Adequate Capital and Stress Testing for Operational Risks*

Reimer Kühn[1] and Peter Neu[2]

[1]Department of Mathematics, King's College London
Strand, London WC2R 2LS, UK
kuehn@mth.kcl.ac.uk, +44 (20) 7848 1035

[2]Group Risk Control, Dresdner Bank AG, Jürgen-Ponto-Platz 1
D-60301, Frankfurt, Germany
Peter.Neu@Dresdner-Bank.com, +49 (69) 263 18582

June 2, 2004

Abstract

We describe how the notion of sequential correlations naturally leads to the quantification of operational risk. Our main point is that functional dependencies between mutually supportive processes give rise to non-trivial temporal correlations, which can lead to the occurrence of collective risk events in the form of bursts and avalanches of process failures, and crashes of process networks. We show how the adequate capital for operational risk can be calculated via a stochastic dynamics defined on a topological network of interacting processes. One of the main virtues of the present model is the suitability for capital allocation and stress testing of operational risks.

*The views presented in this paper are those of the authors and do not necessarily represent models or policies of Dresdner Bank AG.

1 Industry Trends and Practices

The banking industry has become increasingly more complex in recent years. Reasons are the growing sophistication of financial products such as CDOs and credit derivatives, growing activities of banks across several legal and regulatory systems in course of a general globalization, growing reliance on globally integrated IT-systems, and growing complexity in organization due to large-scale mergers or outsourcing of clearing and settlement systems. Clearly in some of the listed examples the chance on short-term profit is traded for operational risk.

Operational risk (OR) is classified as "the risk of losses resulting from inadequate or failed internal processes, people and systems or from external events" [1, 2]. Possible OR-categories are [3]:

1. human processing errors, e.g., mishandling of software applications, reports containing incomplete information, or payments made to incorrect parties without recovery
2. human decision errors, e.g., unnecessary rejection of a profitable trade or wrong trading strategy due to incomplete information
3. (software or hardware) system errors, e.g., data delivery or data import is not executed and the software system performs calculations and generates reports based on incomplete data
4. process design error, e.g., workflows with ambiguously defined process steps
5. fraud and theft, e.g., unauthorized actions or credit card fraud
6. external damages, e.g., fire or earthquake

Since the first consultative paper for the New Basel Accord on Capital Adequacy has been issued by the Basel Committee on Banking Supervision in 2001 (see Ref. [2] for the current consultative papers), known as Basel II, it is clear that regulators will demand banks to hold regulatory capital against operational risks.

The Basel Committee on Banking Supervision has proposed three alternative approaches to operational risk measurement [2]: The "Basic-Indicator Approach (BIA)", "The Standardized Approach (TSA)", and the "Advanced Measurement Approach (AMA)". In the BIA the required capital for operational risk is determined by multiplying a single financial indicator, which is the average annual gross income over the last three years, GI, by a fixed percentage (called the α-factor). The TSA differs from the latter in that banks are allowed to choose business line specific weight factors, β_k, for the gross income indicator, GI_k, of the k^{th} business line. The total regulatory capital charge, RC, is the simple sum of the capital required per business line. Standard business lines defined by the Committee are: (1) Corporate Finance, (2) Trading and Sales, (3) Retail Banking, (4) Commercial Banking, (5) Payment and Settlement, (6) Agency Services, (7) Asset Management, and (8) Retail Brokerage. The factors loadings α and β_k lie between 12% and 18% [2].

Under the AMA, the regulatory capital requirement is set by the bank's internal quantitative and qualitative operational risk measurement system, which is subject to supervisory approval. The common scheme of the AMA has been laid out by Georges, Frachot and Roncalli [4]. Within this scheme, banks estimate—based on internal or external loss data basis or expert knowledge—the mean and variation of annual frequencies and loss severities for OR-events per business line.

Based on this knowledge, distribution functions[1] for the loss frequency N and loss severity X are parametrized from which N_{ik} realizations are drawn in a Monte Carlo simulation for each business unit i and for each OR-category k. The loss in such a sample is

$$L_{ik} = \sum_{m=1}^{N_{ik}} X_{ik}^{m} \ . \tag{1}$$

Drawing a histogram of outcomes of L_{ik} provides the loss distribution function per risk category/business line cell from which the capital charge is read-off as the q-quantile. The required capital for the bank as a whole can either be calculated as the simple summation of the capital charges across each of the risk category/business line cell. Alternatively, the quantile can be determined from the histogram of the total loss, $L = \sum_{i,k} L_{ik}$. This accounts for diversification between the risk category/business line cells.

The Committee is not specifying the approach or distribution function used to generate the operational risk measure for regulatory capital purposes provided that "fat tails" of the loss distribution are accounted for, and the adapted approach compares to the internal ratings based approach for credit risk [2]. Supervisors further expect banks to move along these approaches to operational risk as they develop more sophisticated measurement systems and practices. However, they permit banks to use AMA for some parts of its operations and BIA or TSA for other ("partial use"). The Committee further requires that banks apply scenario analysis to assess the impact of deviations from the parameter assumptions embedded in their OR-measurement framework, and "in particular, to evaluate potential losses arising from multiple simultaneous OR-loss event" (see paragraph 635 in [2]).

The last requirement hints at the critical point of any AMA-framework: the treatment of correlations between OR-events. Thinking of OR-categories as "operational risk processes" it is clear that there are *functionally defined sequential dependencies* between individual processes, which all together bring a big organization to work. Consider the following example for illustration: a system error leads to an incomplete data import into a risk calculation engine, resulting in a wrong calculation of risk figures, and eventually to a human decision error by the trader, who closes a possibly profitable position unnecessarily to reduce a risk which in fact does not exist. Besides this there also exist "equal-time correlations" between OR-events. This is most obvious in category 6 listed above.

Models which account for sequential and equal-time correlations in OR-events were first proposed by Kühn and Neu [5] and, later on, by Leippold and Vanini [6]. Similar approaches have also been applied to credit risk modeling [7, 8]. Our model is based on analogies to models of interacting many particle systems in statistical physics, in which phase transitions triggered by interactions can occur and drastically change the overall behavior of a system. Specifically, our model resembles a lattice gas with heterogeneous, functionally defined couplings—combined with an element of annealed disorder concerning the role of loss-distributions. In such a description, bursts and avalanches of process failures correspond to droplet formation associated with a first order phase transition.

In the following we summarize the main ideas of Ref. [5] and present new simulations with particular focus on capital adequacy and stress testing for operational risk. For further details and analytic descriptions of our model we refer to [5].

[1] Popular choices for the loss severity distribution functions are the Lognormal, Gamma, Beta, Weibull distribution. Common choices for the loss frequency distribution function are the Poisson or negative binomial distribution.

2 Functional Correlation Approach for Operational Risk

Our model to quantify operational risk is based on a topological network of operational processes and activities in a bank. Core processes such as trading and sales, risk management and risk control, financial control (P&L, management and financial accounting, planning, budgeting), IT- and data-systems, back-office and settlement/payment systems, retail and commercial banking, corporate finance, asset management, etc., are represented as vertices in this network. Bonds between these vertices exhibit mutual dependencies between core processes.

Such networks can be designed in hierarchical way: each core process can be further detailed in sub-networks which show more specific steps for, e.g., a risk management function. Furthermore, such a hierarchical network can be drawn for branches and subsidiaries. Examples have been presented by Vandenbrink [3] and in the context of sequential correlations by Leippold and Vanini [6].

Such networks represent mutual dependencies between OR-categories, business lines, branches and subsidiaries, and, thereby, also provide a tool for capital allocation. Clearly, they are the key for a successful implementation of the mathematical framework proposed below. Defining a clearly laid out and sufficiently detailed topological process map is in our view a non-trivial and organization-specific task. Here, we provide the mathematical framework for deriving the adequate capital for such a process network, and show how well such networks are suited for stress analysis of operational risks.

2.1 Mathematical Framework

The occurrence of functionally defined sequential dependencies between risk events in the case of OR is obvious. Processes in organization would normally be organized so as to mutually support each other. Thus, if a process fails, this will affect the workings of other processes in adverse ways, if they rely on receiving input or support of some sort from the failing process in question, so that these other processes run a higher risk of failing as well. It is therefore inadequate to model OR-events individually. In the following we describe our extension of the common AMA-approach that takes the *functionally defined sequential dependencies* between processes into account.

As an idealization, we consider a simple two state model here, i.e., a processes can be either up and running or down. For the process corresponding to the OR-event i we designate these states as $n_i = 0$ and $n_i = 1$, respectively. It will become obvious in what follows that generalizations that would include modeling graded degradation of processes are easily formulated using the principles laid out below. For the following, considerations related to separation between business lines is immaterial, and we will skip the business line index k, accordingly.

The interest is in obtaining reliable estimates of the statistics of processes that are down at any time, and of the statistics of losses that are thereby generated. As the loss severity incurred by a given process going into the down state may vary randomly from event to event, solving the latter problem requires convolving the statistics of down-events with the loss severity distribution related to the process failures.

The reliability of individual processes will vary across the set of processes, and so will the degree of functional interdependence. These random heterogeneities constitute an element of frozen disorder, whereas the loss severities incurred by down processes constitute an element of so-called annealed disorder as they are (randomly) determined anew from their distribution each time a process goes down. An appealing consequence of this analysis of operational risks therefore is the *independence* of the dynamical model of the interacting processes and the loss severity model (i.e. the estimate of the PDFs of loss severity incurred by individual process failures.) A commonly adopted assumption for the latter is to take them as being distributed according to a log-normal distribution with suitable parameters for means and variances, which we will choose in the following.

2.1.1 Dynamics

The formulation of the dynamics in the functional approach is based on the observation that all processes need a certain amount of support in order to maintain a functioning state for the time increment $t \to t + \Delta t$ within the risk horizon, $t \in [0, T)$ (think of energy, human resources, information, input from other processes, etc.). Here, only the generic features of the model shall be outlined. Hence, the increment Δt is chosen such that all processes can fully recover within this time interval, i.e., the state n_i of each process can flip each time step. For practical applications, one may decide to model the recovery process more carefully: specific death-period after the failure of the i^{th} process could be considered, and one could differentiate between process failures being discovered and adjusted up to a certain cut-off time, e.g., end-of-day, at which a process would have been completed [3]. These features are not generic and can only be discussed related a specific OR-event under consideration.

Let $h_i(t)$ denote the total support received by process i at time t; we choose it to take the form

$$h_i(t) = \vartheta_i - \sum_j w_{ij} n_j(t) + \eta_i(t) \ . \tag{2}$$

It is composed of (i) the *average* total support ϑ_i that would be provided by a fully operational network of processes (in which $n_i(t) = 0$ for all i). This quantity is (ii) *reduced* by support that is missing because of failing processes which normally feed into the process in question; (iii) lastly, there are fluctuations about the average which we take to be correlated Gaussian white noise with—by proper renormalizing ϑ_i and w_{ij}—zero mean and unit variance. Correlated Gaussian noise is introduced to model equal-time cross correlations between OR-risk categories in analogy to the approach proposed by the Basel Committee on Banking Supervision for credit risk,

$$\eta_i(t) = \sum_{k=1}^{K} \beta_{ik} Y_k(t) + \xi_i \epsilon_i(t) \ , \tag{3}$$

where

$$\xi_i = \left(1 - \sum_{k=1}^{K} \beta_{ik}^2 \right)^{1/2} \ . \tag{4}$$

The $Y_k(t) \sim \mathcal{N}(0,1)$ represent the common risk factors, whereas the $\epsilon_i(t) \sim \mathcal{N}(0,1)$ are process-specific, idiosyncratic risk factors describing fluctuations in the micro-state of each process. The weights $\beta_i = (\beta_{i1}, \ldots, \beta_{iK})$ stand for the sensitivity of the total support $h_i(t)$ received by process i to changes in k^{th} common risk sector (common risk factors may be fluctuations in power supply, frequency of spam or virus attacks, whether conditions affecting human resources, etc.). Without loss of generality we assume the common risk sectors to be orthogonal, i.e., the $Y_k(t)$ to have zero correlation.[2] Equal-time correlations are then given by $\rho_{ij} = \sum_k \beta_{ik}\beta_{jk}$ for $i \neq j$. It is understood that the dynamics of some common factors $Y_k(t)$ may be slow on the time scale set by Δt. Note that non-linear effects could be included by modifying (2) to $h_i(t) = \vartheta_i - \sum_j w_{ij} n_j(t) - \sum_{j,k} w_{ijk} n_j(t) n_k(t) - \ldots + \eta_i(t)$.

Process i will fail in the next time instant $t + \Delta t$, if the total support for it falls below a critical threshold. By properly renormalizing ϑ_i, we can choose this threshold to be zero, thus (Θ is the step-function: $\Theta(x) = 1$ for $x \geq 0$ and 0 else)

$$n_i(t + \Delta t) = \Theta\Big(- \vartheta_i + \sum_j w_{ij} n_j(t) - \eta_i(t) \Big) \ , \tag{5}$$

where $\eta_i(t)$ is a zero mean, unit variance Gaussian white noise with decomposition given in (3). The losses incurred by process i are then updated according to

$$L_i(t + \Delta t) = L_i(t) + n_i(t + \Delta t) X^i_{t+\Delta t} \ , \tag{6}$$

where $X^i_{t+\Delta t}$ is randomly sampled from the loss severity distribution for process i. Note that the *process dynamics* is *independent* of assumptions concerning their loss severity distributions within the present model.

One can integrate over the distribution of *idiosyncratic* noises to obtain the conditional probability for failure of process i given a configuration $n(t) = \{n_i(t)\}$ of down-processes and a realization of the common factor $Y(t)$ at time t,

$$\begin{aligned} \langle n_i(t + \Delta t) \rangle_{n(t),Y(t)} &\equiv \text{Prob}\left(n_i(t + \Delta t) = 1 \Big| \, n(t), Y(t) \right) \\ &= \Phi \left(\frac{\Phi^{-1}(\text{PD}_{i,\Delta t}) + \sum_j w_{ij} \, n_j(t) - \sum_k \beta_{ik} Y_k(t)}{\sqrt{1 - \sum_k \beta_{ik}^2}} \right) \ . \end{aligned} \tag{7}$$

Here, $\Phi(x)$ denotes the cumulative normal distribution. Note that up to the functional term "$\sum_j w_{ij} \, n_j(t)$" this formula equals the conditional default probability for credit defaults derived in the one-factor Vasicek approach. This formalism is the methodological basis for the Basel II capital charge for credit risk [2].

In (7) we have used the fact that $\vartheta_i = -\Phi^{-1}(\text{PD}_i)$, where $\text{PD}_{i,\Delta t}$ is the *unconditional* expected probability for process failures within the time-increment Δt. This identity is obtained by setting $n_j(t) = 0$ for all j, and by integrating over the noise $\eta_i(t)$ in (5).

The couplings w_{ij} can be determined by considering the transition probabilities, $\text{PD}_{ij,\Delta t}$, for process i failure within the time-increments Δt, given that in the configuration at time t process j is down,

$$\text{PD}_{ij,\Delta t} = \text{Prob}\left(n_i(t + \Delta t) = 1 \Big| \, n_j(t) = 1, \{n_k(t) = 0, k \neq j\} \right) = \Phi\Big(\Phi^{-1}(\text{PD}_{i,\Delta t}) + w_{ij} \Big) \ . \tag{8}$$

This leads to

$$w_{ij} = \Phi^{-1}(\text{PD}_{ij,\Delta t}) - \Phi^{-1}(\text{PD}_{i,\Delta t}) \ . \tag{9}$$

[2] This can always be assured by a simple rotation of common risk factors. The $Y_k(t)$ would then be linear combinations of those assumed initially.

Analogous identities would be available for determining higher order connections w_{ijk}, if nonlinear effects were taken into account. Probabilities for process failure only depend on the increment Δt and not on the time t due to the stationarity of the dynamics. To illustrate how these parameter are fixed in practice, consider the following. Either from a historical loss database or from an expert assessment the following two questions must be answered per OR-risk category and business line:

1. What is the expected period, $\langle \tau_i \rangle$, until process i fails for the first time in a fully operative environment, and
2. given that only process j has failed, what is the expected period, $\langle \tau_{ij} \rangle$, for process i to fail also?

Noting that with Prob(failure at $z\Delta t$) $= \big(1 - \mathrm{PD}_{i,\Delta t}\big)^{z-1} \mathrm{PD}_{i,\Delta t}$ one finds that

$$\langle \tau_i \rangle = \sum_{z=1}^{\infty} z\Delta t \left(1 - \mathrm{PD}_{i,\Delta t}\right)^{z-1} \mathrm{PD}_{i,\Delta t} = \frac{\Delta t}{\mathrm{PD}_{i,\Delta t}} \ , \tag{10}$$

and analogously,

$$\langle \tau_{ij} \rangle = \sum_{z=1}^{\infty} z\Delta t \left(1 - \mathrm{PD}_{ij,\Delta t}\right)^{z-1} \mathrm{PD}_{i,\Delta tj} = \frac{\Delta t}{\mathrm{PD}_{ij,\Delta t}} \ . \tag{11}$$

These identities express the $\mathrm{PD}_{i,\Delta t}$ and $\mathrm{PD}_{ij,\Delta t}$ in terms of estimated average times of failure, and are used to fix the model parameters completely. Note that according to (8) $\mathrm{PD}_{ij,\Delta t}$ can be interpreted as a non-equal time correlation for process failures.

The dynamics (5) resembles that of a lattice gas (on a graph rather than on a lattice), the n_i denoting occupancy of a vertex, the w_{ij} interactions, and ϑ_i taking the role of chemical potentials regulating a-priori occupancy of individual vertices. The system is heterogeneous in that (i) the ϑ_i vary from site to site, (ii) the couplings w_{ij} have a functional rather than regular geometric dependence on the indices i and j designating the vertices of the graph. In the physics context, one usually assumes noise sources other than Gaussian so that cumulative probabilities are described by Fermi-functions rather than cumulative normal distributions as above. The quantitative difference is minute, however.

The model dynamics as such cannot be solved analytically for a general heterogeneous network. We shall resort to Monte Carlo simulations to study its salient properties. As the presence of the common factor expressed by the β_{ik}-term in Eq. (3) would influence only quantitative details of the system's behavior, we will further present the analysis without correlation to the common factors by setting $\beta_{ik} \equiv 0$.

2.2 Main Features

The key features of the collective behavior of networks of interacting processes can easily be anticipated either directly from a discussion of the dynamic rules, or from the analogy with the physics of lattice gasses.

In a network in which the unconditional probabilities for process failures, $\mathrm{PD}_{i,\Delta t}$, are small, but process interdependence is large and consequently *conditional* probabilities for process failures,

$\mathrm{PD}_{ij,\Delta t}$, are sizable, spontaneous failure of individual processes may induce subsequent failures of other processes with sufficiently high probability so as to trigger a breakdown of the whole network. If, on the other hand, process interdependence remains below a critical threshold value, individual spontaneous failures will not have such drastic consequences, and the whole network will remain in a stable overall functioning state.

Of particular interest for the risk manager is the case, in which process interdependence is low enough to make a *self-generated break-down* of the network extremely unlikely, but parameters are nevertheless such that a stable overall functioning state of the network *coexists* with a phase in which nearly the complete network is in the down state (two phase coexistence). In such a situation, it may be *external strain* which can induce a transition from a stably functional situation to overall breakdown. Analogous mechanisms are believed to be responsible for occasional catastrophic breakdowns in bistable ecosystems [9].

With increasing unconditional probabilities for process failures it becomes meaningless to distinguish between an overall functioning and a non-functioning phase of the network. The two-phase coexistence ceases to exist—as in (lattice) gasses—at a critical point.

3 Simulations

In the following we illustrate the validity of our intuitions about global network behavior using Monte Carlo simulations.

Monte Carlo dynamics can either be performed as parallel dynamics (all n_i are at each time step simultaneously updated according to (5) or (7)), or as (random) sequential dynamics (only a single n_i is (randomly) selected for update according to (5) in any given time-step, in which case the time increment must scale with the number N of processes in the net as $\Delta t \sim N^{-1}$).

For the analysis of operational risks, losses are accumulated during a Monte Carlo simulation of the process dynamics over the risk horizon T. Runs over *many* risk horizons then allow to measure loss distribution functions for individual processes within the network of interacting processes, or of business units or the full network by appropriate summations.

In what follows, we are not looking at a specific organization, but rather choose a random setting to illustrate the generic features of our model. That is, unconditional failure probabilities are taken to be homogeneously distributed in the interval $[0, p^{\max}]$ and we determine random conditional failure probabilities as $\mathrm{PD}_{ij,\Delta t} = \mathrm{PD}_{i,\Delta t}(1 + \varepsilon_{ij})$, with ε_{ij} homogeneously distributed in $[0, \varepsilon^{\max}]$, which fixes the ratio $(\mathrm{PD}_{ij,\Delta t}/\mathrm{PD}_{i,\Delta t})^{\max}$.

3.1 Going Concern Analysis

Fig. 1 shows a situation where a functional network coexists with a situation in which the network is completely down, and parameters are such that spontaneous transitions between the phases are not observed during a simulation.

The loss distribution for the *functional* network is unimodal with a bulk of small losses and a fat tail of extreme losses, which are driven by the loss severity distribution.

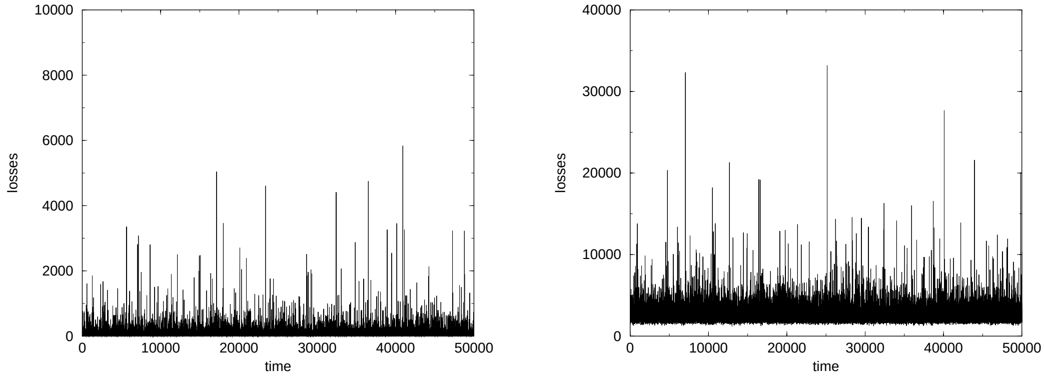

Figure 1: Loss record for a system of $N = 50$ interacting processes with (first panel) $p^{\max} = 0.025$ and $(\mathrm{PD}_{ij,\Delta t}/\mathrm{PD}_{i,\Delta t})^{\max} = 1.6$. For identical system parameters, the low-loss situation coexists with a high-loss situation (shown in the right panel: losses are elevated at all times as witnessed by the lower bound of the loss record in the right panel). Although a spontaneous total breakdown of the operational system (left) into the non-operational high-loss phase (right) does not occur during the simulation, external influences may well induce such a transition.

By increasing the functional interdependence at unaltered unconditional failure probabilities, the functioning state of the network becomes unstable as shown in Fig. 2. A spontaneous transition into the 'down' state is observed during a single run of 50,000 Monte Carlo steps (note that a time step can for many OR categories be associated with a day; thus though the system appears to be stable over a very long time span, it is, in fact not). Two interesting features about this transition to complete breakdown deserve mention: (i) the time to breakdown can vary within very wide limits (not shown here), (ii) there are no detectable precursors to the transition; it occurs due to large spontaneous fluctuations carrying the system over a barrier, in analogy to droplet formation associated with first order phase transitions.

We should like to emphasize that realistically the system dynamics after an overall break-down of a process network would no longer be the spontaneous internal network dynamics: recovery efforts would be started, increasing support for each process by a sufficient amount such as to reinitialize the network in working order.

3.2 Stress Analysis

One of the critical lessons for Risk Control from our analysis is the possible metastability of networks of interacting processes: the organization would not necessarily realize the potential of big losses due to bursts and avalanches of process failures, as there are no detectable precursors to such transitions. With a basically unchanged process setup the network could collapse and cause significant losses, either due to external strain or rare fluctuations of internal dynamics. Owing to the stability of the metastable states, the bank will then have to spend a lot of efforts in order to bring the network back to a functional state, which will cause additional costs. To assess the metastability, one will have to perform stress tests. Indeed, the suitability for stress tests is one of

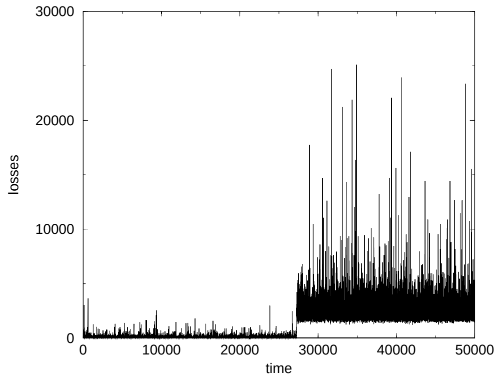

Figure 2: Loss record in a system with the same unconditional default probabilities as in Fig. 1, but $(\mathrm{PD}_{ij,\Delta t}/\mathrm{PD}_{i,\Delta t})^{\max}$ increased to 1.8. A spontaneous breakdown of the system is observed after about 27,000 time steps

the main virtues of the present model.

Fig. 3 illustrates the results of a stress simulation. In each case, the system, if in the operational state, is repeatedly put under external strain by turning off 5 randomly selected functioning processes every $1{,}000^{\mathrm{th}}$ time step, and letting the system evolve under its internal dynamics thereafter. Such a disturbance can either trigger a breakdown of the system or not. In the former case, if the system is found fully down 1,000 time steps later, it is reinitialized in the fully operational state and once more disturbed 1,000 steps later.

One observes that the operational low-loss phase is resilient against disturbances of the kind described above when $(\mathrm{PD}_{ij,\Delta t}/\mathrm{PD}_{i,\Delta t})^{\max} = 1.7$, although the low-cost phase coexists with a phase of catastrophic break-down, whereas at $(\mathrm{PD}_{ij,\Delta t}/\mathrm{PD}_{i,\Delta t})^{\max} = 1.8$ external strain occasionally succeeds to trigger breakdown of the net; on the other hand, at $(\mathrm{PD}_{ij,\Delta t}/\mathrm{PD}_{i,\Delta t})^{\max} = 1.8$ breakdown under external strain of the given strength is the typical response of the system (with a few exceptions and occasional spontaneous recoveries).

Such a stress analysis would also provide a tool to deal with errors in the parameter estimation.

3.3 Capital Adequacy and Capital Allocation

Of interest to the risk manager in the end are the total losses accumulated over a risk horizon, T,

$$L(T) = \sum_i L_i(T) \; ; \tag{12}$$

more specifically, the corresponding probability density function. Fig. 4 presents such distribution of accumulated losses for a network that remains operational throughout the simulation. For this simulation we took $T = 365\Delta t$. The loss distribution has an extended tail (barely visible on the scale of the data, with a 99.5% quantile at 3.07×10^4 and the largest aggregated loss observed during the simulation over a time span of T at 2.5×10^5, i.e., more than an order of magnitude

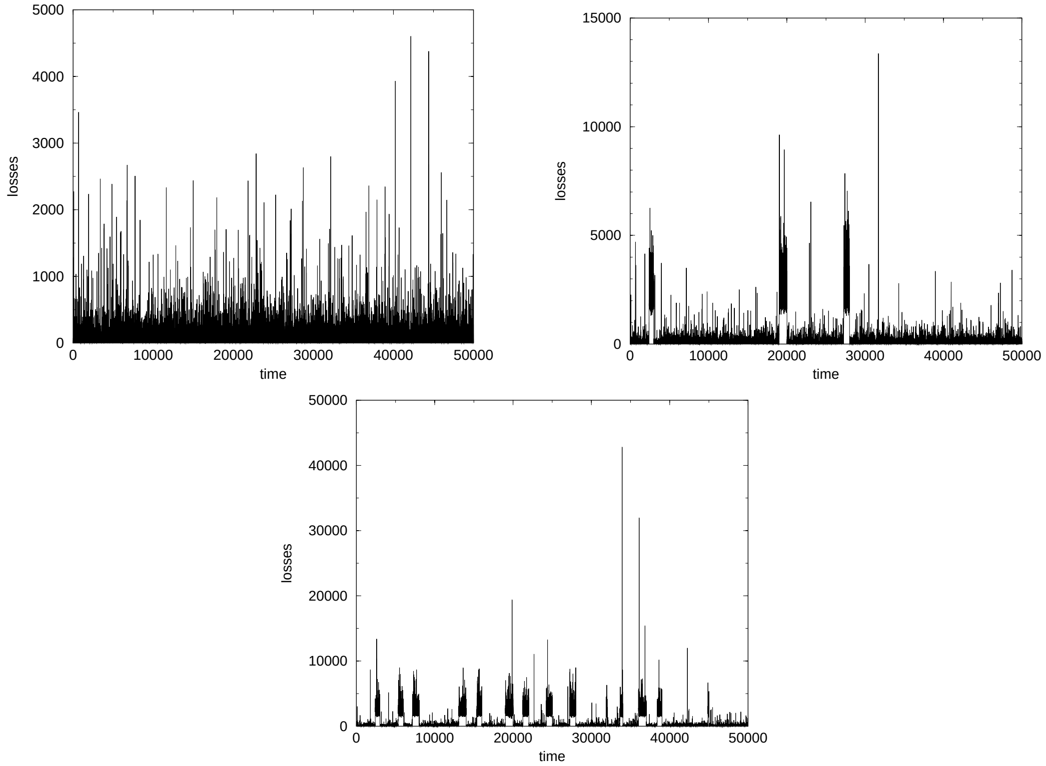

Figure 3: Stress simulations described in the main text. The parameters are $p^{\max} = 0.025$ as in the previous figures and (clockwise) $(\mathrm{PD}_{ij,\Delta t}/\mathrm{PD}_{i,\Delta t})^{\max} = 1.7,\ 1.72,\ 1.8$.

larger than the expected loss for the chosen risk horizon T in the present case. A scatter-plot also shown in the figure reveals that it is the spread in the loss-severity distributions which is primarily responsible for the extended tail of the loss distribution.

The histogram reveals the adequate capital to be allocated to the process map underlying the simulation. Possible risk measures are the q-quantile (in excess of the expected loss, if this is accounted for elsewhere) or the expected shortfall. By repeating the simulations with different process maps for different business lines, branches or subsidiaries, the bank can decide which capital amount is best allocated to these sub-units.

4 Conclusion

We have described how ideas from physics of collective phenomena and phase transitions can naturally be applied to modeling operational risk in financial institutions, or indeed any other form of organization. Our main point was that functional dependencies between mutually supportive processes give rise to non-trivial temporal correlation, which can lead to the occurrence of collective

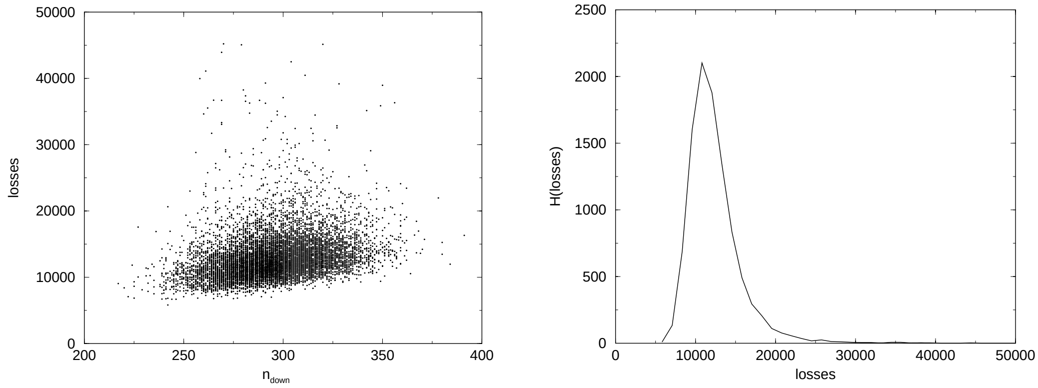

Figure 4: Scatter-plot of number of risk-events during a risk horizon of $T = 365\Delta t$ vs. total amount of incurred losses (left panel); loss distribution (right panel). Total time covered was $10^4 T$. The histogram is not normalized. Parameters of the system are $N = 50$, $p^{\max} = 0.025$, and $(\mathrm{PD}_{ij,\Delta t}/\mathrm{PD}_{i,\Delta t})^{\max} = 1.6$, loss severity distributions are taken as log-normal, with means randomly spread over an interval [0,100] and volatilities chosen randomly as a factor of their respective means, the maximum factor being 0.1.

risk events in the form of burst, avalanches and crashes. For risks associated to process failure (operational risks) a functional dependence seems to be the appropriate way for modeling sequential correlations.

We have shown how the adequate capital for operational risk can be calculated based on a dynamics which is defined on a topological process map. The key parameters of the dynamics, the unconditional and conditional loss probability and the loss severity, can be obtained for realistic situations via expert assessment or from loss databases. Defining these maps on different hierarchical levels within an organization allows for appropriate allocation of capital to sub-units.

The critical lessons for Risk Control from our analysis is the possible metastability of networks of interacting processes: The organization would not necessarily realize the potential of big losses due to bursts and avalanches of process failures, as there are no detectable precursors to such transitions. With a basically unchanged process setup the network could collapse and cause significant losses, either due to external strain or rare fluctuations of internal dynamics. To assess the metastability, one will have to perform stress tests. Indeed, the suitability for stress tests is one of the main virtues of the present model.

References

[1] Basel Committee on Banking Supervision, *Sound Practices for the Management and Supervision of Operational Risk*, URL: `http://www.bis.org`, (Basel, February 2003).

[2] Basel Committee on Banking Supervision, *The New Basel Capital Accord*, The Thrid Consultative Paper, URL: `http://www.bis.org`, (Basel, April 2003).

[3] G.-J. van den Brink, *Operational Risk, the new challenge for banks*, Palgrave, (Hampshire (UK), 2001); *Die Bedeutung operativer Risiken für Eigenkapitalunterlegung und Risikomanagement*, Tagungsbericht vom Duisburger Bank-Symposium, Rolfes (Editor) (Duisburg, 2002).

[4] P. Georges A. Frachot and T. Roncalli, *The loss distribution approach for operational risk*, Working Paper, Credit Lyonnais (2000)

[5] R. Kühn and P. Neu, *Functional correlation approach to operational risk in banking organizations*, Physica **A 322**, 650-666 (2003)

[6] M. Leippold and P. Vanini, *The quantification of operational risk*, working paper, University of Southern Switzerland (2003)

[7] D. Egloff, M. Leippold, and P. Vanini, *Credit risky portfolios with micro- and macro-economic dependencies*, working paper, University of Southern Switzerland (2003)

[8] P. Neu and R. Kühn, *Credit risk enhancement in a network of interdependent firms*, accepted for publication in Physica **A** (2004)

[9] M. Scheffer, S. Carpenter, J. A. Foley, C. Folke, and B. Walker, *Catastrophic shifts in ecosystems*, Nature, **413** 591 (2001).