overline{z})$ is the value of $u$ which *minimizes* $U_{\rm eff}(u)$ for given values of $z_0$, $z_1$, and $\overline{z}$, and $\mathcal{D}z_1$ is a Gaussian measure. Figure 1 displays the resulting distribution of $d_1$ *conditioned* on $d_2$.

The RSB result is *qualitatively* different from the RS result and from what is assumed in phenomenological models, in that small $d_1$ and thus symmetric tunneling systems are

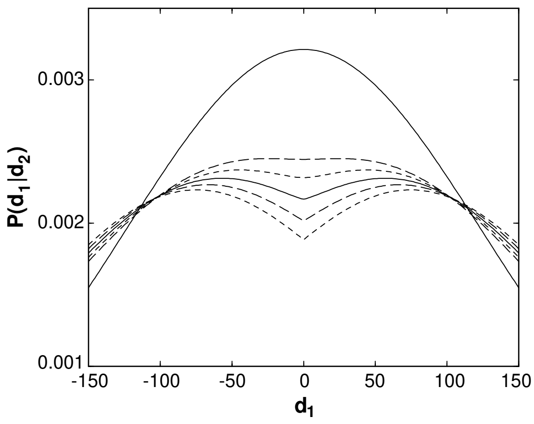


Fig. 1

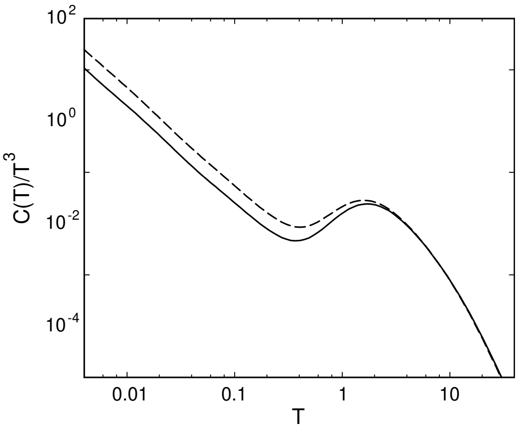


Fig. 2

Fig. 1 – Central part of $P(d_1|d_2)$, for $J = 50$ and $g = 1$. From top to bottom we have RS (independent of $d_2$) and 1RSB with $d_2 = 10, 5, 0, -5$ and $-10$, respectively.

Fig. 2 – Low-$T$ specific heat for $J = 50$ in RS (dashed line) and 1RSB (full line).

systematically suppressed. In fact, in the DWP region for negative $d_2$ and small $d_1$, the form is $P(d_1, d_2) \simeq \pi_0(d_2)(1 + \pi_1(d_2)|d_1|)$, so $P(d_1, d_2)$ is *non-analytic* at $d_1 = 0$. Yet, due to the smallness of $\pi_1(d_2)$, RSB effects do not have strong influence on functional forms of thermodynamic and response functions at low temperatures. This was observed before for the specific heat in a non-translationally invariant model [10], and is confirmed here.

*Low-temperature anomalies.* – Glassy low-temperature anomalies follow from low-energy excitations in the ensemble of single-site problems $H_{\rm eff} = \frac{p^2}{2m} + U_{\rm eff}(u)$. The energy of tunneling excitations in DWPs is determined by an asymmetry $\Delta$ and a tunneling matrix element $\Delta_0$ as $E = \sqrt{\Delta^2 + \Delta_0^2}$, which are in turn expressed through $d_1$ and $d_2$ as $\Delta = E_* d_1 \sqrt{2(|d_2| - 1)}$ and $\Delta_0 = E_* \sqrt{2}|d_2|^{3/2} \exp[\frac{2}{3}(1 - |d_2|^{3/2})]$ [13]. Frequencies of higher (quasi-harmonic) excitations in SWPs or DWPs are given by the curvature of the potential in its minimum, $U''_{\rm eff}(u_{\rm min}) = m\omega^2$ [12].

By averaging over $P(d_1, d_2)$, one obtains tunneling and vibrational density of states and the specific heat as usual. Tunneling systems give rise to a slightly *super-linear* specific heat at low temperatures, $C(T) \sim T^{1+\varepsilon}$, with $\varepsilon$ decreasing with $J$ (hence $T_{\rm c}$), $\varepsilon \simeq 0.1$, 0.05 and 0.01 for $J = 25$, 50, and 100, respectively, whereas the strongly peaked vibrational density of state is the origin of a Bose peak in our model, see fig. 2. RSB effects are small in $C(T)$, the dominant effect being a reduction of the tunneling density of states as compared to RS. Except for the super-linearity at low $T$, which can be traced down to the translational invariance of the interactions, $C(T)$ is qualitatively as in [10], both in RS and 1RSB.

It is interesting to compare with assumptions of the STM at the level of the distribution $P(\Delta, \lambda)$, where $\lambda = \ln(E_*/\Delta_0)$. In the STM $P^{\rm STM}(\Delta, \lambda) = P_0$ is assumed. We have

$$P(\Delta, \lambda) = \frac{P(d_1, d_2)}{E_* \sqrt{2(|d_2| - 1)}\left[\sqrt{|d_2|} - 3/(2|d_2|)\right]} \tag{8}$$

for $d_2 < -(3/2)^{2/3}$, with $d_1 = d_1(\Delta, \lambda)$ and $d_2 = d_2(\Delta, \lambda)$ instead. The results shown in fig. 1 imply that, unlike in the STM or the SPM, symmetric tunneling systems are systematically suppressed. If one were to *fit* our results to an STM parameterization, one would associate

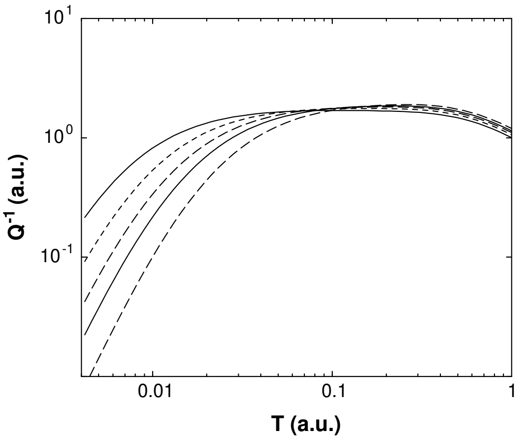


Fig. 3 – Internal friction as a function of $T$ on a double logarithmic scale for various driving frequencies, $\omega = 0.33$, 1.03, 2.52, 5.03 and 14.0 kHz (top to bottom).

$P_0 \simeq P(0, \lambda^*)$ for some $\lambda^* = \mathcal{O}(1) \Leftrightarrow |d_2^*| = \mathcal{O}(10)$. Interestingly, for reasonable energy scales (associated with tunneling of complexes like $SiO_2$ over atomic distances, and giving $E_*$ between 1 and 5 K), we would estimate $P_0 = 10^{-7} \dots 10^{-6}\,\mathrm{K}^{-1}$/atom for the system with $J = 50$, which is the right order of magnitude for typical glasses. Also, using the fact that the low-$T$ limit of the fixed-point equations entails $q_d$, $q \propto J$, and $C \propto J^{-1}$ in RS, and similarly $q_d$, $q_1$ and $q_0 \propto J$, $C \propto J^{-1}$, and $D \propto J^{-2}$ in 1RSB, we can combine this with $T_\mathrm{c} \propto J^2$ and the expression for $P(d_1, d_2)$ to predict the scaling $P_0 \sim T_\mathrm{c}^{-5/4}$ for large $T_\mathrm{c}$ both in RS and 1RSB, our second result relating low-$T$ properties with a property of the glass-transition in our model. A detailed comparison with experiments is difficult, as a large change in glass transition temperature requires changing composition of the glass, and thus interactions in non-trivial ways. Yet the trend expressed by this result appears to be correct.

Next, we discuss dynamics due to a coupling between local degrees of freedom $u$ and the strain field $e$ generated by Debye phonons, $H_\mathrm{SB} = \gamma u e$, where $\gamma$ is a deformation potential and $e = \frac{1}{\sqrt{V}} \sum_{\boldsymbol{q},s} \sqrt{\frac{\hbar}{2\rho\omega_{\boldsymbol{q},s}}}\, iq(b_{\boldsymbol{q},s} - b^\dagger_{\boldsymbol{q},s})$ the strain-field; the sum is over transversal and longitudinal modes and the tensorial nature of the strain field is neglected. Quantities like internal friction are obtained by averaging the dynamical susceptibility over the ensemble of single-site problems,

$$Q^{-1} = \frac{\gamma^2}{\rho v^2} \overline{\chi''_{uu}(\omega)}\,. \tag{9}$$

At low $T$, and for $\hbar\omega \ll k_\mathrm{B}T$, the dominant contribution is the relaxational contribution of tunneling systems in DWPs within the ensemble of single-site potentials. For these a two-level approximation for $H_\mathrm{eff}$ is appropriate and (9) reduces to the well-known expression for two-level tunneling systems, when the internal coordinate $u$ is approximated by a two-state variable in terms of a Pauli matrix, $u = u_0\sigma_z$. Figure 3 shows the internal friction of the present model for $J = 50$ and driving frequencies as in [9] computed in 1RSB (except for a different global overall pre-factor, RS results are functionally hardly distinguishable). The dominant asymptotics is $Q^{-1} \sim T^{3+\varepsilon}(1 + c_1 T)$ for $T \ll T^*$ and $Q^{-1} \simeq \frac{\pi}{2}\frac{P_0\gamma^2}{\rho v^2}(\hbar\omega/k_\mathrm{B}T)^{\varepsilon/2}(1 + c_2 T)$ for $T \gg T^*$ —the influence of the $c_\alpha = \mathcal{O}(10^{-3})$ due to RSB effects being basically indiscernible. The exponent $\varepsilon$ is the one used to quantify the super-linearity of the low-$T$ specific heat, and $\varepsilon \simeq 0.05$ for the $J$-value chosen. Note that i) there is a weak frequency dependence $\omega^{\varepsilon/2}$ of the plateau height, *correlated* with the low-$T$ specific-heat exponent (the extra $T^{-\varepsilon/2}$ factor is indiscernible for the small range of temperatures considered), and that ii) the asymptotic low-$T$ exponent is slightly larger than 3. However, the crossover region from plateau to low-temperature asymptotics is so large that effective exponents in accessible temperature ranges are still *smaller* than 3, the effect being stronger for lower frequencies. Both findings are well in line with recent experiments [9], though observed effective exponents tend to show a somewhat stronger frequency dependence and be smaller than ours.

*Universality.* – Lastly, we turn to the universality issue. In [5], universality is explained within a renormalization group approach as a collective effect due to interactions *of* quantized low-energy excitations. Here, as in [10], it is understood as a property of the interaction-generated glassy potential energy landscape, thus as a collective effect *leading to* a particular spectrum of quantized low-energy excitations. The mechanism is robust and may therefore justly be expected to be insensitive to details. Specifically, this insensitivity follows from a scaling argument based on the observation that the parameters $d_1$ and $d_2$ of the family of effective potentials can generally be written in terms of some $\mathcal{O}(1)$ random variables $z_1$ and $z_2$ and the interaction scale $J$ as $d_1 = J^a z_1$ and $d_2 = J^b z_2 - J^c \overline{z}_2$, with some positive exponents $a$, $b$, and $c$, with $a = 3/2$ and $b = c = 1/2$ for the model specifically considered in the present letter. (We have explicitly kept track of averages.) Then, if $\rho(z_1, z_2)$ denotes the (material-dependent) joint probability density function of $z_1$ and $z_2$ (which for microscopically realistic models is likely to be different from the one computed here), we have $P(d_1, d_2) = J^{-(a+b)} \rho(J^{-a} d_1, J^{-b} d_2 + J^{c-b} \overline{z}_2)$. Low-temperature phenomena are dominated by effective potentials with $d_1 = \mathcal{O}(1)$ and $d_2 = \mathcal{O}(10)$, for which $P(d_1, d_2)$ can at large $J$ be approximated by $P(d_1, d_2) \simeq J^{-(a+b)} \rho(0, J^{c-b} \overline{z}_2)$, sampling the fundamental distribution $\rho$ only at a single point, and eliminating any dependence on its shape at large, which would encode the material-dependent properties. Corrections to this may be contemplated, if need be.

Interestingly, there is within the present theory also a rather simple explanation for the remarkably weak dependence of values for the sound attenuation plateau (and, related to it, of thermal conductivity [6]) on substance or system parameters. It simply follows from combining the scaling $P_0 \sim J^{-5/2}$ with the plausible supposition $\gamma \sim J$ expressing the fact that deformation potential and original interaction are of the same origin, and the elasticity theory scaling $\rho v^2 \sim J$ for the sound velocity, which together entail $\frac{P_0 \gamma^2}{\rho v^2} \sim J^{-3/2} \sim T_c^{-3/4}$. This weak parameter dependence due to cancellations implies, *e.g.*, that the internal friction plateau is changed by only a factor 8 when interaction energies are changed such as to increase $T_c$ from 100 to 1600 K! Note that similar cancellations, and weak dependence on detail, must be expected in general, with details of course depending on the exponents $a$, $b$ and $c$ introduced above.

*Summary and discussion.* – In summary, we have proposed and analyzed a translationally invariant glass model. It exhibits typical glassy low-temperature anomalies of specific heat and acoustic attenuation. The super-linearity of the low-$T$ specific heat is quantitatively related with a weak frequency dependence of the internal friction plateau. Both features are in good agreement with experiment. Within the present modeling they are due to the non-degenerate barrier-height distribution, which in turn can be shown to follow solely from translational invariance (a feature that was absent in our original proposal [10]). Within our approach we can also correlate low- and high-temperature properties. Universality is understood as a consequence of collective effects. Most importantly, we also have for the first time a simple explanation of the mysterious quantitative universality [6], an issue which has been open since it was first raised more than 15 years ago in a paper by Freeman and Anderson [14].

It goes without saying that alternative approaches have been proposed for many of the phenomena described here. *E.g.*, a linear specific heat at low $T$ has been found in a spherical quantum $p$-spin model [15]. The Bose peak in particular has recently received considerable attention [13,16–19]. Our line of reasoning, starting from a microscopic model, basically goes along [13]; some overlap with the ideas presented in [19] appears to exist. Proposals explaining the Bose peak as due to an (incipient) mechanical instability occurring in a disordered *harmonic* system [16,17] appear to follow a line of reasoning entirely different from ours; however, in order to fully assess the differences between the approaches, the fate of the phonon bath in the Bose-peak frequency region within *our* line of reasoning would require closer investigation; indeed, significant restructuring of the phonon bath in this energy range is to be expected. A recent paper by Gurevich *et al.* [18] goes some way in the direction of addressing this issue. Finally, if perhaps less directly related, properties of energy landscapes are also being widely discussed in connection with the dynamics of super-cooled liquids [20].

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