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Representation and processing of novel stimuli by an associative neural network

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Overview

- Motivation

- The Model

- Collective Properties

  - Firing Rate Distributions

  - Stimuli, Mutual Information, Correlation

- Perspective:
  - Optimise Representations
  - Dynamics

Motivation

- Representation of stimuli in an associative neural networks

  — neural firing patterns, rates and their distributions

- Of interest: differences in firing rate distributions, if new stimulus is correlated with pre-learnt patterns or not.

- For new stimuli correlated with pre-learnt patterns:
  - mutual information between firing rates and novel stimulus
  - dependence on correlation with pre-learnt patterns
  - dependence on other parameters
    (number of memories, thresholds, neural gain-function)

- Theoretical framework for interpretation of recordings using trained-untrained scenarios?
  - untrained animals represent stimuli in existing cognitive structure
  - coginitive structure changes in response to new stimuli
- Results – at least in principle – experimentally accessible

Context

- A Treves et al (Neural Computation, 1999)
  Recordings from inferior temporal cortex, visual stimuli
  rate-distributions non-exponential, fits to assumed current distributions

- N Brunel (J. Comp. Neurosci., 2000)
  Dynamics of sparsely connected networks of leaky IF neurons with uniform synaptic strengths: study of collective network states

- J Hertz et al (Neurocomputing, 2003); q-bio.NC/0402023
  Dynamics of sparsely connected networks of leaky IF neurons with uniform synaptic strengths: computation of Fano factors

The Model

- Graded response neurons, Kirchhoff equations for coupled leaky integrators

$$C_i \frac{dU_i}{dt} = -\frac{U_i}{R_i} + \sum_{j=1}^{N} J_{ij}\, \nu_j + I_i$$

- Firing rates ν_i via voltage-to-rate transduction-function

$$\nu_i = g(U_i - \vartheta_i)$$

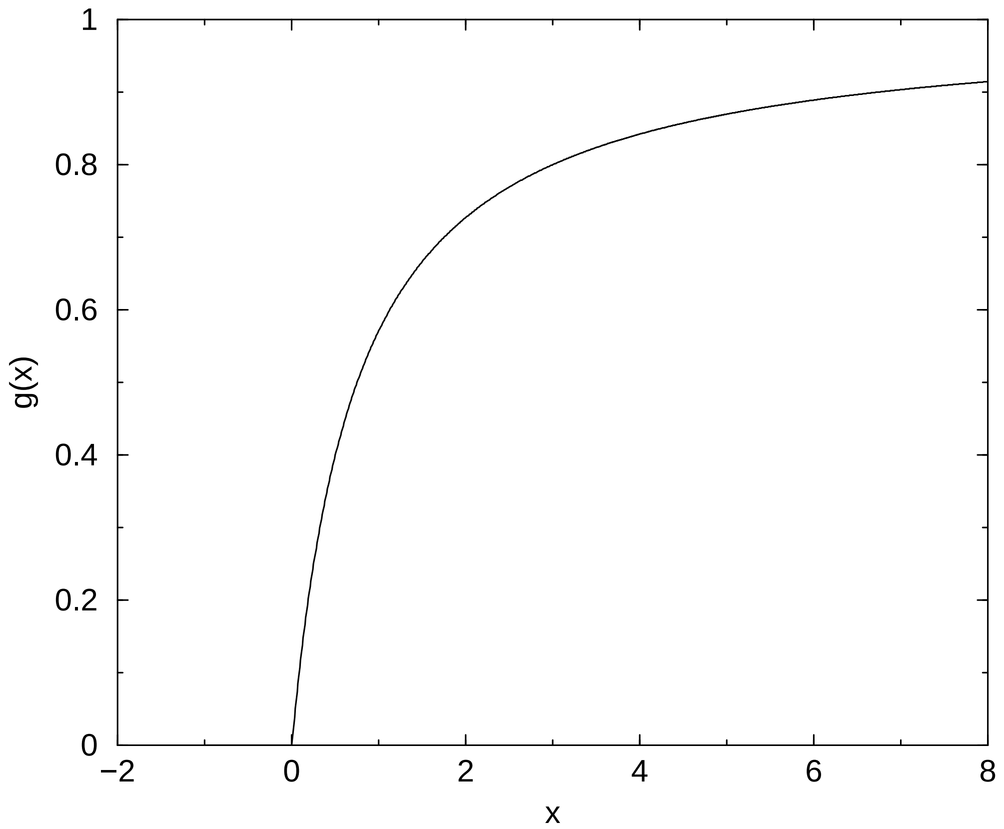
- Synaptic couplings from Hebbian covariance learning rule

$$J_{ij} = \frac{1}{Na(1-a)} \sum_{\mu=1}^{p} (\eta_i^\mu - a)(\eta_j^\mu - a)$$

$$\eta_i^\mu = \begin{cases} 1; & \text{with prob. } a, \\ 0; & \text{with prob. } 1-a, \end{cases} \qquad \text{hence :} \quad \langle \eta_i^\mu \rangle = a$$

- Voltage-to-rate transduction-function for present setup:

$$g(x) = \nu_{\text{max}} \, \frac{x}{U_0 + x} \, \Theta(x)$$



Voltage-to-rate transduction-function, $x = U - \vartheta$, $U_0 = 0.75$, $\nu_{\text{max}} = 1$.

- Present talk: only long time stationary response

- Symmetric couplings: dynamics governed by Lyapunov function

$$H_N(\nu) = -\frac{1}{2} \sum_{i,j=1}^{N} J_{ij}\nu_i\nu_j + \sum_{i=1}^{N} G(\nu_i) - \sum_{i=1}^{N} (I_i - \vartheta_i)\nu_i$$

  with

$$G(\nu) = \int^{\nu} d\nu' g^{-1}(\nu')$$

  $\Longrightarrow$ stationary response from minima of H_N

- Characterisation of attractors (minima of H_N):

  $T \rightarrow 0$-limit of free energy corresponding to H_N

  (RK, S. Bös J. Phys. A, 1993)

Collective Properties

- Stationary limit from equilibrium statistical mechanics.
  - Partition function

$$Z_N = \int \prod_i d\nu_i \,\mathrm{exp}[-\beta H_N(\nu)]$$

  - Free energy

$$f_N(\beta) = -(\beta N)^{-1} \log Z_N$$

  - $T \to 0 \Leftrightarrow \beta \to \infty$-limit: only minima of H_N contribute.
  - A technical point: randomness due to $\{\eta_i^\mu\}$ $\quad \Rightarrow \quad \overline{f_N(\beta)}$
  - Macroscopic characterization of system — order parameters:

$$m^\mu = \frac{1}{N} \sum_i \frac{\eta_i^\mu - a}{a(1-a)} \overline{\langle \nu_i \rangle}\,, \quad q = \frac{1}{N} \sum_i \overline{\langle \nu_i \rangle^2}\,, \quad c = \frac{\beta}{N} \sum_i [\overline{\langle \nu_i^2 \rangle} - \overline{\langle \nu_i \rangle^2}]$$

- Self-consistency equations, $T = 0$-limit:

$$m = \left\langle\!\!\left\langle \frac{\eta-a}{a(1-a)}\,\hat{\nu} \right\rangle\!\!\right\rangle_{\eta,z,I}$$

$$c = \frac{1}{\sqrt{\alpha r}} \left\langle\!\!\left\langle z\hat{\nu} \right\rangle\!\!\right\rangle_{\eta,z,I} \qquad\qquad r = \frac{q}{(1-c)^2}$$

$$q = \left\langle\!\!\left\langle \hat{\nu}^2 \right\rangle\!\!\right\rangle_{\eta,z,I}$$

with

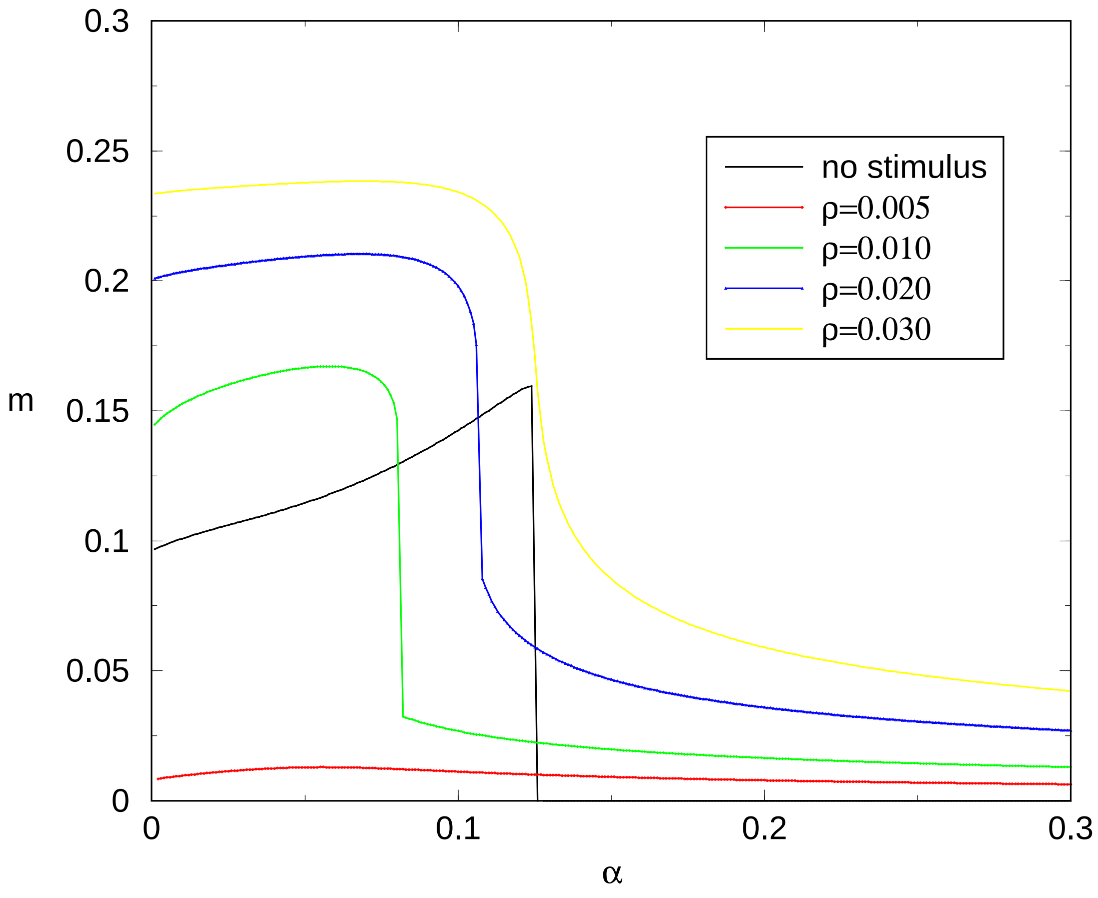
$$\hat{\nu} = \hat{\nu}(\eta,z,I) = g\Big(m(\eta-a) + \sqrt{\alpha r}\; z + \frac{\alpha c}{1-c}\hat{\nu} + I - \vartheta\Big) \qquad (*)$$

- Input currents (stimuli) Gaussian: $I_i = \rho\eta_i + \sigma\xi_i$ with $\xi_i \sim \mathcal{N}(0,1)$

- Firing rate distribution, parameterised by m, c, q; from (*):

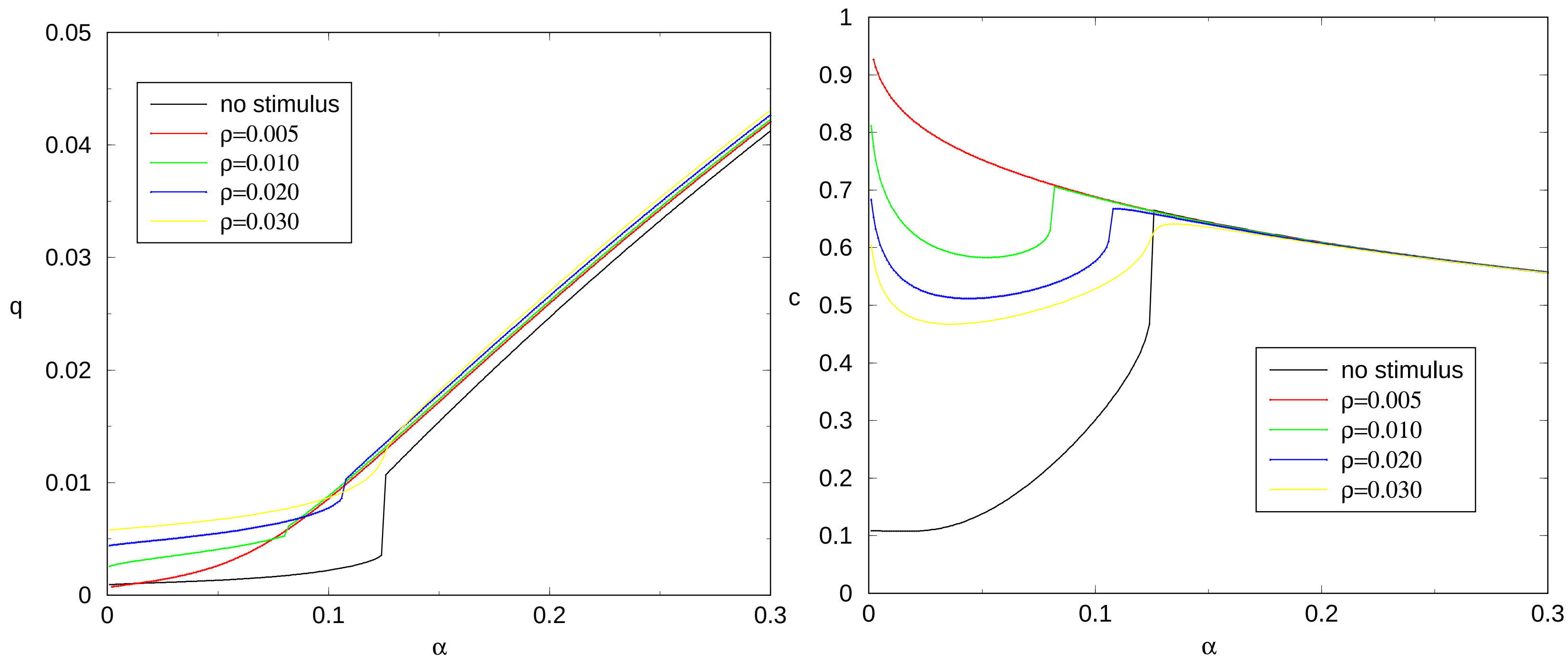
$$p(\nu,I,\eta) = \frac{1}{N}\sum_i \delta_{\eta,\eta_i}\;\delta(I-I_i)\;\Big\langle \delta(\nu-\nu_i)\Big\rangle = \Big\langle \delta(\nu-\hat{\nu})\Big\rangle\Big|_{I,\eta}$$

Order Parameters



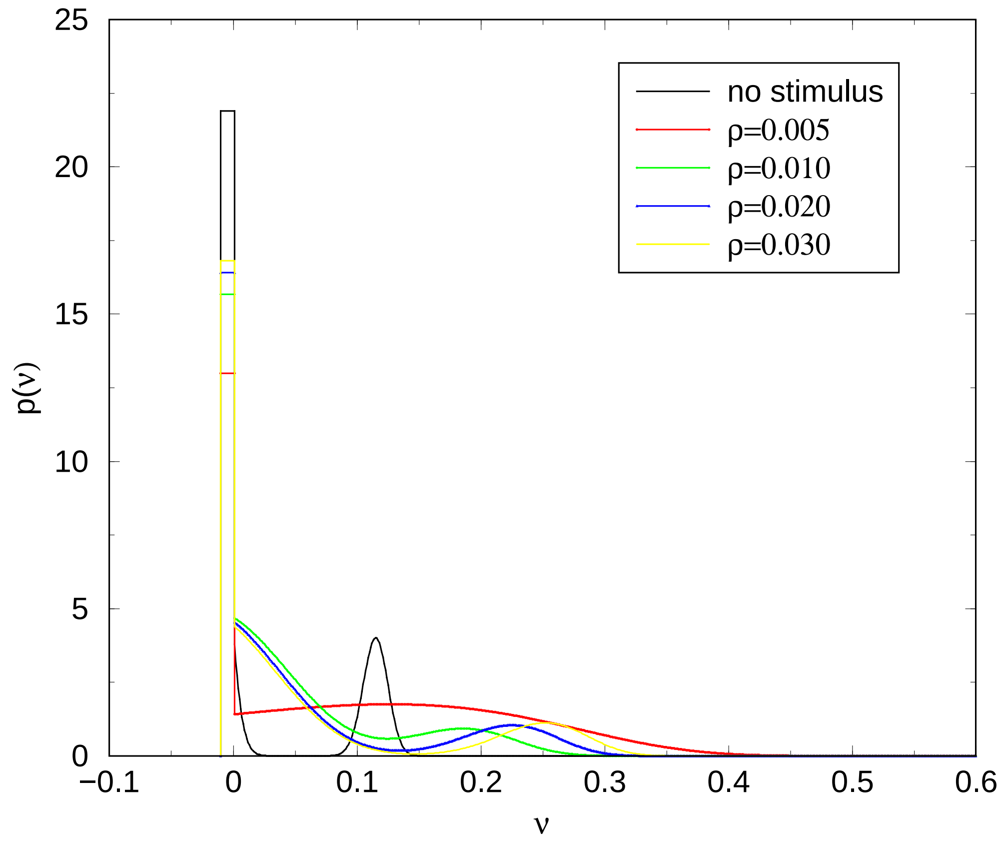
Overlap as function of loading level, without stimulus, and for $\rho = 0.005$, $\rho = 0.01$, $\rho = 0.02$, $\rho = 0.03$, and $\sigma = 0.03$.

Order Parameters



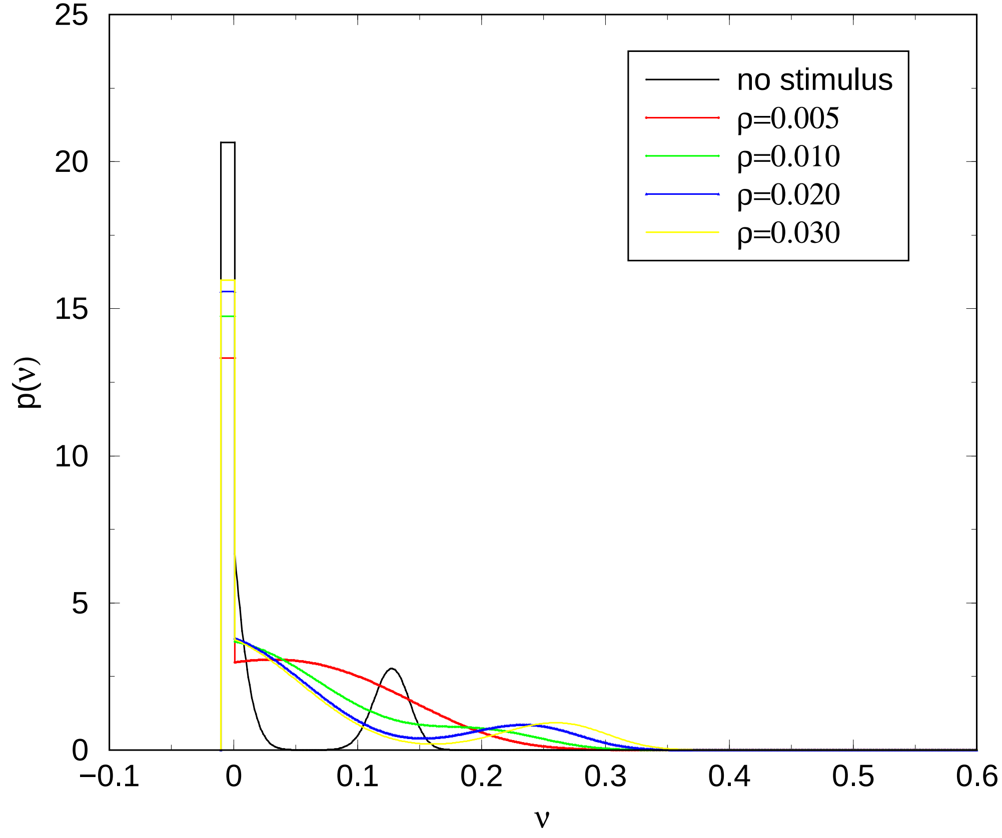
Spin-glass order parameter q and susceptibility c as functions of loading level, without stimulus, and for $\rho = 0.005$, $\rho = 0.01$, $\rho = 0.02$, $\rho = 0.03$, and $\sigma = 0.03$.

Firing Rate Distributions



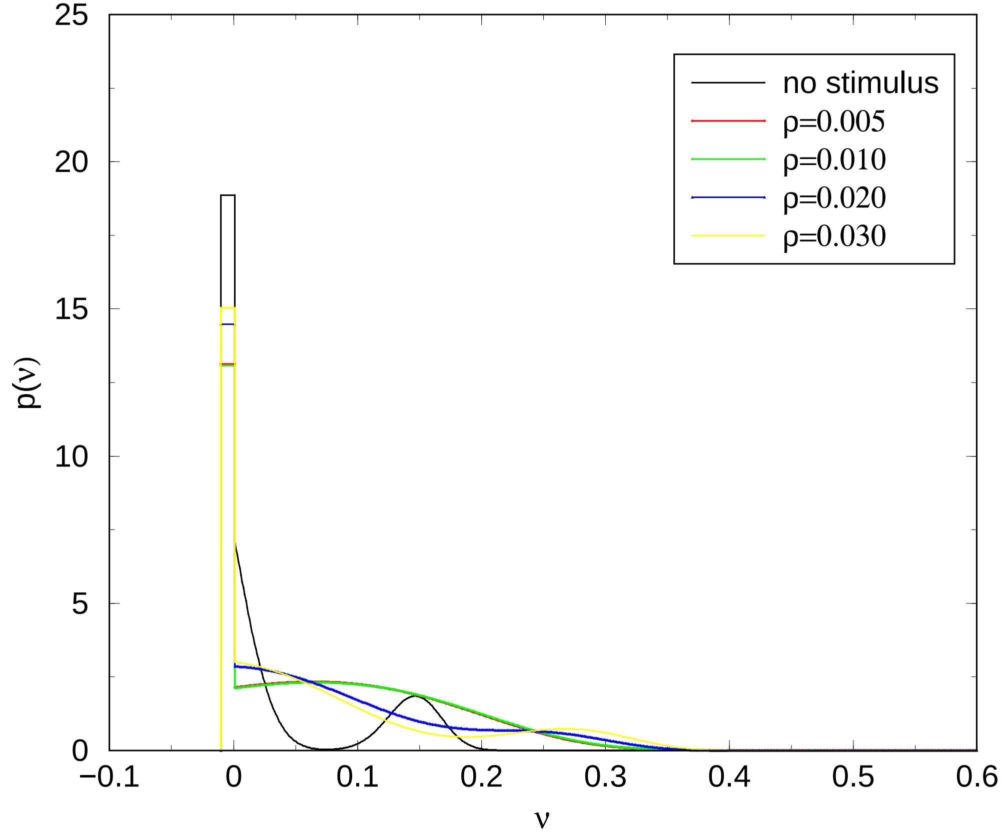
Firing rate distribution at $\alpha = 0.005$, without stimulus, and for $\rho = 0.005$, $\rho = 0.01$, $\rho = 0.02$, $\rho = 0.03$, and $\sigma = 0.03$.

Firing Rate Distributions



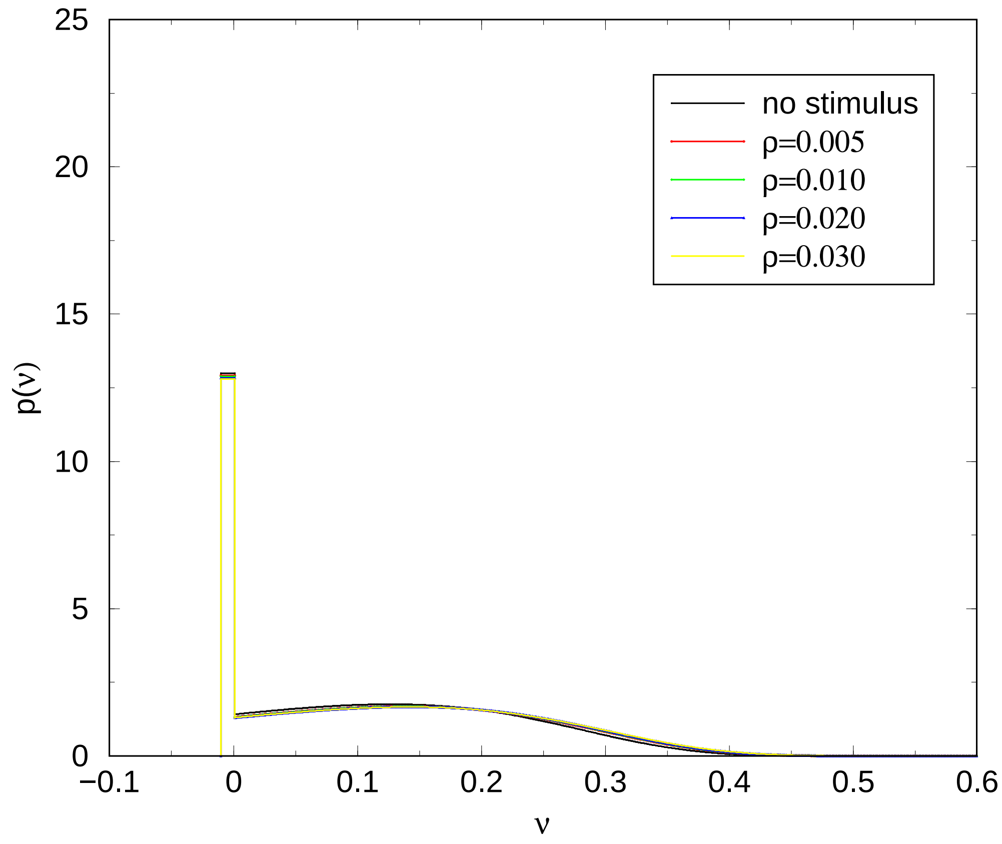
Firing rate distribution at $\alpha = 0.075$, without stimulus, and for $\rho = 0.005$, $\rho = 0.01$, $\rho = 0.02$, $\rho = 0.03$, and $\sigma = 0.03$.

Firing Rate Distributions



Firing rate distribution at $\alpha = 0.10$, without stimulus, and for $\rho = 0.005$, $\rho = 0.01$, $\rho = 0.02$, $\rho = 0.03$, and $\sigma = 0.03$.

Firing Rate Distributions



Firing rate distribution at $\alpha = 0.15$, without stimulus, and for $\rho = 0.005$, $\rho = 0.01$, $\rho = 0.02$, $\rho = 0.03$, and $\sigma = 0.03$.

Performance Measures

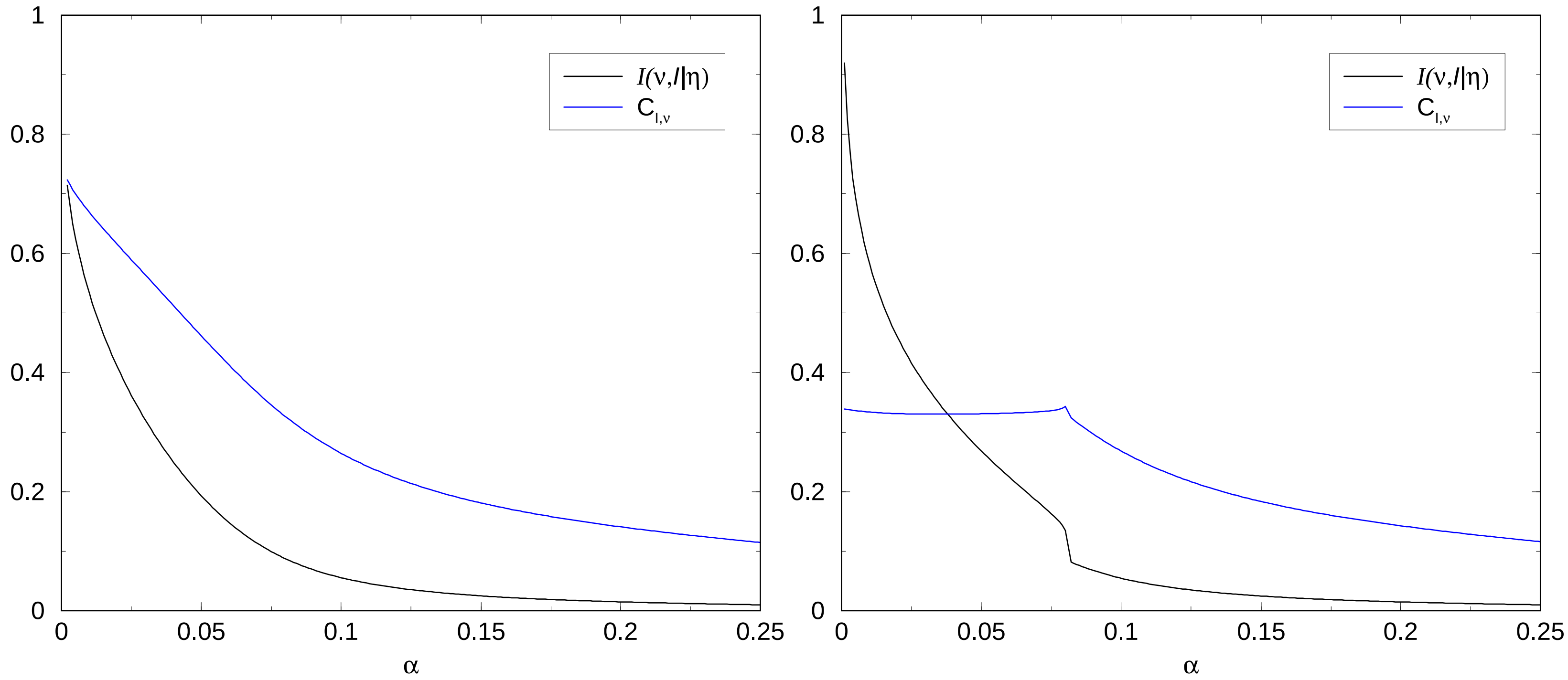
- Mutual conditional information of firing rate distribution and input currents

$$\tilde{I}(\nu, I|\eta) = \sum_{\eta} \int \mathrm{d}\nu \mathrm{d}I \; p(\nu, I, \eta) \; \log_2 \frac{p(\nu, I|\eta)}{p(\nu|\eta)p(I|\eta)}$$
$$= \sum_{\eta} p(\eta) \int \mathrm{d}\nu \mathrm{d}I \; p(\nu|I\eta)p(I|\eta) \; \log_2 \frac{p(\nu|I,\eta)}{p(\nu|\eta)}$$

- Normalised correlation between firing rates and currents

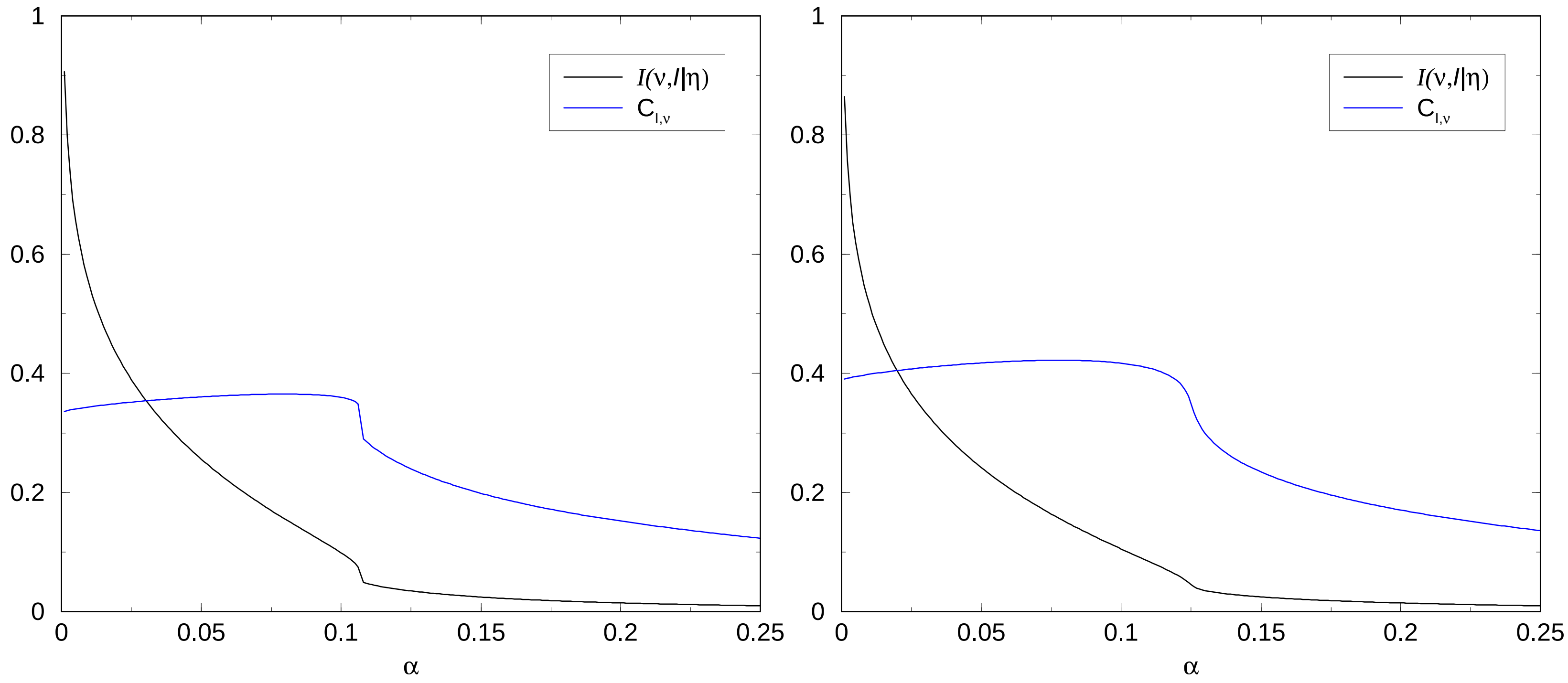
$$C_{I,\nu} = \frac{1}{\sqrt{\sigma_\nu^2 \sigma_I^2}} \Big[\langle \nu \; I \rangle - \langle \nu \rangle \langle I \rangle \Big]$$

Mutual Information and Correlation



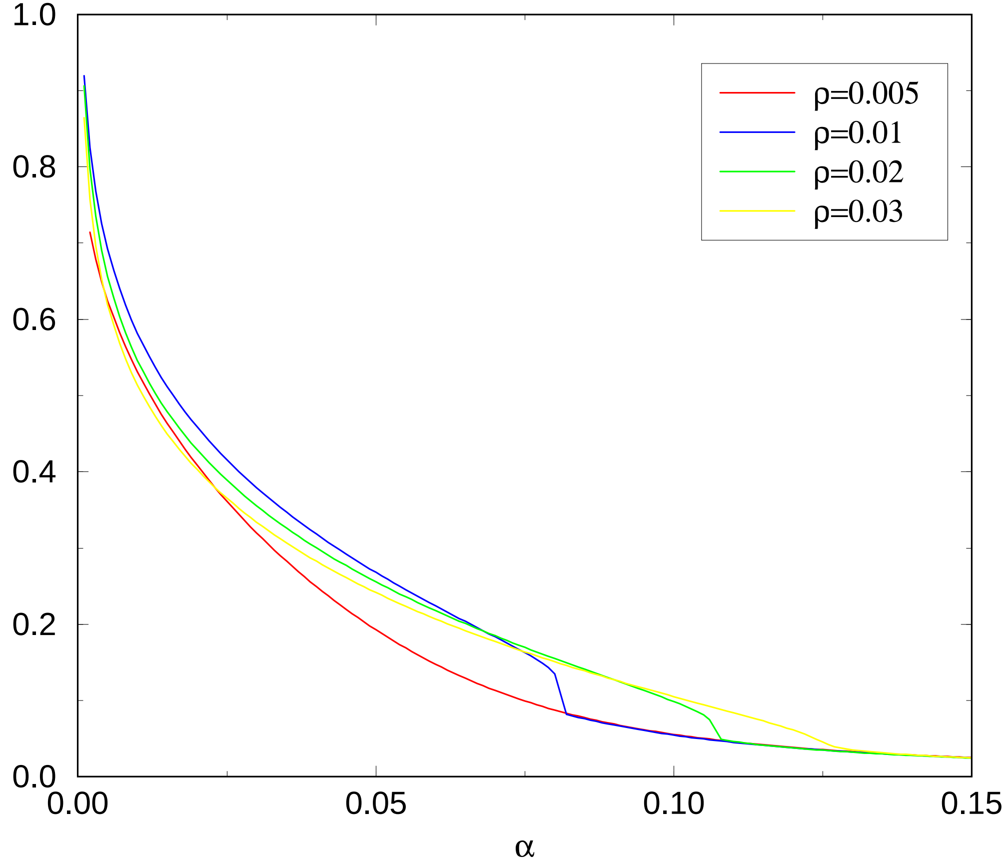
Mutual conditional information and current-rate correlation at $\sigma = 0.03$ and $\rho = 0.005$ (left) and $\rho = 0.01$ (right) as functions of α.

Mutual Information and Correlation



Mutual conditional information and current-rate correlation at $\sigma = 0.03$ and $\rho = 0.02$ (left) and $\rho = 0.03$ (right) as functions of α.

Mutual Information



Mutual conditional information as a function of α for various degrees of correlation.

Summary and Outlook

- Computed firing rate distributions in analogue neuron systems

- Studied dependence on degree of correlation between stimulus and pre-learnt patterns

- Started systematic evaluation of information theoretic performance measures

- Results – at least in principle – experimentally accessible

- Useful as theoretical framework for interpretation of recordings using trained-untrained scenarios?

- Within reach and/or to be done
  - Graded patterns in the learning rule
  - Pattern distribution that maximises mutual information?
  - Asymmetric couplings
  - Non-stationary effects, dynamics $\Rightarrow$ J. Hatchett (KCL)
  - More realistic neuron models