The Second King's Workshop on Random Graphs and Random Processes

Tuesday 17th April 2018

Bush House Lecture Theatre 1

Bush House (North), King's College London

Programme

The workshop looks at recent work in the area of random structures and algorithms and random processes on networks. In particular threshold behavior, the short term dynamics of processes during approach to equilibrium, the time taken to reach equilibrium and algorithmic efficiency.

Morning Session (10:00–12:45)
Coffee (from 9:40)

10:00-10:40	Pietro Caputo Universitá Roma Tre	Random walks on sparse random digraphs: invariant measure and mixing time
10:40-11:20	Tim Rogers University of Bath	Heterogeneous micro-structure of percolation in networks

Coffee

11:40-12:20	Alan Frieze Carnegie Mellon University	Cuckoo hashing using random walks
12:20-12:45	Colin Cooper King's College London	A triangle-creation process on regular graphs

Lunch Break (12:45–14:00)

Afternoon Session (14:00–17:15)

14:00-14:40	Guillem Perarnau University of Birmingham	Random colourings of bounded degree graphs
14:40-15:10	Nicolás Rivera University of Cambridge	Dispersion processes
15:10-15:35	Nan Kang King's College London	The best of three voting process

Coffee

15:55-16:35	Annika Heckel University of Oxford	Colouring random graphs
16:35-17:15	Artur Czumaj Warwick University	Generating random permutations using switching networks

The entrance to Bush House (North Building) is at 30 Aldwych, London, WC2R 4BG

Coffee etc is supplied, but not lunch. There is a café on the Ground floor (If entering from The Strand, Floor 0) of Bush House and a cafeteria on the 8th floor that does food at reasonable prices.

To plan how many people might want coffee etc, it would be helpful if you Register (non-obligatory)

Links to related papers and talks.

Pietro Caputo. Random walks on sparse random digraphs: invariant measure and mixing time.
Abstract: We analyse the convergence to stationarity for a class of random walks on random directed graphs. Examples include the configuration model with prescribed in- and out-degree sequences. The invariant measure has a nontrivial shape and is characterized via recursive distributional equations. The mixing time is given by the entropy of the equilibrium distribution divided by the average one-step entropy of the Markov chain. Moreover, the chain has a sharp cutoff behavior around this time. The results are extended to a large family of sparse Markov chains whose transition matrix has exchangeable random rows satisfying a sparsity assumption. Based on joint work with Charles Bordenave and Justin Salez. See also: Paper 1, Paper 2

Colin Cooper:A triangle-creation process on regular graphs.
Abstract: The process inserts or deletes an edge in the neighbourhood of a randomly chosen vertex as follows. If w, x are neighbours of the chosen vertex v and the edge wx is present, it is deleted by switching with a randomly chosen edge yz to give edges wy, xz. Similarly, the edge wx would be inserted by using the reverse switch. We prove that provided every move of this type is accepted with non-zero probability, chains of this type are ergodic on the space of 3-regular graphs. Examples of such chains produce 3-regular graphs with a number of triangles linear in the number of vertices (w.h.p.). This is obtained in near linear time, irrespective of the starting graph.

Artur Czumaj. Generating Random Permutations Using Switching Networks.
Abstract: We consider the problem of designing a simple, oblivious scheme to generate (almost) random permutations. We use the concept of switching networks and show that almost every switching network of logarithmic depth can be used to almost randomly permute any set of $(1-\epsilon)n$ elements with any $\epsilon > 0$(that is, gives an almost $(1-\epsilon)n$–wise independent permutation). Furthermore, we show that the result still holds for every switching network of logarithmic depth that has some special expansion properties, leading to an explicit construction of such networks. Our result can be also extended to an explicit construction of a switching network of depth $O(\log^2 n)$ and with $O(n \log n)$ switches that almost randomly permutes any set of n elements.

Our results are obtained using a non-trivial coupling approach to study mixing times of Markov chains which allows us to reduce the problem to some random walk-like problem on expanders.

Alan Frieze. Cuckoo hashing using random walks.
Abstract: We discuss the expected time for Cuckoo Hashing to insert items via a random walk. See also: Paper 1, Paper 2, Paper 3

Annika Heckel. Colouring random graphs.
Abstract: A (proper) colouring of a graph is a vertex colouring where no two neighbouring vertices are coloured the same, and the chromatic number is the least number of colours where this is

possible. Determining the chromatic number of $G(n,p)$ is one of the classic challenges in random graph theory. For the case where p is constant, we will establish upper and lower bounds which are the first to match each other up to a term of size o(1) in the denominator. In particular, these bounds determine the average colour class size in an optimal colouring almost completely, answering a question by Kang and McDiarmid. We also consider a closely related graph parameter, the equitable chromatic number of the dense random graph $G(n,m)$, which can be determined exactly on a subsequence of the integers. See also: Paper 1, Paper 2

Nan Kang: The best of three voting process.
Abstract: We introduce a Best-of-Three voting protocol on connected finite graphs, in which every vertex initially has an opinion. At each time step, each vertex randomly samples three neighbours with replacement and changes its opinion according to the majority of their samples. For a class of almost-regular graphs with minimum degree $d = n^{\alpha}$, where $\alpha = O\left(1/\log\log n\right)$, the consensus time of this protocol such that the final opinion of the graph is consistent with the initial majority, is $O\left(\log_2\log_2 n\right) + O\left(\log\left(\delta^{-1}\right)\right) + \Theta(1)$ with high probability.

Guillem Perarnau: Random colourings of bounded degree graphs
Abstract: Given a graph G with maximum degree Δ, a well-known conjecture in graph colouring states that the Glauber dynamics on the set of proper k-colorings of G rapidly mixes provided that $k \geq \Delta + 2$. In 1999, Vigoda showed that the flip dynamics on the set of proper k-colourings of G has rapid mixing provided that $k > 11\Delta/6$, implying polynomial time mixing for Glauber dynamics under the same constraints. The conjecture has attracted a lot of attention in the literature and better results are known for certain classes of graphs. In this talk, we improve Vigoda's bound for general graphs by showing that there exists $\eta > 0$ such that the Glauber dynamics mixes in polynomial time for $k \geq (11/6 - \eta)\Delta$. Similar to Vigoda's proof, the proof is based on path coupling. Our novel contribution is to use a pre-metric that takes into account the local structure of the colourings. (Joint work with Michelle Delcourt and Luke Postle.) See also: Paper 1

Nicolás Rivera. Dispersion processes.
Abstract: In a synchronous dispersion process N particles are initially placed at a distinguished origin vertex of a graph G. At each time step, at each vertex v occupied by more than one particle at the beginning of this step, each of these particles moves to a randomly chosen neighbour of v. The dispersion process ends once the particles have all stopped moving, that is, at the first step when each vertex has at most one particle.

Tim Rogers: Abstract: Heterogeneous micro-structure of percolation in sparse networks.
We examine the heterogeneous responses of individual nodes in sparse networks to the random removal of a fraction of edges. Using the message-passing formulation of percolation, I will show that there is often considerable variation across a network in the probability of a particular node to remain part of the giant component, and in the expected size of small clusters containing that node. In the vicinity of the percolation threshold, weakly non-linear analysis reveals that node-to-node heterogeneity is captured by the recently introduced notion of non-backtracking centrality. Interpreted in terms of the application of percolation to real-world processes, our results shed light on the heterogeneous exposure of different nodes to cascading failures, epidemic spread, and information flow.

Organization. The workshop is held jointly between the Department of Informatics and Depart-

ment of Mathematics at King's College, and the School of Computing, University of Leeds; and was organized by C. Cooper, M. Dyer, R. Kuehn, A. McDowell and T. Radzik. We gratefully acknowledge support from the Department of Mathematics and Department of Informatics at KCL. The research groups of the organizers are:
Algorithms and Data Analysis Group, Department of Informatics, KCL.
Disordered Systems Group, Department of Mathematics, KCL.
Algorithms and Complexity Group, School of Computing, University of Leeds