Energy imbalance market call options and the valuation of storage

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Motivation: System balancing

- System balancing (mentioned earlier) is increasingly challenging
  - Increasing renewable penetration
  - Decommissioning of fossil generation
- Innovation is tapping new resources
  - New programmes from National Grid
  - New industry players eg. virtual power plants

What's the opportunity?

Businesses of all shapes and sizes can make money, reduce their bills and cut their carbon footprint by getting involved in demand side response (DSR).

This fast-growing market is all about using energy more intelligently. It provides flexibility that enables National Grid to balance Britain's electricity system cost-effectively, while our energy landscape changes rapidly. If your business has the flexibility to increase, decrease, or shift its electricity use, then the power is in your hands to take full advantage.
Are new contracts needed?

Untapped resources have constraints:

Constraints due to having other primary uses (e.g. HVAC, fridges, EVs)
- temporal constraints (e.g. on availability)
- physical constraints (e.g. being energy limited)

They need contracts which:
1. do not conflict with device constraints
2. incentivise participation
3. avoid unintended consequences (e.g. grid destabilisation, profiteering)
Contract design

We study contracts for balancing service designed for battery storage which:
(BO = battery operator, SO = system operator)

1. Do not conflict with device constraints
   Storage is energy limited, so the contract:
   - protects BO by only committing to limited balancing service
   - protects SO by requiring physical cover

2. Incentivise participation
   Financial parameters of the contract can be chosen so that:
   - BO makes a profit on average
   - SO never faces a guaranteed loss

3. Avoid unintended consequences
   We can calculate and hence anticipate:
   - BO’s average profits, and check they are bounded
   - BO’s optimal strategy, check it doesn’t destabilise the grid
Market assumption

The contract we propose would operate in an energy imbalance market (EIM):

- EIMs aim to reduce cost by pooling operating reserve
- Operating eg. in Germany and California
- Typically, bids and offers accepted in price order until real-time balancing requirement is met (for each time period)

Idea: Introduce American style call option contracts to the EIM:

1. BO buys and stores one unit of energy at the EIM price
2. BO sells one call option to the SO (fixed price $p_c$, like the ‘option premium’)
3. SO requests the unit of energy from the BO for balancing when needed (fixed cost $K_c$, like the ‘strike price’)

![Diagram of energy imbalance market and call option contracts](image-url)
The contract would be an **American style call option**: 

- **Load storage**
- **Sell option**
- **Pay \( p_c \)**
- **Request delivery when price hits \( x^* \)**
- **Pay \( K_c \)**

**Battery operator**  
**System operator**
Checking the contract design (1)

1. Do not conflict with device constraints

Storage is energy limited, so the contract:
- protects BO by only committing to limited balancing service – Contract is for a single balancing action
- protects SO by requiring physical cover – Must load store before selling option

2. Incentivise participation

Financial parameters of the contract can be chosen so that:
- BO makes a profit on average
- SO never faces a guaranteed loss

3. Avoid unintended consequences

We can calculate and hence anticipate:
- BO’s average profits, and check they are bounded
- BO’s optimal strategy, check it doesn’t destabilise the grid
The probability model

- Most general model we use is specified by stochastic differential equation. It is a regular diffusion process \((X_t)\):

\[
    dX_t = \mu(X_t)dt + \sigma(X_t)dW_t
\]

modelling the EIM price, with natural boundaries \(a \in \mathbb{R} \cup \{-\infty\} \) and \(+\infty\).

  - Allows negative prices
  - Allows rather general stochastic dynamics

- This model is in the style of mathematical finance - where it was very successful, beginning with Black-Scholes in 1970’s

- But our analysis in the next slides is certainly not Black-Scholes – because this problem does not satisfy the Black-Scholes assumptions (not even close!)

- Our analysis models cashflows under the above probability model and aims to find the unique optimal strategy for the BO
The probability model: Example

- Ornstein-Uhlenbeck process \((X_t)_{t \geq 0}\) captures the observed ‘stylised fact’ of mean reversion (below, bottom panel) in the imbalance price (time in days):
  \[
dX_t = \theta(D - X_t)dt + \sigma dW_t
  \]

- When fitted to UK data from 07/2011-03/2014, 8am: baseline level \(D = 60\), \(\theta = 0.77\), \(\sigma = 20.81\)
  \[
dX_t = 0.77(60 - X_t)dt + 20.81dW_t
  \]
**Objective in this work**

To calculate the BO's optimal strategy and average payoff from

1. one contract
2. an infinite sequence of back-to-back contracts ('lifetime problem')

**Parameter restriction:** The option provides a discount relative to the EIM price:

\[ p_c + K_c < x^*. \]

(We prove this condition is equivalent to requiring that the contract never leads to a certain financial loss for the system operator.)

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**Note:** This is not the way options are constructed in Black-Scholes. Important because 'delta hedging' involves buying high and selling low, so amplifies imbalance

**Related:**

- Carmona, Ludkovski (2010): numerics for pump-storage management
- Cruise et al. (2014): control of storage in a deterministic model with price impact
- Gast et al. (2014): combined storage with wind generation (semi-stochastic model)
- Finance (buy low – sell high): Zhang and Zhang (Automatica 2008), Zervos et al. (Mathematical Finance 2013)
1. Do not conflict with device constraints

Storage is energy limited, so the contract:
- protects BO by only committing to limited balancing service
- protects SO by requiring physical cover

2. Incentivise participation

Financial parameters of the contract can be chosen so that:
- BO makes a profit on average
- SO never faces a guaranteed loss – SO obtains a discount relative to direct use of EIM

3. Avoid unintended consequences

We can calculate and hence anticipate:
- BO’s average profits, and check they are bounded
- BO’s optimal strategy, check it doesn’t destabilise the grid
Recent results in this framework

- We (M. & Palczewski) provide exhaustive solution to single and lifetime problems
- (Palczewski, work in progress) Solution to *ergodic* problem
- (Martyr, Szabó) Solutions for corresponding put option problem
- (Szabó) Numerical solutions to extended models
  - Options with finite time horizon
  - BO can write either put or call options, provided battery is in correct state, plus can buy and sell in EIM (optimal switching problem)

Notation: we construct optimal solutions from hitting times:

$$\tau_x = \inf\{t \geq 0 : X_t = x\}$$

The symbol $\mathbb{E}^x\{\cdot\}$ will denote the expectation (average) operator
Optimal stopping problem for single contract

Recall the contract:

1. BO buys and stores one unit of energy at the EIM price $X_\tau$
2. BO sells one call option to the SO (fixed price $p_c$, like the ‘option premium’)
3. SO requests the unit of energy from the BO for balancing when needed (fixed cost $K_c$, like the ‘strike price’)

- Easy to see that step 2 should follow immediately after step 1 NB: not true for put option (Martyr and Szabó)

- Optimally timing the simultaneous step 1+2 (ie. the purchase of electricity and selling the contract) corresponds to solving an optimal stopping problem:

$$V_c(x) = \sup_{\tau} E^x \{ e^{-r\tau} h(X_\tau) \},$$

where

$$h(x) = -x + p_c + \mathbb{E}^x \{ e^{-r\hat{\tau}_e} K_c \}.$$

and

$$\hat{\tau}_e = \inf \{ t \geq 0 : X_t \geq x^* \}.$$
Fixed point for lifetime problem

- Idea of proof: introduce a continuation value $\xi(x)$ per unit of storage – representing the value of the remaining lifetime sequence of back-to-back options.
- Fixed point argument: Adding the value function $\xi$ to the single contract, the resulting value function should again be $\xi$ (since both represent the lifetime value)
- Mathematically the optimal stopping problem becomes

$$T \xi(x) = \sup_{\tau} E^x \{ e^{-r\tau} ( - X_\tau + p_c + h_\xi(X_\tau)) \}, \quad (1)$$

where

$$h_\xi(x) = -x + p_c + E^x \{ e^{-r^*} (K_c + A \xi(X_{^*})) \}$$

and

$$^*_e = \inf\{t \geq 0 : X_t \geq x^*\}.$$

- Value of one option is $T_0(x)$, two options: $T^2_0(x), \ldots$ and the lifetime value function is:

$$\hat{V}(x) = \lim_{n \to \infty} T^n_0(x)$$

**Theorem**

*If $V_c = T_0 \notin \{0, \infty\}$, then $\hat{V}$ is finite, non-zero and

$$T \hat{V} = \hat{V}.$$*
Checking the contract design (3)

1. Do not conflict with device constraints

Storage is energy limited, so the contract:
- protects BO by only committing to **limited** balancing service
- protects SO by requiring **physical cover**

2. Incentivise participation

Financial parameters of the contract can be chosen so that:
- BO makes a profit on average – just check value of single option $\neq 0$
- SO never faces a guaranteed loss

3. Avoid unintended consequences

We can calculate and hence anticipate:
- BO’s average profits, and check they are **bounded** – just check value of single option $\neq \infty$
- BO’s optimal strategy, check it doesn’t destabilise the grid – just check the threshold of the BO’s strategy is sufficiently low
Conclusions

- Proposed a new contract for energy imbalance markets
- Based on the American call option in mathematical finance
- Designed for battery storage to provide balancing service:
  - Avoid conflict with device constraints (i.e., batteries are energy limited)
  - Incentivise participation (by both BO and SO)
  - Avoid unintended consequences for SO
- Mathematically we have derived the option value and optimal strategy for single option and lifetime problem
- Other recent results in this framework
- Presentation based on:
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Thank you!


