Designing Agents for Derivatives Markets: a preliminary framework

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Abstract

In this paper we present a preliminary framework for a model of agents which are able to trade in futures and options derivatives markets. We design a basic logical model for the exchange process and extend it to depict futures and options contracts; we then evince an example scenario for our model to show the applicability of the work.

1 Introduction

Economics has attracted many Artificial Intelligence researchers since the beginning of research in AI.[1] Major topics have included auctions and stock markets, especially in modelling the dynamics between the different entities that participate in those markets. Hakman *et al.* ([2]) give a survey of multiagent system models of stock markets.

However, there have been few published efforts in modelling derivatives markets, that is, markets for products whose values derive from the values of other products. Maza *et al.* ([1]) ran a series of experiments with an economic model adapted to simulate futures markets with nonrational agents, while King *et al.* ([5]) also describe a model of derivatives markets, experimenting with the interactions of agents undertaking options trading.

Efforts on modelling derivatives markets are generally focused on the interactions between the agents in the market, with little if no effort being made towards modelling the internal states of the agents participating in the system. In this work we aim to model a rational agent able to participate in a derivatives market model. This agent will be modelled using a BDI architecture with a reasoning engine drawing on an epistemic/doxastic and a temporal Logic.

[1] For example, the work of Herbert Simon ([7]) or one of his other 973 listed publications at `http://www.psy.cmu.edu/psy/faculty/hsimon/hsimon.html`.

As the economist John Maynard Keynes first noted ([4]), a successful participant in a commodity market does not need to accurately forecast the direction of future prices of the product being bought or sold, but only the direction which the majority of other participants currently forecast. This observation leads naturally to notions of belief, belief-about-beliefs, common knowledge, etc, and so we begin our research by asking whether it is possible to use formal logic to adequately represent the agents participating in futures and options markets. Since moreover the agents' decisions are driven by desires (utilities), and, in order to perform an exchange, they need to formulate their intentions, a good starting point for us seems to be to assume that our agents' mental states are similar to these of BDI-agents, i.e., they comprise beliefs, desires and intentions. Moreover, since Futures and Options by definition refer to the temporal future, we also incorporate a notion of time in our formal language.

The formal part of our framework, as presented in this paper, is still rather limited and incomplete. In particular, we do not aim at presenting a full axiomatic system, and we do not propose any complicated properties of the mental states of our agents – like whether their beliefs are introspective, or whether their desires are consistent with their beliefs. Neither have we as yet developed formal semantics for the language that we use. In this paper we try to demonstrate that our logical language is appropriate to reason about agent's decisions in derivatives markets. Our ultimate objectives are to define the logical preconditions which would ensure that a transaction occurs between agents in a derivatives market, or that can at least explain rational behaviour of agents in such markets, and to characterize the conditions which guarantee (or not) a market equilibrium.

In Section 2 we present a formal definition of a transaction, and then proceed to define an options and futures exchange. This is followed, in Section 3, with a description of the internal states of an agent in our model. We list the agent rules which the agent will use to buy or sell a good and finish by giving an example scenario of an exchange. Finally, we then present the future work to be developed and the conclusions that we have gained until now, in Section 4.

2 Building a derivatives market model

Derivatives are economic securities whose values depend on the performance of another security or asset. Futures Contracts and Options are two of the most common derivatives; they are used by investors to hedge the risk of an investment and to increase their overall wealth [3].

Futures markets are increasingly common. These are centralized market exchanges for buyers and sellers with future contracts being the securities exchanged in this market. Typically, a futures market has a central regulatory body which defines the standard features of a futures contract, including statements of prices and dates of the contracts. A future contract is an agreement to buy or sell an asset for delivery in a specific place and time in the future. Because contracts are themselves traded, possibly many times before the future dates, most futures contract trades do not lead to actual delivery of any goods.

An option is a contract which gives the buyer the right to buy or to sell an asset at a specific price and future date. When an option trader gets the option to buy, the contract is named a *call* option, when it gets the option to sell, the contract is called a *put* option. Unlike futures contracts, options do not usually rely on a central regulator and so the price and dates of exchange are determined by the traders themselves [3].

To perform successfully in an exchange market, an agent needs to have an accurate view of the future prices of the asset being traded. Information about current and future prices will be used by an agent to form its beliefs about the price trend of a certain asset, beliefs which the agent will use to execute a transaction.

We start by defining a simple exchange transaction as a tuple $\mathcal{E} = \langle \mathcal{A}, \mathcal{B}, a, p, t \rangle$ defining:

- $\mathcal{A}$: The agent-role selling the asset.
- $\mathcal{B}$: The agent-role buying the asset.
- a: The asset being exchanged.
- p: The price at which the asset will be traded.
- t: The time when the exchange will be performed.

This definition shows a simple buyer - seller exchange, based on the price information the agents have. We assume that agents can both act as a seller and a buyer at the same time, but, to separate both roles, we will specify the seller's role for an agent called $\mathcal{A}$, and a buyer's role for agent $\mathcal{B}$.

We assume the buyer agent utility is proportional to the assets he owns, thus he prefers to buy an asset instead of not buying it; likewise, the seller agent utility is proportional to the money he has, so he prefers to sell his owned assets rather than doing nothing.

Before going further we must define the terms used in our examples. $B_{\mathcal{A}}\varphi$ represents that agent $\mathcal{A}$ believes the expression φ holds and p^t is the price of our example asset at time t.

In the first example scenario, in which we only consider two time points t_0 and t_1, the seller agent $\mathcal{A}$ believes the price will decrease at the next time step; and the buyer agent $\mathcal{B}$ believes the price of the asset will increase. Hence, we state:

$$t_0 \models B_{\mathcal{A}}[p^0 \geq p^1]$$

$$t_0 \models B_{\mathcal{B}}[p^0 \leq p^1]$$

According to the previous statements, the buyer believes at time t_0 that buying the asset in the time t_0 at price p^0 is the best action to take (because the agent believes the price will increase). Also, the seller agent believes that selling the asset at time t_0 is better than selling it afterwards. This scenario, by itself, will result in the exchange of the asset for money being performed.

Another example scenario will be when the buyer agent believes the price of the asset will decrease in the future and the seller agent believes the price will increase; we shall state that as:

$$t_0 \models B_{\mathcal{A}}[p^0 \leq p^1]$$
$$t_0 \models B_{\mathcal{B}}[p^0 \geq p^1]$$

This implies the buyer agent believes it is better to buy the asset at a future time than to buy it in t_0 and the seller believes it is better to sell the asset also in a future time. In this example the transaction would not be performed.

In these two scenarios, agents beliefs are expressed very strongly, that is, they believe the price of a good will always rise or never rise. These are severe and unrealistic beliefs. In the first scenario, we can guarantee the exchange will be done, because the agents do not need to consider any other issue unlike the second example where the agents will not exchange the asset.

So far, we have exposed a basic interaction between two agents within a simple buying and selling transaction. To extend this model to an options exchange we will recall the definition of an option contract which is *a contract which gives the holder the right to buy or sell an asset by a certain date for a certain price* [3].

We defined previously an exchange $\mathcal{E} = \langle \mathcal{A}, \mathcal{B}, a, p, t \rangle$. We now let $a = \mathcal{E}'$, where $\mathcal{E}'$ is an asset exchange transaction where the good will be exchanged and $\mathcal{E}$ is the option exchange transaction, where the buyer agent will get the option to buy or sell the good at a later time. Thus, we can describe the option contract as:

$$\mathcal{E}_{option} = \langle \mathcal{A}, \mathcal{B}, \mathcal{E}', p, t \rangle$$

where: $\mathcal{E}' = \langle \mathcal{A}, \mathcal{B}, a, p, t' \rangle$ or $\mathcal{E}' = \langle \mathcal{B}, \mathcal{A}, a, p, t' \rangle$, the first choice for $\mathcal{E}'$ represents the scenario in which the option buyer agent got an option to buy a good (*call* option) and the second choice represents the situation in which the agent got an option to sell a good (*put* option).

Although at first sight, this may look odd, with one symbol, $\mathcal{E}$, on both sides of the equality sign, this expresses the recursive nature of an options contract — a transaction at the present time in which both parties agree to a possible transaction at a future time.

We had also specified the definition of a futures contract, because the execution of a future contract requires a mediator (an exchange), which is the entity responsible for pricing and for stating the times of contracts. the basic definition of the futures contract can be stated as an exchange between the buyer agent and the mediator, and in the same way, an exchange between the seller and the mediator. This can be stated as:

$$\mathcal{E}_{future} \langle \mathcal{A}, \mathcal{M}, \mathcal{B}, f, p, t \rangle$$

where $\mathcal{M}$ is the mediator and f is the future being exchanged. The price p will be specified by the mediator according to the properties of the future being exchanged. Both the buyer $\mathcal{A}$ and seller $\mathcal{B}$ agent will make the transaction directly with the mediator.

3 Agent model for goods exchange

Modeling an agent capable of executing a derivatives contract is a non-trivial task since it requires the agent to have several abilities. Firstly, the agent must be able to generate beliefs based on its predictions of the asset that is referenced by the derivative. Secondly, it must also be aware of the direct derivative market behaviour to decide correctly. Also, an agent capable of performing an option contract needs to have beliefs about the future price of the good it is going to trade, and base its decision in the outcomes expected. A successful such agent must be able to employ strategies like buying a derivative or asset to sell at a future time, forecasting the price of the asset and acting accordingly to maximize his utility.

Our logical model of the agent contains the beliefs, desires and intentions it has to participate in the market. We also give the agent a set of inference rules which provide the means for generating new beliefs, desires and, at the end, some intentions which result in actions. It is worth noting the rules stated next are strong and specific, and so this allows the agent little flexibility in decision making. We developed these as a first approach to modelling the agent inference engine.

3.1 Market model specification

The market world is composed of several agents where $\mathcal{A}$ and $\mathcal{B}$ denote a specific agent. There are several goods to be exchanged in the market, and $G = \{g_1, g_2, ..., g_n\}$ stands for the set of all the goods available in the system. At the beginning of the simulation, each agent $\mathcal{A}$ owns a set of goods $G_{\mathcal{A}}$ and an initial amount of cash $c_{\mathcal{A}}$. Also, every good g_k is assigned to one of the agents thus $G = G_{\mathcal{A}} \cup G_{\mathcal{B}} \cup ... \cup G_{\mathcal{N}}$ where $G_x \cap G_y = \varnothing$ for any pair of agents x, y in the system.

Each agent has a utility function $U_{\mathcal{A}}(\cdot)$ which measures the agent's assigned utility for the evaluated statement. This statement can be for example to own certain good, a statement denoted by the predicate $O^t_{\mathcal{A}}(\cdot)$, which specifies whether the agent owns certain good at time t.

Each good g_k has a specific market price p_k; this price varies over time. Agents are able to see the price of all the goods at current time (p^t_k), and they may estimate the price at future times; for example, $B_{\mathcal{A}}[p^{t+1}_k = c]$ states that the price for the good g_k for time $t + 1$ estimated by agent $\mathcal{A}$ is the value c. We also use the predicate $done(e)$ from the definition in [6] as a state formula where e is an event that will occur immediatly.

Agents have beliefs, desires and intentions; the operators $B_{\mathcal{A}}, D_{\mathcal{A}}, I_{\mathcal{A}}$ are used to denote them accordingly; $\Gamma^t_{\mathcal{A}}$ is the set of goods that agent $\mathcal{A}$ desires to buy at time t; $\Pi^t_{\mathcal{A}}$ stands for the set of prices for items being sold by agent $\mathcal{A}$ at time t. Finally, $\mathcal{E}(\mathcal{A}, \mathcal{B}, g_k, p^t_k, t)$ will mean the exchange $\mathcal{E}$ takes place as a transition between t and $t+1$. To be more precise, if the selling agent $\mathcal{A}$ has an intention that a good g_k be sold at time t for a price p^t (which we will denote as $I_{\mathcal{A}} done[\mathcal{E}(\mathcal{A}, \beta, g_k, p^t_k, t)]$) and a buying agent $\mathcal{B}$ has at the same time an intention to buy the same good for the same price (denoted $I_{\mathcal{B}} done[\mathcal{E}(\alpha, \mathcal{B}, g_k, p^t_k, t)]$), then we assume the *system* will unify the two intentions and guarantee that indeed the exchange will take place.

Subscript indexes referring to particular agents, $\mathcal{A}$ and $\mathcal{B}$, are omitted when not explicitly necessary to increase reading simplicity.

3.2 Buyer agent rule set

First, we specify the set of rules used by an agent $\mathcal{B}$ to deliberate about the possiblity of buying a good.

1 $B\big[UO^{t+1}c_{\mathcal{B}} < UO^{t+1}g_k + UO^{t+1}(c_{\mathcal{B}} - p^t_k)\big] \rightarrow B[UO^{t+1}g_k > U\neg O^{t+1}g_k]$

2 $B[UO^{t+1}g_k > U\neg O^{t+1}g_k] \rightarrow DO^{t+1}g_k$

3 $DO^{t+1}g_k \wedge BO^t p^t_k \rightarrow D\big[done\big(\mathcal{E}(\alpha, \mathcal{B}, g_k, p^t_k, t+1)\big)\big]$

4 $D\big[done\big(\mathcal{E}(\alpha, \mathcal{B}, g_k, p^t_k, t+1)\big)\big] \wedge B \bigwedge_{g \in \Gamma^t_{\mathcal{A}}, g \neg g_k} \big[UO^{t+1}g_k \geq UO^{t+1}g\big] \rightarrow$
$I_{\mathcal{B}}\big[done\big(\mathcal{E}(\alpha, \mathcal{B}, g_k, p^t_k, t+1)\big)\big]$

5 $DO^{t+1}g_k \wedge B\neg O^t p_k \rightarrow DO^{t+1}p^t_k \wedge DO^{t+2}g_k$

The intended meaning of the preceding rules is as follows.

In the first rule the agent tests if his utility in the next time step for owning the cash he owns now is less than the utility of owning a specific good plus the utility of owning his expected cash after subtracting the expected price of the good in the next time step. If the antecedent holds then the agent believes the utility of owning the good at next time step is better than not owning it.

The second rule specifies that if the agent believes the utility of owning the good g_k at next time step is better than the utility of not owning it then the agent acquires the desire of owning the good at the next time step.

The third rule makes sure the agent owns, at the current time, enough cash to buy the good, if it is the case and if he also desires to own the good at next time step then he will obtain the desire to make an exchange. The formula $done\big(\mathcal{E}(\alpha, \mathcal{B}, g_k, p^t_k, t+1)\big)$ denotes that an exchange action will have been done between agent $\mathcal{B}$ as buyer and any other agent α as seller, trading the good g_k for the price p_k at time $t+1$.

In the fourth rule, the agent verifies if the good that he desires to buy is the one which will give him more utility from the set of all the goods that he can buy at the next time step. If it is true then he will generate an intention to make the actual exchange.

The fifth and last rule is used when the agent desires to own the good g_k but he does not have enough cash to buy it; when that is the case, then the agent will gain the desire to own the money at the next time and, he will also get a desire to own the good g_k at a future time.

3.3 Seller agent rule set

Next, we list the rules used by a seller agent $\mathcal{A}$ to chose a good to sell.

6 $B\big[UO^{t+1}g_k + UO^{t+1}c_{\mathcal{A}} < UO^{t+1}(c_{\mathcal{A}} + p_k^t)\big] \rightarrow B[U\neg O^{t+1}g_k > UO^{t+1}g_k]$

7 $B[U\neg O^{t+1}g_k > UO^{t+1}g_k] \rightarrow D\neg O^{t+1}g_k$

8 $D\neg O^{t+1}g_k \wedge BO^t g_k \rightarrow D[done\big(\mathcal{E}(\mathcal{A}, \beta, g_k, p_k^t, t+1)\big)]$

9 $D\big[done\big(\mathcal{E}(\mathcal{A}, \beta, g_k, p_k^t, t+1)\big)\big] \wedge B \bigwedge_{p \in \Pi_{\mathcal{A}}^t} [UO^{t+1}p_k^t \geq UO^{t+1}p] \rightarrow$
$I\big[done\big(\mathcal{E}(\mathcal{A}, \beta, g_k, p_k^t, t+1)\big)\big]$

10 $DO^{t+1}p_k^t \wedge B\neg O^t g_k \rightarrow DO^{t+1}g_k \wedge DO^{t+2}p_k^t$

In the rule 6, the agent compares the utility of owning certain good g_k plus the utility of owning the cash he owns $c_{\mathcal{A}}$ against the utility of owning the cash he owns plus the price of the good. If the antecedent holds, then the agent will believe the utility of not owning the good is better than the utility of owning it; that is, he will believe it is better for him to own the money than the good itself.

In rule 7, the agent tests if he prefers to own the price instead of the good g_k, if that is true then the agent will get a desire to do not own the good at the next time step.

The agent uses rule 8 to test if he desires not to own the good at next time step $(t+1)$ and if he believes he owns that good at time t; if that is true then the agent will desire to execute an exchange with any other agent β.

The 9th rule verifies whether the agent desires to do an exchange and if he believes the utility of the price of the good p_k is greater or equal than the utility of the price of the other goods he can sell $(\Pi_{\mathcal{A}}^t)$, then he will acquire the intention to exchange the good g_k.

The 10th rule is used when the agent desires to own a specific amount of cash (p_k) which is also the price of a certain good and he does not owns that good, then he will desire to own the good in the next time step, and he will get the desire to own the cash at a future time. This rule is specified to allow the agent more complex behaviours such as buying and selling goods based on future price predictions.

3.4 Merged rule set

The rules defined above can be merged to build the complete agent model. This allows an agent to buy or sell depending on his goals. As we will see later, the definition of the utility function is what will characterize the behaviour of the agent making him chose between buying or selling at any particular time. This shows that our agent has the basic structure to participate in a simple market and therefore is useful as a basis for the options and futures market participant agent.

3.5 Agent deliberation cycle

To participate in an exchange, the agent will get the information about the goods prices, then it will use the new acquired information and with each good, it will test each of the rules in order to acquire new beliefs and generate his desires and intentions. A time step t ends when the agent has tested each good in the world with his rules. At this time, the agent could or not have an intention.

Next, the system verifies if the agents have an intention; if this is the case then it will try to find two matching intentions and execute the transactions. In this work we are not interested in how the actual exchange is executed: we just assume the overall system is able to find 'matching' intentions for exchanges.

After the system has matched the intentions, the agents will again get the new prices of the goods and reason about their current knowledge developing his beliefs, desires and intentions. This cycle is repeated until the end of time t.

3.6 Exchange scenario example

In this section we show an example scenario using the model previously built. We start by specifying the initial world setup as shown in table 1.

$G = \{g_1, g_2, g_3\}$	The set of all the goods in the system
$p_1 = p(g_1) = \$15$	The price for the good g_1
$p_2 = p(g_2) = \$15$	The price for the good g_2
$p_3 = p(g_3) = \$10$	The price for the good g_3
$U_{\mathcal{A}} O_{\mathcal{A}} c = c * 2$	The utility function of agent $\mathcal{A}$ for owning a specific amount of cash c.
$U_{\mathcal{B}} O_{\mathcal{B}} c = c/2$	The utility function of agent $\mathcal{B}$ for owning a specific amount of cash c.

Table 1: World Initial State

The model will have two agents (a buyer and a seller agent). The agents initial state parameters are described in tables 2 and 3 ; each agent has a specific initial knowledge; we define knowledge as the facts the agents explicitly know to be true. The agents also have beliefs, desires and intentions that will be developed during their life.

As we will see in the example, the agent $\mathcal{A}$ will act as a seller, and agent $\mathcal{B}$ will act as a buyer agent. This behaviour will emerge based in the cash utility function of each agent and the utility for owning the goods. In this extreme

Agent $\mathcal{A}$	
Initial state	Initial Knowldege
$G_{\mathcal{A}} = \{g_1, g_2\}$	$K_{\mathcal{A}} O_{\mathcal{A}} G_{\mathcal{A}}$
$c_{\mathcal{A}} = 100$	$K_{\mathcal{A}} O_{\mathcal{A}} c_{\mathcal{A}}$
$U_{\mathcal{A}} O_{\mathcal{A}} g_1 = 10$	$K_{\mathcal{A}}[U_{\mathcal{A}} O_{\mathcal{A}} g_1 = 10]$
$U_{\mathcal{A}} O_{\mathcal{A}} g_2 = 5$	$K_{\mathcal{A}}[U_{\mathcal{A}} O_{\mathcal{A}} g_2 = 5]$
$U_{\mathcal{A}} O_{\mathcal{A}} g_3 = 15$	$K_{\mathcal{A}}[U_{\mathcal{A}} O_{\mathcal{A}} g_3 = 15]$
$U_{\mathcal{A}} O_{\mathcal{A}} c_{\mathcal{A}} = c_{\mathcal{A}} * 2 = 200$	$K_{\mathcal{A}}[U_{\mathcal{A}} O_{\mathcal{A}} c_{\mathcal{A}} = 200]$
$U_{\mathcal{A}} = U_{\mathcal{A}} O_{\mathcal{A}} G_{\mathcal{A}} + U_{\mathcal{A}}$ $O_{\mathcal{A}} c_{\mathcal{A}} = 215$	$K_{\mathcal{A}}[U_{\mathcal{A}}(O_{\mathcal{A}} g_1 \wedge O\mathcal{A} g_2 \wedge O\mathcal{A} g_3 \wedge O_{\mathcal{A}} c_{\mathcal{A}}) = 215]$

Table 2: Seller agent initial state

Agent $\mathcal{B}$	
Initial state	Initial Knowldege
$G_{\mathcal{B}} = \{g_3\}$	$K_{\mathcal{B}} O_{\mathcal{B}} G_{\mathcal{B}}$
$c_{\mathcal{B}} = 50$	$K_{\mathcal{B}} O_{\mathcal{B}} c_{\mathcal{B}}$
$U_{\mathcal{B}} O_{\mathcal{B}} g_1 = 20$	$K_{\mathcal{B}}[U_{\mathcal{B}} O_{\mathcal{B}} g_1 = 20]$
$U_{\mathcal{B}} O_{\mathcal{B}} g_2 = 15$	$K_{\mathcal{B}}[U_{\mathcal{B}} O_{\mathcal{B}} g_2 = 15]$
$U_{\mathcal{B}} O_{\mathcal{B}} g_3 = 30$	$K_{\mathcal{B}}[U_{\mathcal{B}} O_{\mathcal{B}} g_3 = 30]$
$U_{\mathcal{B}} O_{\mathcal{B}} c_{\mathcal{B}} = c_{\mathcal{B}}/2 = 25$	$K_{\mathcal{B}}[U_{\mathcal{B}} O_{\mathcal{B}} c_{\mathcal{B}} = 25]$
$U_{\mathcal{B}} = U_{\mathcal{B}} O_{\mathcal{B}} G_{\mathcal{B}} + U_{\mathcal{B}} O_{\mathcal{B}} c_{\mathcal{B}} = 45$	$K_{\mathcal{B}}[U_{\mathcal{B}}(O_{\mathcal{B}} g_3 \wedge O_{\mathcal{B}} c_{\mathcal{B}}) = 45]$

Table 3: Buyer agent initial state

Seller agent $\mathcal{A}$		
Step	Acquired mental state	Testing Rules $[t = 1, p_k = p_1]$
1	$K_{\mathcal{A}}[p_1^1 = \$15]$	[Rule 6] $B_{\mathcal{A}}\big[U_{\mathcal{A}}O^2_{\mathcal{A}}g_1 + U_{\mathcal{A}}O^2_{\mathcal{A}}c_{\mathcal{A}} < U_{\mathcal{A}}O^2_{\mathcal{A}}(c_{\mathcal{A}} + p_1^1)\big] \rightarrow B_{\mathcal{A}}[U_{\mathcal{A}}\neg O^2_{\mathcal{A}}g_1 > UO^2g_1]$ $B_{\mathcal{A}}\big[10+200 < U_{\mathcal{A}}O^2_{\mathcal{A}}(\$100+\$15)\big] \rightarrow B_{\mathcal{A}}[U_{\mathcal{A}}\neg O^2_{\mathcal{A}}g_1 > U_{\mathcal{A}}O^2_{\mathcal{A}}g_1]$ $B_{\mathcal{A}}\big[10 + 200 < 230\big] \rightarrow B_{\mathcal{A}}[U_{\mathcal{A}}\neg O^2_{\mathcal{A}}g_1 > U_{\mathcal{A}}O^2_{\mathcal{A}}g_1]$ $true \rightarrow B_{\mathcal{A}}[U_{\mathcal{A}}\neg O^2_{\mathcal{A}}g_1 > U_{\mathcal{A}}O^2_{\mathcal{A}}g_1]$ therefore: $B_{\mathcal{A}}[U_{\mathcal{A}}\neg O^2_{\mathcal{A}}g_1 > U_{\mathcal{A}}O^2_{\mathcal{A}}g_1]$
2	$B_{\mathcal{A}}[U_{\mathcal{A}}\neg O^2_{\mathcal{A}}g_1 > U_{\mathcal{A}}O^2_{\mathcal{A}}g_1]$	[Rule 7] $B_{\mathcal{A}}[U_{\mathcal{A}}\neg O^2_{\mathcal{A}}g_1 > U_{\mathcal{A}}O^2_{\mathcal{A}}g_1] \rightarrow D_{\mathcal{A}}\neg O^2_{\mathcal{A}}g_1$ $true \rightarrow D_{\mathcal{A}}\neg O^2_{\mathcal{A}}g_1$ therefore: $D_{\mathcal{A}}\neg O^2_{\mathcal{A}}g_1$
3	$D_{\mathcal{A}}[\neg O^2_{\mathcal{A}}g_1]$	[Rule 8] $D_{\mathcal{A}}\neg O^2_{\mathcal{A}}g_1 \wedge B_{\mathcal{A}}O^1_{\mathcal{A}}g_1 \rightarrow D_{\mathcal{A}}\big[done\big(\mathcal{E}(\mathcal{A}, \beta, g_1, p_1^1, 2)\big)\big]$ $true \wedge true \rightarrow D_{\mathcal{A}}\big[done\big(\mathcal{E}(\mathcal{A}, \beta, g_1, p_1^1, 2)\big)\big]$ therefore: $D_{\mathcal{A}}\big[done\big(\mathcal{E}(\mathcal{A}, \beta, g_1, p_1^1, 2)\big)\big]$
4	$D_{\mathcal{A}}\big[done\big(\mathcal{E}(\mathcal{A}, \beta, g_1, p_1^1, 2)\big)\big]$	[Rule 9] $D_{\mathcal{A}}\big[done\big(\mathcal{E}(\mathcal{A}, \beta, g_1, p_1^1, 2)\big)\big] \wedge B_{\mathcal{A}}\bigwedge_{p\in\Pi^1_{\mathcal{A}}}[UO^2p_1^1 \geq UO^2p] \rightarrow I_{\mathcal{A}}\big[done\big(\mathcal{E}(\mathcal{A}, \beta, g_1, p_1^1, 2)\big)\big]$ $true \wedge B_{\mathcal{A}}[10 \geq 5] \rightarrow I_{\mathcal{A}}\big[done\big(\mathcal{E}(\mathcal{A}, \beta, g_1, p_1^1, 2)\big)\big]$ $true \wedge true \rightarrow I_{\mathcal{A}}\big[done\big(\mathcal{E}(\mathcal{A}, \beta, g_1, p_1^1, 2)\big)\big]$ therefore: $I_{\mathcal{A}}\big[done\big(\mathcal{E}(\mathcal{A}, \beta, g_1, p_1^1, 2)\big)\big]$
5	$I_{\mathcal{A}}\big[done\big(\mathcal{E}(\mathcal{A}, \beta, g_1, p_1^1, 2)\big)\big]$	

Table 4: Example Scenario for good g_1 for seller agent

case, agent $\mathcal{A}$ clearly prefers to own money and on the other hand, agent $\mathcal{B}$ gives little weight to cash.

The scenario is shown in tables 4 and 5 which show the agents actions in one time step.

The second column in the tables shows the different knowledge, beliefs, desires and intentions the agent gets trough his reasoning process. The third column shows the rule being tested with the result of the test. After the agent has finished testing a rule, it will acquire another belief, desire or intention. It is important to point the new information acquired by the agent will be added to the initial agent state. Thus when the agent gets the price of a good (e.g. p_1^1), this price is added to his initial knowledge (specified in table 3).

Table 4 shows the reasoning performed by the seller agent at the time t_1. The agent will test the rules for the good g_1 and will conclude with an intention to exchange the good for its price; likewise the buyer agent (table 5) will test the rules for good g_1 and it will conclude with an intention to get the good.

At step 1, the agent seller obtains the price of the good g_1 at the current time ($t = 1$); this is p_1^1. Then, it tests the rule number 6 to verify if the antecedent holds. Because the utility for owning g_1 and his cash c_A is less than the utility it could have owning c_A plus the p_1^1, it acquires the belief that it is better for him not to own the good at the next time step (time $t = 2$).

Following this, in step 2, the agent has acquired a new belief ($B_{\mathcal{A}}[U_{\mathcal{A}}\neg O_{\mathcal{A}}^2 g_1 > U_{\mathcal{A}} O_{\mathcal{A}}^2 g_1]$); it then tests rule number 7, in this case the reasoning is straightforward because of the previous obtained belief. For this, the agent gets the desire not to own the good at next time step.

In step 3, the agent has got the new desire not to own the good at next time point ($D_{\mathcal{A}}[\neg O_{\mathcal{A}}^2 g_1]$). The agent tests rule number 8 to see if it also owns the good at the present time ($t = 1$). Because the agent owns the good, it then gets a desire to make an exchange, where $done\big(\mathcal{E}(\mathcal{A}, \beta, g_1, p_1^1, 2)\big)$ implies that it wants the event of an exchange to be *done* at time step 2.

Next, in step 4 the agent acquires the desire to do the exchange, and proceeds to test rule number 9. Here, the agent verifies that it desires to make the exchange and considers ($B_{\mathcal{A}} \bigwedge_{p \in \Pi_{\mathcal{A}}^1} [UO^2 p_1^1 \geq UO^2 p]$) if there is another good it wants to sell which will yield more utility than the good g_1. In this example, the only other good the agent would be able to sell is g_2 which utility for owning is 5; for this reason the agent will conclude with the intention to make the exchange.

The last step only adds the intention of the exchange to the agent. After this step the agent will finish its reasoning process and proceed to act according to the generated intentions, which in this case is the intention to do an exchange with some other agent for the good g_1.

The same process can be followed to visualize the buyer's agent reasoning process displayed in Table 5. At the end of time t_1, both agents will have the intention to exchange the good g_1.

4 Future work and conclusions

We have presented work-in-progress for a logical model of trading agents able to trade in derivatives markets. At this time our model is still very basic, but we have showed that it is possible to define such agents using formal logic for their reasoning mechanisms. We depicted a basic formal model for the exchange process and then extended it to provide a formal definition of futures and options contracts. To create the derivatives trading agent we need a multi-agent framework which will allow us to build the structure of the market in an easy way and yet it must be flexible enough to let us modify it to comply with the Future and Option markets constraints to be modelled.

We are currently considering use of the technology developed by Vasconcelos *et al.* ([8]), which would allow us to create an electronic multi-agent market. An advantage of using this framework is that it would lead to a market model with wide generality, i.e., not limited only to auctions or any other particular

Buyer agent $\mathcal{B}$		
Step	Acquired mental state	Test Rules $[t = 1, p_k = p_1]$
1	$K_\mathcal{B}[p_1^1 = \$15]$	[Rule 1] $B_\mathcal{B}\big[U_\mathcal{B}O_\mathcal{B}^2 c_\mathcal{B} < U_\mathcal{B}O_\mathcal{B}^2 g_1 + U_\mathcal{B}O_\mathcal{B}^2(c_\mathcal{B} - p_1^1)\big] \rightarrow$ $B_\mathcal{B}[U_\mathcal{B}O_\mathcal{B}^2 g_1 > U_\mathcal{B}\neg O_\mathcal{B}^2 g_1]$ $B_\mathcal{B}[25 < 20 + (17.5)]$ therefore: $B_\mathcal{B}[U_\mathcal{B}O_\mathcal{B}^2 g_1 > U_\mathcal{B}\neg O_\mathcal{B}^2 g_1]$
2	$B_\mathcal{B}[U_\mathcal{B}O_\mathcal{B}^2 g_1 > U_\mathcal{B}\neg O_\mathcal{B}^2 g_1]$	[Rule 2] $B_\mathcal{B}[U_\mathcal{B}O_\mathcal{B}^2 g_1 > U_\mathcal{B}\neg O_\mathcal{B}^2 g_1] \rightarrow D_\mathcal{B}O_\mathcal{B}^2 g_1$ $true \rightarrow D_\mathcal{B}O_\mathcal{B}^2 g_1$ therefore: $D_\mathcal{B}O_\mathcal{B}^2 g_1$
3	$D_\mathcal{B}[O_\mathcal{B}^2 g_1]$	[Rule 3] $D_\mathcal{B}O_\mathcal{B}^2 g_1 \wedge B_\mathcal{B}O_\mathcal{B}^1 p_1^1 \rightarrow D_\mathcal{B}\big[done\big(\mathcal{E}(\alpha, \mathcal{B}, g_1, p_1^1, 2)\big)\big]$ $true \wedge B_\mathcal{B}O_\mathcal{B}^1(\$15) \rightarrow D_\mathcal{B}\big[done\big(\mathcal{E}(\alpha, \mathcal{B}, g_1, p_1^1, 2)\big)\big]$ $true \wedge true \rightarrow D_\mathcal{B}\big[done\big(\mathcal{E}(\alpha, \mathcal{B}, g_1, p_1^1, 2)\big)\big]$ therefore: $D_\mathcal{B}\big[done\big(\mathcal{E}(\alpha, \mathcal{B}, g_1, p_1^1, 2)\big)\big]$
4	$D_\mathcal{B}[done\big(\mathcal{E}(\alpha, \mathcal{B}, g_1, p_1^1, 2)\big)]$	[Rule 4] $D_\mathcal{B}\big[done\big(\mathcal{E}(\alpha, \mathcal{B}, g_1, p_1^1, 2)\big] \wedge B \bigwedge_{g \in \Gamma_\mathcal{B}^1, g \neg g_1} \big[UO^2 g_1 \geq UO^2 g\big] \rightarrow I_\mathcal{B}\big[done\big(\mathcal{E}(\alpha, \mathcal{B}, g_1, p_1^1, 2)\big)\big]$ $true \wedge B_\mathcal{B}\big[20 \geq 15\big] \rightarrow I_\mathcal{B}\big[done\big(\mathcal{E}(\alpha, \mathcal{B}, g_1, p_1^1, 2)\big)\big]$ $true \wedge true \rightarrow I_\mathcal{B}\big[done\big(\mathcal{E}(\alpha, \mathcal{B}, g_1, p_1^1, 2)\big)\big]$ therefore: $I_\mathcal{B}\big[done\big(\mathcal{E}(\alpha, \mathcal{B}, g_1, p_1^t, 2)\big)\big]$
5	$I_\mathcal{B}[done\big(\mathcal{E}(\alpha, \mathcal{B}, g_1, p_1^1, 2)\big)]$	

Table 5: Example Scenario for good g_1 for buyer agent

matching mechanism. It would therefore allow us to build an *e-institution* that satisfied the properties we wish to model of Futures and Options markets.

Our future work will also focus on the development of the agent rule set to allow agents to participate in options exchange markets. This will require us to generalize the current rules so that agents may predict the future prices of goods, and the beliefs and intentions of other agents, with resulting modifications of their own behaviour. We also need to make precise the underlying properties of beliefs, knowledge, desires and intentions, and validate our framework by providing it with a proper semantics. Lastly, we will need to expand our definitions of futures and options contracts to build the market in which our agents will be tested; this will include defining the rules for the futures market mediator and specifying in more detail the contracts information needed to simulate the market.

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