The Third King's Workshop on Random Graphs and Random Processes
Tuesday 9th April 2019
Bush House Lecture Theatre 1
Bush House (North), King's College London
Programme

The workshop looks at recent work in the area of random structures and algorithms and random processes on networks. Of particular interest are properties of discrete random structures, random walks and interacting particle systems, threshold behaviour, the short term dynamics of processes during approach to equilibrium, the time taken to reach equilibrium and algorithmic efficiency.

Morning Session (10:00–12:50)
Coffee (from 9:40)

10:00-10:40	Adrian Kosowski INRIA Paris	Universal protocols for information dissemination and related topics
10:40-11:20	Richard Pymar Birkbeck College	Mixing times of exclusion processes on regular graphs

Coffee

11:40-12:20	Petra Berenbrink University of Hamburg	Introduction to Opinion Dynamics
12:20-12:50	Richard Mycroft University of Birmingham	Oriented trees in directed graphs

Lunch Break (12:50–14:00)

Afternoon Session (14:00–17:20)
Coffee from 13:45

14:00-14:40	James Gleeson University of Limerick	Cascade dynamics on networks
14:40-15:10	Jonathan Hermon University of Cambridge	Entropy decay and concentration for Strong Rayleigh measures via couplings
15:10-15:40	Pierpaolo Vivo King's College London	Top eigenpair statistics for weighted sparse graphs

Coffee

16:00-16:40	Cécile Mailler University of Bath	The monkey walk: a random walk with random reinforced relocations and fading memory
16:40-17:20	Alan Frieze Carnegie Mellon University	On the rank of random binary matrices

The entrance to Bush House (North Building) is at 30 Aldwych, London, WC2R 4BG. See block Q on the map.

Coffee etc. is supplied, but not lunch. There is a café on the Ground floor (If entering from The Strand, Floor 0) of Bush House a cafeteria on Floor (-1) and a vegan cafeteria on Floor 8.

To plan how many people might want coffee etc, it would be helpful if you Register (non-obligatory)

Abstracts

Petra Berenbrink, University of Hamburg

Alan Frieze, CMU: On the rank of a random binary matrix

We study the rank of a random $n \times m$ matrix $A_{n,m;k}$ with entries from $GF(2)$, and exactly k unit entries in each column, the other entries being zero. The columns are chosen independently and uniformly at random from the set of all $\binom{n}{k}$ such columns.

We obtain an asymptotically correct estimate for the rank as a function of the number of columns m in terms of c, n, k, and where $m = cn/k$. The matrix $A_{n,m;k}$ forms the vertex-edge incidence matrix of a k-uniform random hypergraph H. The rank of $A_{n,m;k}$ can be expressed as follows. Let $|C_2|$ be the number of vertices of the 2-core of H, and $|E(C_2)|$ the number of edges. Let m^* be the value of m for which $|C_2| = |E(C_2)|$. Then w.h.p. for $m < m^*$ the rank of $A_{n,m;k}$ is asymptotic to m, and for $m \geq m^*$ the rank is asymptotic to $m - |E(C_2)| + |C_2|$.

In addition, assign i.i.d. $U[0, 1]$ weights $X_i, i \in 1, 2, ...m$ to the columns, and define the weight of a set of columns S as $X(S) = \sum_{j \in S} X_j$. Define a basis as a set of $n - \mathbb{1}(k \text{ even})$ linearly independent columns. We obtain an asymptotically correct estimate for the minimum weight basis. This generalises the well-known result of Frieze [On the value of a random minimum spanning tree problem, Discrete Applied Mathematics, (1985)] that, for $k = 2$, the expected length of a minimum weight spanning tree tends to $\zeta(3) \sim 1.202$. Joint work with Colin Cooper and Wesley Pegden. Relevant paper

James Gleeson, University of Limerick, Ireland: Cascade dynamics on networks

Abstract: Network models may be applied to describe many complex systems, and in the era of online social networks the study of dynamics on networks is an important branch of computational social science. Cascade dynamics can occur when the state of a node is affected by the states of its neighbours in the network, for example when a Twitter user is inspired to retweet a message that she received from a user she follows, with one event (the retweet) potentially causing further events (retweets by followers of followers) in a chain reaction. In this talk I will review some simple models that can help us understand how social contagion (the spread of cultural fads and the viral diffusion of information) depends upon the structure of the social network and on the dynamics of human behaviour. Although the models are simple enough to allow for mathematical analysis, I will show examples where they can also provide good matches to empirical observations of cascades on social networks.

Jonathan Hermon, University of Cambridge: Entropy decay and concentration for Strong Rayleigh measures via couplings.

Abstract: Together with Justin Salez we establish universal modified log-Sobolev inequalities for reversible Markov chains on the Boolean lattice $\{0,1\}^n$, under the only assumption that the invariant law π satisfies a form of negative dependence known as the stochastic covering property. This condition is strictly weaker than the strong Rayleigh property, and is satisfied in particular by all determinantal measures, as well as by the uniform distribution over the set of bases of any balanced matroid and by the occupation measure of the exclusion process. This implies that one can rapidly sample from such distributions, a problem with numerous applications. In the special case where π is k-homogeneous, our results imply the celebrated concentration inequality for Lipschitz functions due to Pemantle & Peres (2014).

Adrian Kosowski, INRIA Paris: Universal protocols for information dissemination and related topics

We consider a population of n agents which communicate with each other in a decentralized manner, through random pairwise interactions. One or more agents in the population may act as authoritative sources of information, and the objective of the remaining agents is to obtain information from or about these source agents. We study two basic tasks: broadcasting, in which the agents are to learn the bit-state of an authoritative source which is present in the population, and source detection, in which the agents are required to decide if at least one source agent is present in the population or not. We focus on designing protocols which meet two natural conditions: (1) universality, i.e., independence of population size, and (2) rapid convergence to a correct global state after a reconfiguration, such as a change in the state of a source agent. Our main positive result is to show that both of these constraints can be met. For both the broadcasting problem and the source detection problem, we obtain solutions with a convergence time of $O(log^2 n)$ rounds, w.h.p., from any starting configuration. The solution to broadcasting is exact, which means that all agents reach the state broadcast by the source, while the solution to source detection admits one-sided error on a ϵ-fraction of the population (which is unavoidable for this problem). Our protocols exploit the properties of self-organizing oscillatory dynamics, and we provide some insights on why designs which do not rely on oscillatory dynamics might not be feasible. Finally, we show how to use the constructed oscillatory dynamics as a clock component in the design of universal population protocols, allowing us to solve computational tasks, such as Leader Election or Majority Consensus, in a polylogarithmic number of rounds. This talk includes results of joint work with Bartek Dudek and Przemek Uznanski. Paper 1, Paper 2

Cecile Mailler, University of Bath: The monkey walk: a random walk with random reinforced relocations and fading memory.

In this joint work with Gerónimo Uribe-Bravo, we prove and extend results from the physics literature about a random walk with random reinforced relocations. The "walker" evolves in $\mathbb{Z}^d$ or $\mathbb{R}^d$ according to a Markov process, except at some random jump-times, where it chooses a time uniformly at random in its past, and instantly jumps to the position it was at that random time. This walk is by definition non-Markovian, since the walker needs to remember all its past.

Under moment conditions on the inter-jump-times, and provided that the underlying Markov process verifies a distributional limit theorem, we show a distributional limit theorem for the position of the walker at large time. The proof relies on exploiting the branching structure of this random walk with random relocations; we are able to extend the model further by allowing the memory of the walker to decay with time. Relevant paper

Richard Mycroft, University of Birmingham: Oriented trees in directed graphs

Abstract: There are many fascinating results and open problems about finding oriented trees within tournaments and other digraphs, and in particular on the notion of unavoidability. We say that an oriented tree on n vertices is *m-unavoidable* if it is contained in every oriented tree on m vertices.

1. Sumner conjectured that every oriented tree on n vertices is $2n-2$-unavoidable. For sufficiently large n this was confirmed by my work with Kühn and Osthus using a randomised embedding algorithm, whilst for small n the best known bound is $21n/8 - 47/16$ by a very recent result of Dross and Havet.

2. Havet and Thomassé made the much stronger conjecture that every oriented tree on n vertices with k leaves is $n+k-1$-unavoidable. This conjecture remains wide open, but the recent work of Dross and Havet gives the best current partial results.

3. Bender and Wormald conjectured that almost all oriented trees on n vertices are n-unavoidable (i.e. appear as a spanning subgraph of all tournaments on n vertices). Naia and I recently proved this by further developing the randomised embedding approach.

We discuss these results and the methods used to achieve them, particularly the randomised embedding strategy. We also describe a very recent application of this method for embedding oriented trees in dense digraphs other than tournaments, and discuss the limitations of this method and scope for further developments. Relevant paper

Richard Pymar, Birkbeck College London: Mixing times of exclusion processes on regular graphs

Place k black particles and n-k white particles on the vertices of an n vertex graph, with one per vertex. Suppose each edge rings at rate 1 independently, and when an edge rings particles at the end-points switch positions. Oliveira conjectured that this k-particle exclusion process has mixing time of order at most that of k independent particles. Together with Jonathan Hermon we prove a bound for regular graphs which is in general within a log log n factor from this conjecture when $k > n^c$ and which, in certain cases, verifies the conjecture. As a result we obtain new mixing time bounds for the exclusion process on expanders and the hypercube. Relevant paper

Pierpaolo Vivo, KCL: Top eigenpair statistics for weighted sparse graphs.

We re-adapt a formalism originally developed by Kabashima et al. to compute the statistics of the largest eigenpair of weighted sparse graphs with Poissonian connectivity and bounded maximal degree. Framing the problem in terms of optimization of a quadratic form on the sphere and introducing a fictitious temperature, we employ the cavity and replica methods to find the solution in terms of self-consistent equations for auxiliary probability density functions, which can be solved by population dynamics. This derivation allows us to identify and disentangle the individual contributions to the top eigenvector's components coming from nodes of degree 'k', and is in perfect agreement with numerical diagonalization of large (weighted) adjacency matrices. Known results for the random regular case are effortlessly recovered.

Organization. The workshop is held jointly between the Department of Informatics and Department of Mathematics at King's College, and the School of Computing, University of Leeds; and was organized by C. Cooper, M. Dyer, R. Kuehn, T. Radzik and N. Rivera. We gratefully acknowledge support from the London Mathematical Society, the Department of Mathematics and Department of Informatics at KCL. The research groups of the organizers are:

Algorithms and Data Analysis Group, Department of Informatics, KCL.

Disordered Systems Group, Department of Mathematics, KCL.

Algorithms and Complexity Group, School of Computing, University of Leeds

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