Derivatives and Credit Contagion in Financial Networks

Reimer Kühn

Disordered Systems Group

Department of Mathematics, King's College London, UK

http://www.mth.kcl.ac.uk/~kuehn

Heidelberg, July 7, 2011

This Talk

- Part of research programme devoted to understanding

  influence of interactions on systemic risk

  So far looked at

  - Operational Risk (OR): interacting processes
  - Market Risk (MR): interacting Geometric Brownian Motions
  - Credit Risk (CR): economic interaction and credit contagion

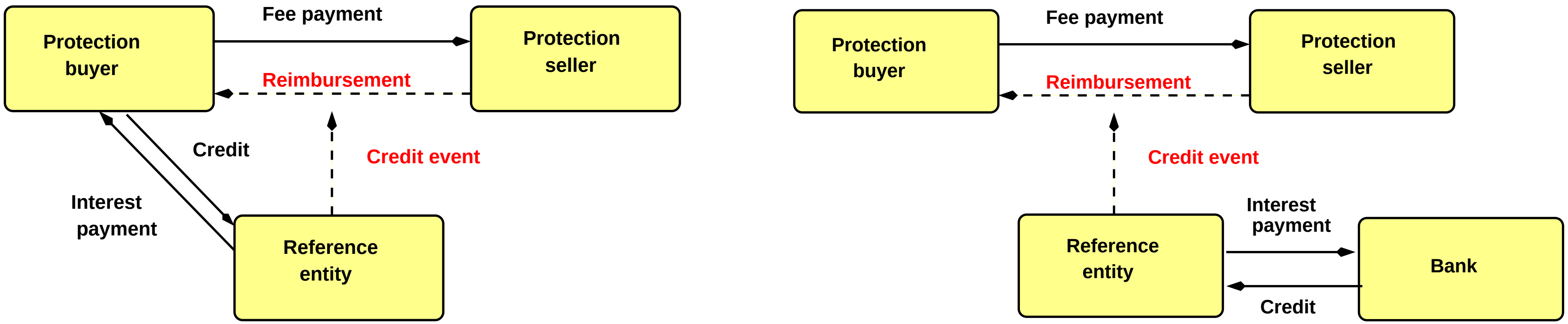
- Here: **interactions due to Credit Default Swap contracts**

  Work done with Sebastian Heise, now @ Economics Dept., Yale.

Motivation

- Credit defaults clustered around times of economic stress
  - dependency on macro-economic factors
  - credit contagion
- Contagion dynamics radically changed in the last decade through Credit Default Swaps
- Role of of counter-party risk in pricing derivative contracts looked at in Hull and White (2001); Errais, Giesecke and Goldberg (2007); Haworth and Reisinger (2007); Haworth, Reisinger and Shaw (2008); Frey and Backhaus (2008); Brigo and Chourdakis (2009); Frey and Backhaus (2010)
- Hardly any research on influence of CDS on contagion dynamics, though recent crisis has clearly highlighted their significance (Lehman – AIG)

Mechanics of CDS



Mechanics of CDS contracts used for hedging and speculation.

- CDS
  - used to manage credit risk (hedging), and for speculation
  - create ‘three-particle’ contagion channels
  - or every CDS position there is a counter-position ($J^i_{kj} = J^k_{ij}$)

Method — Take-Home Message

- Introduce CDS into existing model of credit contagion

  P Neu & RK, Physica A (2004), JPL Hatchett & RK, J Phys A (2006), Quant. Fin. (2009)

- While CDS can help to reduce losses under normal conditions, they cannot eliminate risk completely, and may amplify contagion and losses in times of stress, if used to expand loan books.

Credit Risks — Interacting Companies Model

P Neu & RK, Physica A (2004), JPL Hatchett & RK, J Phys A (2006), Quant. Fin. (2009)

- Two state model:

  company up and running ($n_i = 0$), or defaulted ($n_i = 1$)

- Probabilities of default and mutual impacts of defaults (exposures) heterogeneous across the set of companies (quenched disorder); connectivity functionally defined

  $\Longrightarrow$ lattice gas model defined on random graph

- Losses determined (randomly: recovery process) when a company defaults (annealed disorder)

Wealth Dynamics I: Firms

- Companies need "orders" (support, cash inflow) to maintain wealth and avoid default

- W_{it} wealth position of firm i at time t,

$$W_{it} = \vartheta_i - L_{it} - \eta_{it} = \vartheta_i - \sum_{j \in F} J_{ij} n_{jt} - \eta_{it}$$

- Noise η_{it} idiosyncratic & economy-wide (minimal Basel II)

$$\eta_{it} = \sigma_i \left(\sqrt{\rho_i}\, \eta_0 + \sqrt{1 - \rho_i}\, \xi_{it} \right)$$

- Company i defaults, if the total wealth falls below zero

$$n_{it+1} = n_{it} + (1 - n_{it}) \Theta(-W_{it})$$

- No recovery within 'risk horizon' T: $n_i = 1$ is absorbing state. Time unit: 1 month; $T = 12 \Leftrightarrow 1$ year. $\Rightarrow$ no equilibrium dyn.

Relatiing Parameters to Default Probabilities

- Probability of failure in given 'situation' ($\sigma_i = 1$)

$$\mathsf{Prob}\Big(n_{it+\Delta t} = 1 \Big| \boldsymbol{n}(t)\Big) = \mathsf{Prob}\Big(W_{it} < 0\Big) = \Phi\left(\sum_j J_{ij}\, n_{jt} - \vartheta_i\right)$$

with

$$\Phi(x) = \int_{-\infty}^{x} \frac{\mathrm{d}z}{\sqrt{2\pi}} \mathrm{e}^{-z^2/2}$$

- unconditional and conditional probability of failure

$$\begin{aligned} p_i &= \Phi(-\vartheta_i) \\ p_{i|j} &= \Phi(J_{ij} - \vartheta_i) \end{aligned}$$

$$\Rightarrow \quad \vartheta_i = -\Phi^{-1}(p_i)\ , \quad J_{ij} = \Phi^{-1}(p_{i|j}) - \Phi^{-1}(p_i)$$

Wealth Dynamics II: Banks and Insurers

- Banks and insurers engage in several types α of interaction among each other, and with firms:
  - direct exposures (d),
  - unhedged loans (u),
  - hedged loans (hb),
  - protection selling for hedged loans (hs),
  - speculative buying/selling (sb/ss) of CDS

$$W_{it} = \vartheta_i - \sum_{\alpha} L_{it}^{\alpha} - \eta_{it}$$

- Wealth dynamics as for firms:

$$n_{it+1} = n_{it} + (1 - n_{it})\Theta(-W_{it})$$

Types of Losses

- Direct exposures: material impact of default (as for firms)

$$L_{i,t}^{(d)} = \sum_{j} J_{ij}^{(d)} n_{j,t}$$

- Unhedged loans: losses through defaults, income from interest payments

$$L_{i,t}^{(u)} = \sum_{j \in F} J_{ij}^{(u)} \sum_{\tau=1}^{t} [(n_{j,\tau} - n_{j,\tau-1}) - \epsilon_{ij,\tau}]$$

- Hedged loans: losses through (coincident) defaults & fees, income from interest

$$L_{i,t}^{(hb)} = \sum_{j \in F} \sum_{k \in B,I} J_{ij}^{k} \sum_{\tau=1}^{t} \left[(n_{j,\tau} - n_{j,\tau-1}) n_{k,\tau} + f_{ij,\tau}^{k} - \epsilon_{ij\tau}\right]$$

- Protection selling: Losses through credit events, fee income

$$L_{i,t}^{(hs)} = \sum_{j \in F} \sum_{k \in B} J_{ij}^{k} \sum_{\tau=1}^{t} \left[(n_{j,\tau} - n_{j,\tau-1})(1 - n_{i,\tau}) - f_{ij,\tau}^{k}\right]$$

- Speculative protection buying: income from credit events, fee-payments

$$L_{i,t}^{(sb)} = -\sum_{j \in F} \sum_{k \in B} K_{ij}^{k} \sum_{\tau=1}^{t} \left[(n_{j,\tau} - n_{j,\tau-1})(1 - n_{k,\tau}) - f_{ij,\tau}^{k}\right]$$

- Speculative protection selling: losses from credit events, fee income

$$L_{i,t}^{(ss)} = \sum_{j \in F} \sum_{k \in B} K_{ij}^{k} \sum_{\tau=1}^{t} \left[(n_{j,t} - n_{j,\tau-1})(1 - n_{i\tau}) - f_{ij,\tau}^{k}\right]$$

Interest and Fees

- Interest payments in month τ on a unit loan

$$\epsilon_{ij,\tau} = \epsilon_{ss'}(1+\epsilon_{ss'})^{\tau-1}(1-n_{j,\tau}) \ .$$

Interest paid only while creditor is alive; rate chosen to depend only on sectors $s = s(i)$, $s' = s'(j)$.

- Fee payments in month τ for protection of a unit loan

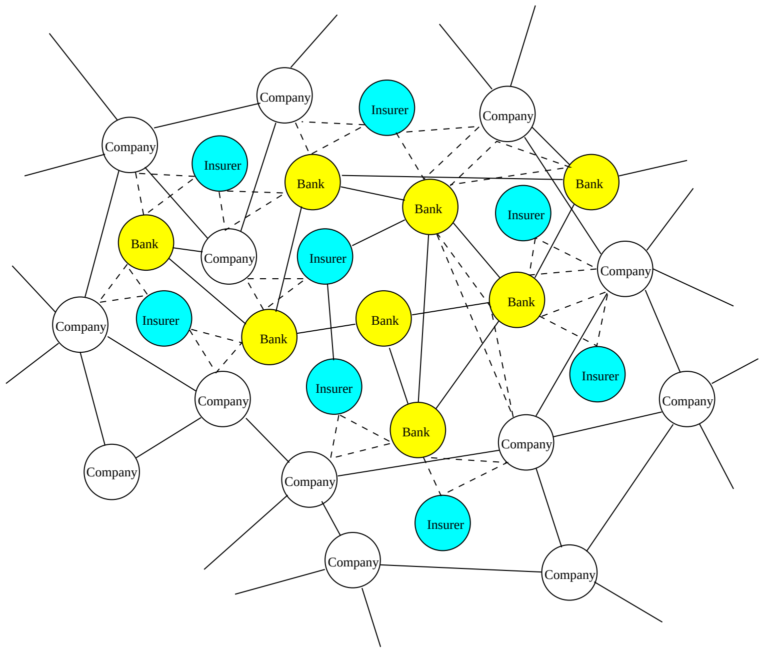
$$f^k_{ij,\tau} = (f_0 + f\, y^k_{ij,\tau})(1-n_{i,\tau})(1-n_{j,\tau})(1-n_{k,\tau}) \ .$$

— paid only as long as all counterparties are still alive
— rates fluctuate (offsetting trades).

- Because of interest and fees and structure of contagion-losses in CDS contracts, dynamics is non-Markovian.

Analysis for a Stochastic Setting

- Interactions J_{ij} and J_{ij}^{k} on a random graph
- Assume: large number of interactions (loans, CDS contracts)



Network of financial dependencies.

Full lines: direct exposures and unhedged loans. Triangles: CDS contracts.

Exposures

- For direct exposures and unhedged loans

$$J_{ij} = c_{ij}\left(\frac{\bar{J}_{rs}}{C_{rs}} + \frac{J_{rs}}{\sqrt{C_{rs}}}\, x_{ij}\right) \ ,$$

with $r = r(i)$ and $s = s(j)$ sectors of counterparties; connectivities such as to create a sparse (modular) Erdös-Renyi random graphs, $1 \ll C_{rs} \ll N_s$.

- For hedged exposures in a similar fashion

$$J_{ij}^{k} = c_{ij}^{k}\left(\frac{\bar{J}_{b,r}^{s}}{C_{b,r}^{s}} + \frac{J_{b,r}^{s}}{\sqrt{C_{b,r}^{s}}}\, x_{ij}^{k}\right)$$

creating Erdös-Renyi random graphs of hyperedges (triangles) $1 \ll C_{b,r}^{s} \ll N_r N_s$

Heuristic Solution (1)

- In dynamical evolution

$$n_{it+1} = n_{it} + (1 - n_{it})\Theta\left(\sum_\alpha L_{it}^\alpha - \vartheta_i + \eta_{it}\right)$$

losses L_{it}^α are Gaussian in large networks (CLT).

$$L_{it}^\alpha = \bar{L}_{st}^\alpha + \sqrt{V_{st}^\alpha}\,\zeta_{it}^\alpha\ , \qquad s = s(i)\ , \qquad \zeta_{it}^\alpha \sim \mathcal{N}(0,1)\ .$$

- Means $\bar{L}_{st}^\alpha$ and variances V_{st}^α (from LLN) depend only on low order statistics of the $\{J_{ij}\}$ and the $\{J_{ij}^k\}$ and on the fractions

$$m_{s,\tau} = \frac{1}{N_s}\sum_{i\in s} n_{i,\tau}\ , \qquad \tau \le t$$

of defaulted companies in the various sectors $s \in \{F, B, I\}$.

Heuristic Solution (2)

- Get

$$m_{s,t+1} = \frac{1}{N_s}\sum_i n_{it+1} = \frac{1}{N_s}\sum_i \left[n_{it} + (1-n_{it})\Theta\left(L_{it} - \vartheta_i + \eta_{it}\right)\right]$$

- Macroscopic dynamics via LLN as average over joint L_{it}, ϑ_i and σ_i distributions (at fixed η_0). With conditional probability of default

$$\mathrm{Prob}(n_{it+1} = 1 | \boldsymbol{n}_t, n_{it} = 0, \eta_0) = \Phi\left(\frac{\sum_\alpha \bar{L}^\alpha_{st} + \sigma_i\sqrt{\rho_i}\,\eta_0 - \vartheta_i}{\sqrt{\sum_\alpha V^\alpha_{st} + \sigma_i^2(1-\rho_i^2)}}\right)$$

find

$$m_{s,t+1} = m_{s,t} + \left\langle \Big(1 - \langle n_t(\vartheta,\sigma)\rangle\Big)\Phi\left(\frac{\sum_\alpha \bar{L}^\alpha_{st} + \sigma\sqrt{\rho}\,\eta_0 - \vartheta}{\sqrt{\sum_\alpha V^\alpha_{st} + \sigma^2(1-\rho)}}\right)\right\rangle_{\vartheta,\sigma}$$

Results: Loss Distribution and Defaults in Banking Sector

- Starting point: no CDS

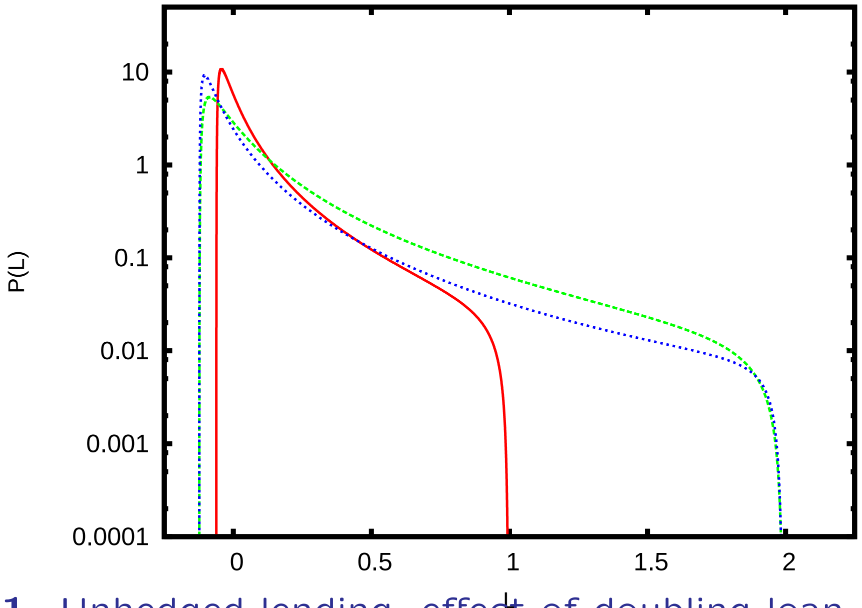


Fig 1. Unhedged lending, effect of doubling loan books
(with firms, half-half firm & inter bank).

- Three scenarios with CDS
  - case 1: B & F, only hedging
  - case 2: B, F & I, only hedging
  - case 3: B, F & I, hedging and speculation

Results: Case 1, Hedging Exposures — Losses

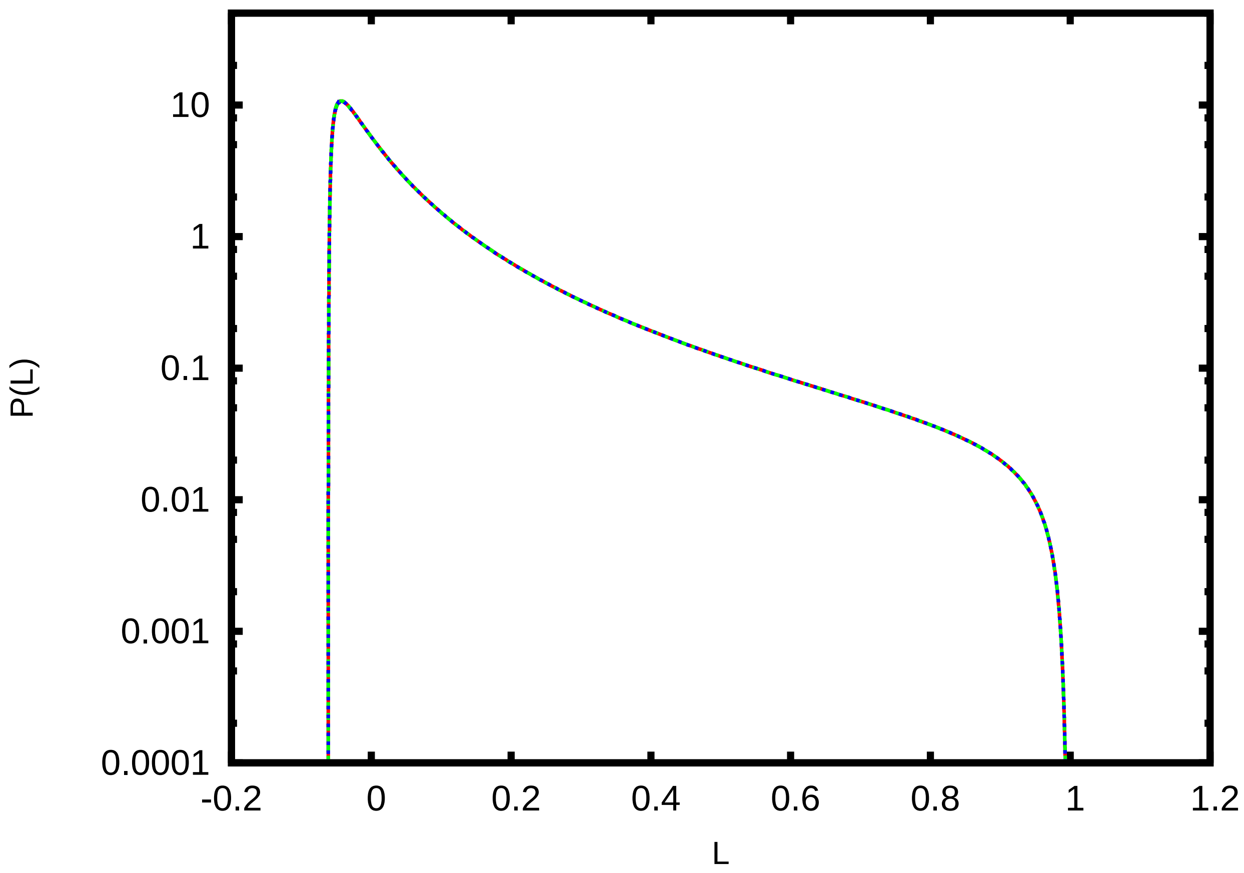


Fig 2a. Case 1: the effect of CDS, hedging exposures **within banking sector**

(unhedged, 1/3 hedged, 2/3 hedged ⇔ **zero-sum game**).

Results: Case 1, Hedging Exposures — Default Rates

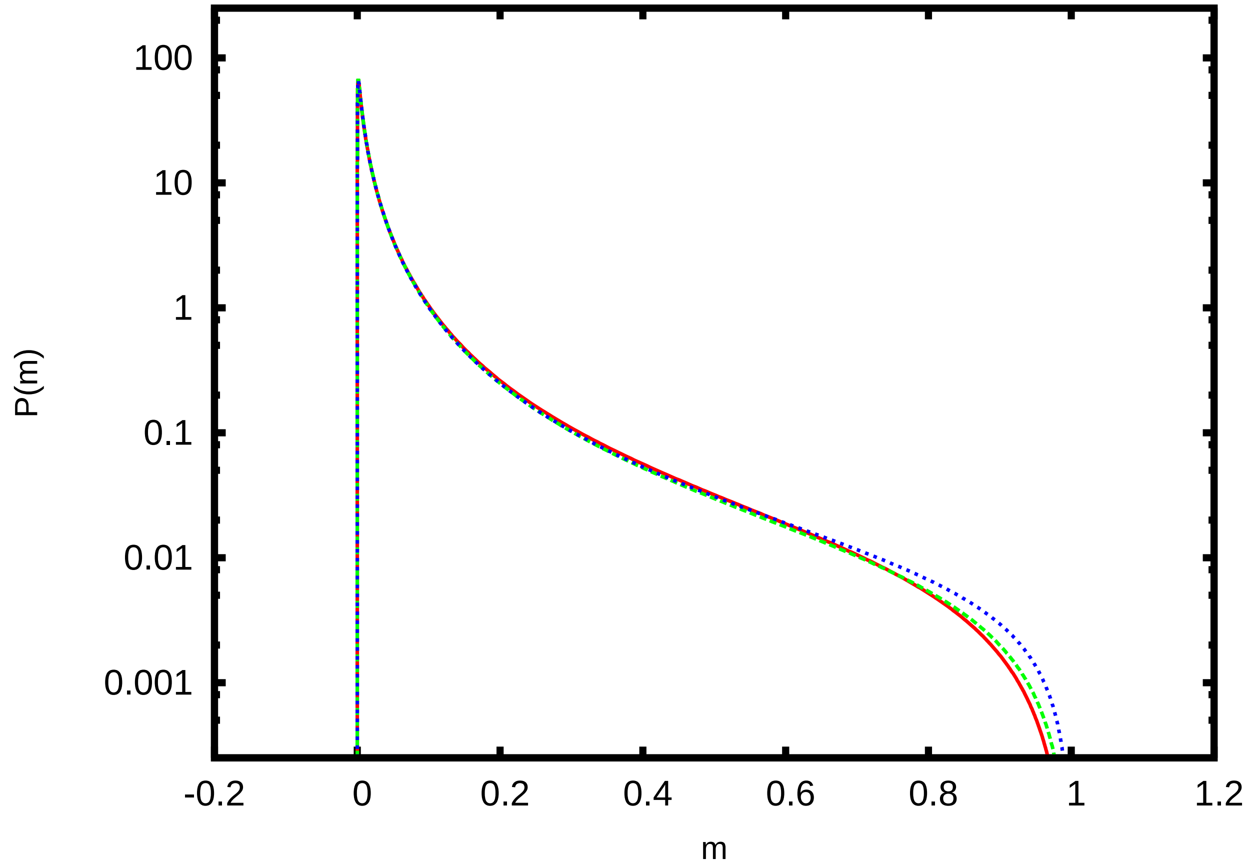


Fig 2b. Case 1: the effect of CDS, hedging exposures **within banking sector**

(unhedged, 1/3 hedged, 2/3 hedged ⇔ **more defaults, despite unchanged loss distribution**).

Results: Case 2, Hedging Exposures with Insurers

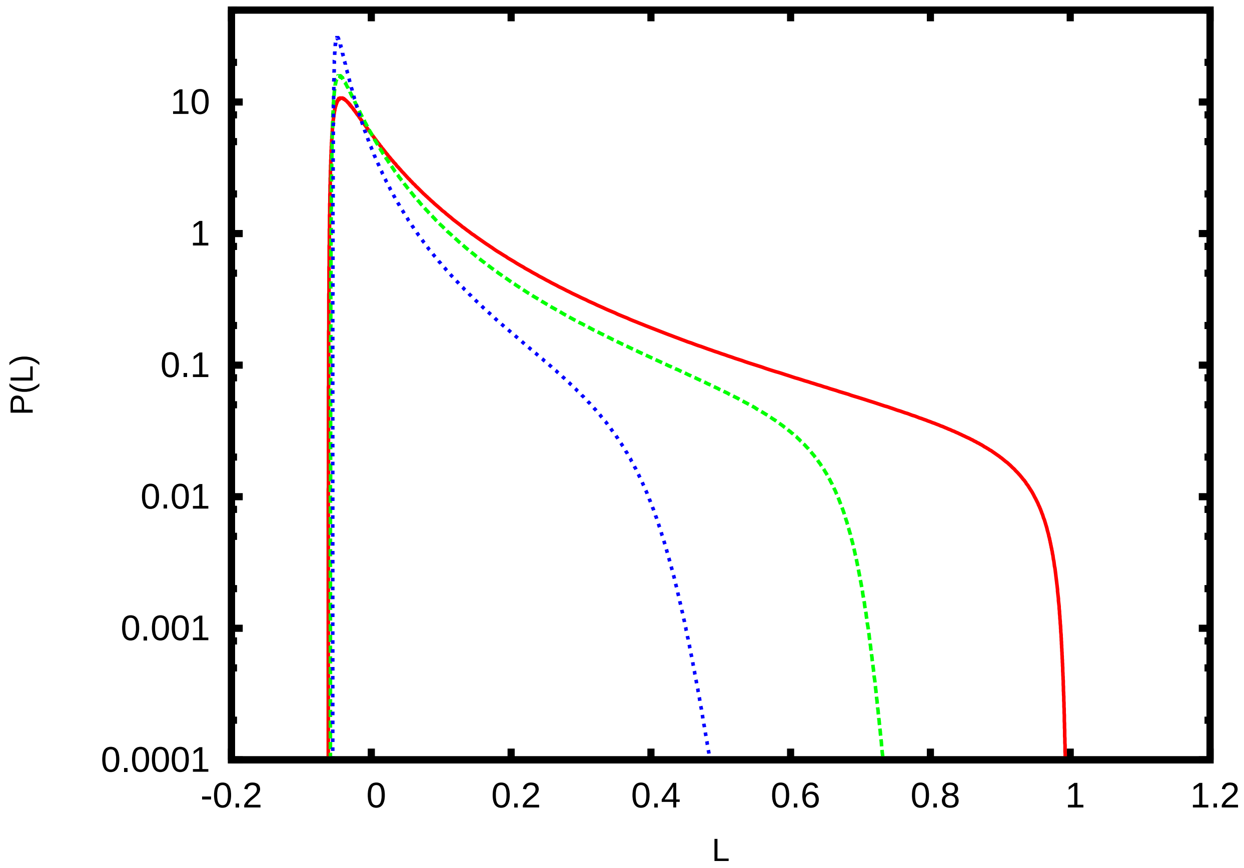


Fig 3. Case 2: the effect of CDS, hedging with insurers.

(unhedged, 1/3 hedged, 2/3 hedged)

Results: Case 2, Hedging Increased Exposures with Insurers

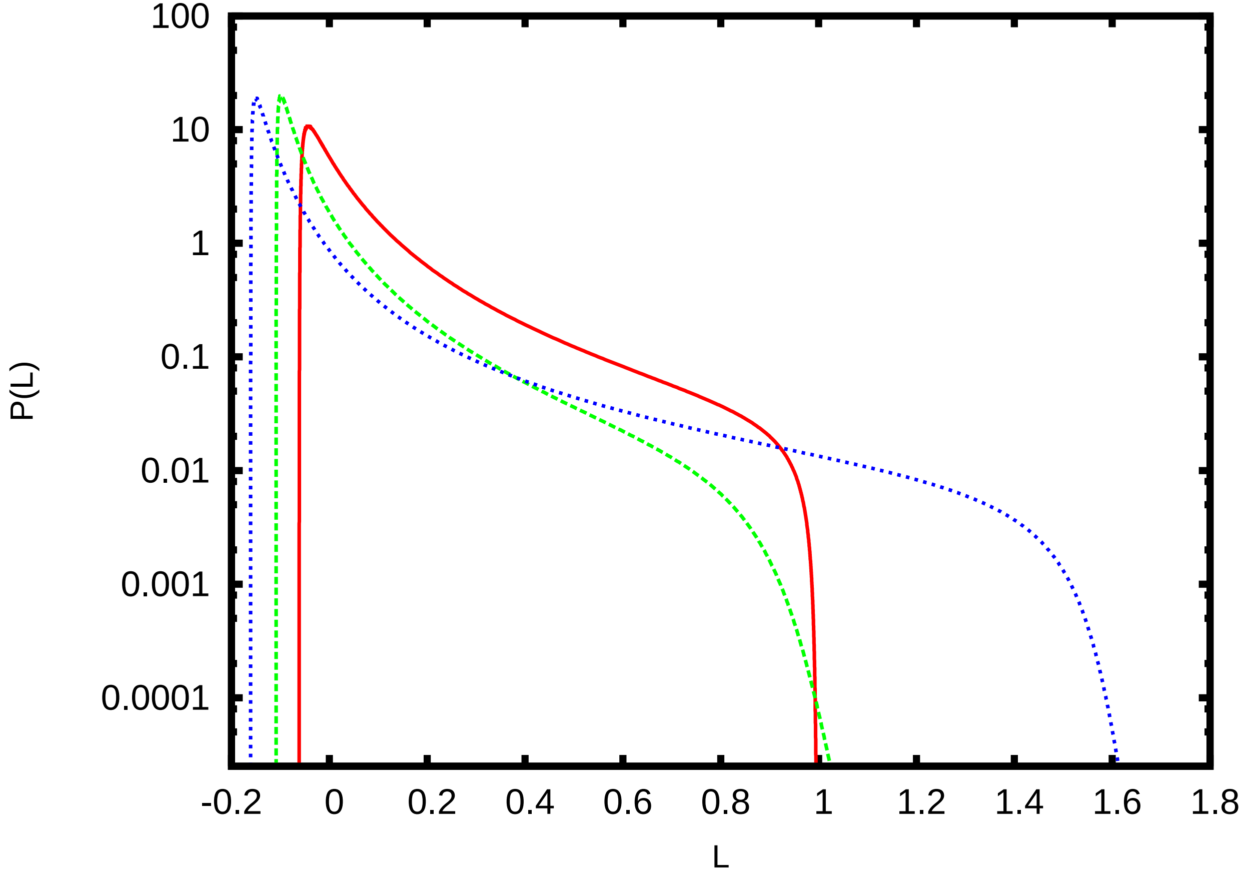


Fig 4. Case 2: the effect of CDS, hedging with insurers @ increased exposures.

Results: Case 3, Adding Speculative CDS

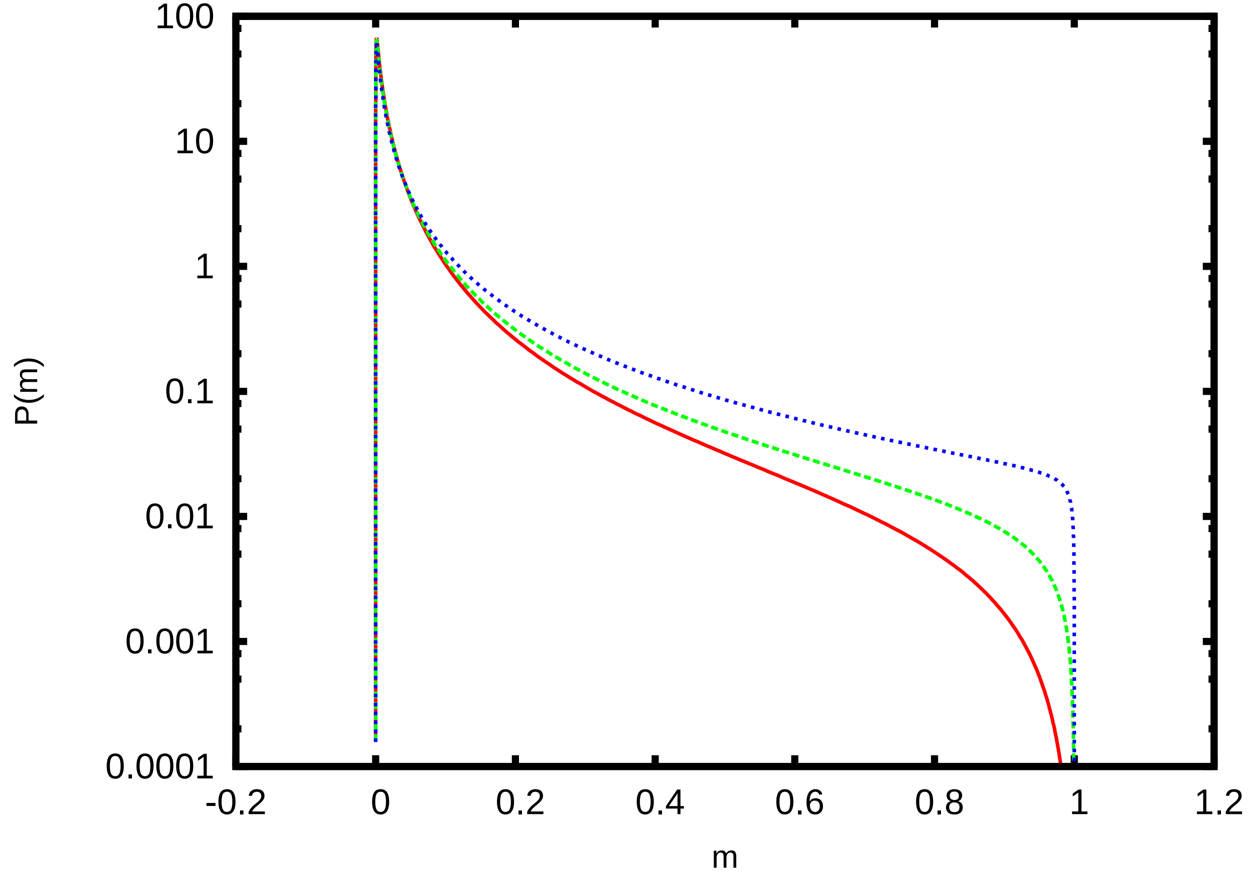


Fig 5a. Case 3: Distribution of the fraction of defaulted banks; the base-line scenario compared with situations where speculative CDS of a volume matching the base-line exposure or twice the volume of the base-line exposure are taken out *inside* the banking sector.

Results: Case 3, Adding Speculative CDS

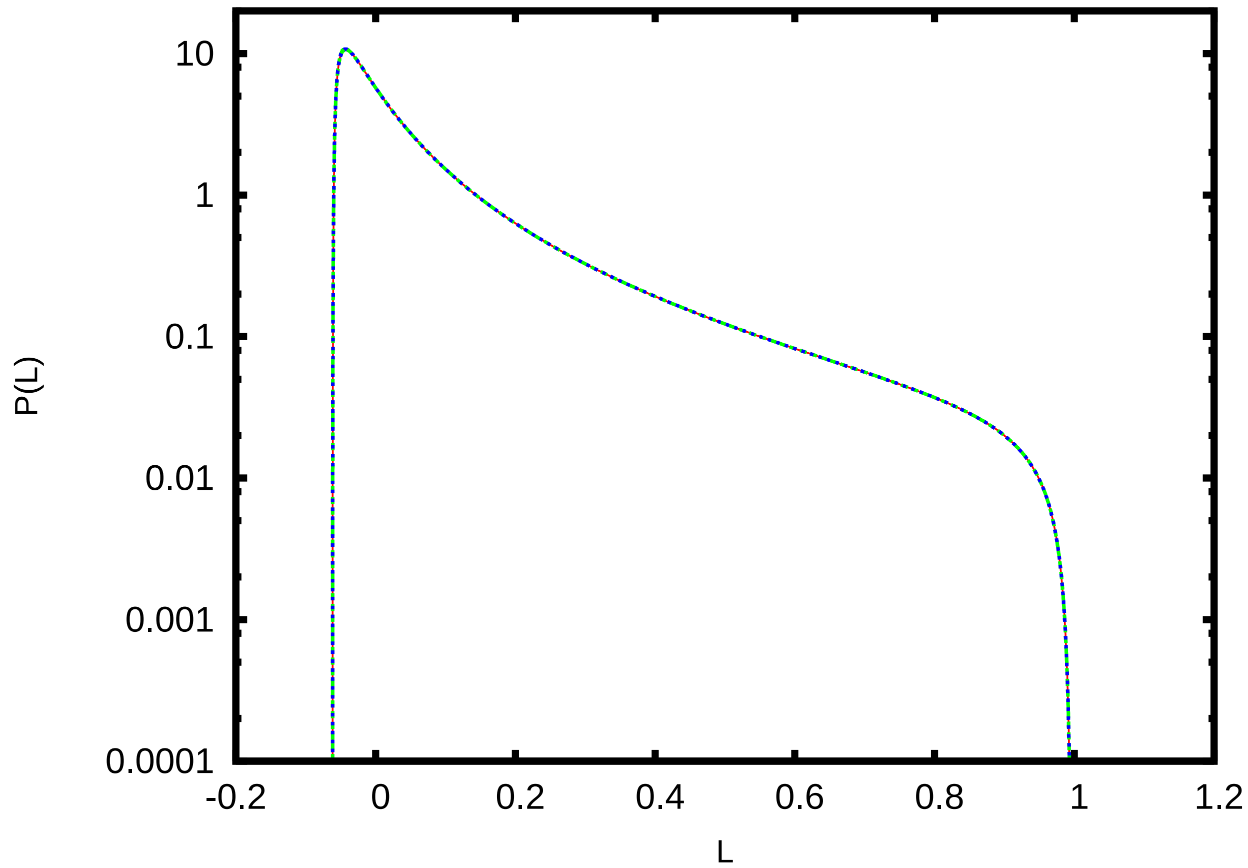


Fig 5b. Case 3: Loss distribution corresponding to the previous slides; Curves are exactly on top of each other (zero-sum game).

Summary

- Looked at stylized model of networks of firms, banks and insurers
- Limit of large number of loans/CDS contracts allows to exploit LLN, CLT to obtain macroscopic dynamics.
- Only low order statistics of interaction effects required.
- Hedging can help to reduce credit risk, but only if size of loan books is not significantly expanded
- CDS contracts provide an additional contagion channel which increases risk of default and losses in times of economic stress.
- Limitations/points for improvement: small banking sector, include feedback of defaulting banks on network of firms, CDS fees correlated with defaults, . . .

Literature

Brigo and Chourdakis (2009) Brigo, D. and Chourdakis, K.,Counterparty Risk for Credit Default Swaps: Impact of spread volatility and default correlation. *International Journal of Theoretical and Applied Finance* **12**, 1007-1026 (2009).

Errais, Giesecke and Goldberg (2007) Errais, E. Giesecke, K., and Goldberg, L., Pricing Credit form the Top Down with Affine Point Processes, Working paper (2007)

Frey and Backhaus (2008) Frey, R. and Backhaus, J., Pricing and Hedging of Portfolio Credit Derivatives with Interacting Default Intensities, *International Journal of Theoretical and Applied Finance*, **11**, 611-634 (2008).

Frey and Backhaus (2010) Frey, R. and Backhaus, J. Dynamic hedging of synthetic CDO-tranches with spread- and contagion risk, *Journal of Economic Dynamics and Control* **34**, 710–724 (2010).

Hatchett and Kühn (2006) Hatchett, J.P.L. and Kühn, R., Effects of economic interactions on credit risk. *Journal of Physics A*, **39**, 2231-2251 (2006).

Hatchett and Kühn (2009) Hatchett, J.P.L. and Kühn, R., Credit contagion and credit risk. *Quantitative Finance*, **9**, 373-382 (2009).

Haworth and Reisinger (2007) Haworth, H. and Reisinger, C., Modeling Basket Credit Defaults Swaps with Default Contagion, *Journal of Credit Risk*, **3** 31-67 (2007).

Haworth, Reisinger and Shaw (2008) Haworth, H., Reisinger, C. and Shaw, W., Modelling Bonds and Credit Default Swaps Using a Structural Model with Contagion, *Quantitative Finance*, **8**, 669-680 (2008)

Hull and White (2001) Hull, J. and White, A., Valuing Credit Default Swaps II: Modeling Default Correlations, *Journal of Derivatives* **8**, 12-22 (2001).

Neu and Kühn (2004) Neu, P. and Kühn, R., Credit risk enhancement in a network of interdependent firms. *Physica A*, **342**, 639-655 (2004).