1. [2004 STEP III question 5]

   Remember that if $\cos p = \cos q$ then you cannot simply conclude that $p = q$.

   (a) Express $\cos x - 7 \sin x$ in the form $R\cos(x - \alpha)$. Then use the first part.

   (b) Similar to the previous part.

2. [2005 STEP III question 7]

   By comparing the two integrals in the question, guess a substitution.

   (a) Guess that this will be an application of the result already proved with a particular value of m and a particular function f.

   (b) The same again (with a different m and f).

3. [2004 STEP II question 9] Mechanics

   The first thing is to draw some diagrams. First simply draw a section though the hemisphere which is a semicircle of radius r and contains the centre of the base of the hemisphere and the centre of mass.

   Next draw the scenario where the small mass is at A, and then draw the scenario where this mass is at B.

   If C is the point of contact between the hemisphere and the horizontal plane, and O is the centre of the 'base' of the hemisphere, make sure the positions of these relative to G are carefully considered.

   To calculate $\tan\alpha$, take moments about C.

   To calculate $\tan\beta$, take moments about the new C.

   Then use a standard formula to get an expression for $\tan(\alpha-\beta)$.

   (Final hint: why is C vertically below O? Use geometry.)

4. [2004 STEP II question 12]

   To show that $k=\dfrac{2a}{1-e^{-a}}$ use the fact that

   $$\int_0^1 kxe^{-ax^2}\mathrm{d}x = 1 \quad \text{(Why must this be true?)}$$

   The mode m is the value of x for which $f(x)$ is a maximum. (Use calculus to examine the sign of $f'(x)$ for $0\leq x\leq 1$.)

   The median h satisfies $F(h)=\frac{1}{2}$ where

   $$F(y)=\int_0^y kxe^{-ax^2}\mathrm{d}x \quad \text{(Why?)}$$

   For the last part, use the formula for conditional probability

   $$P(X>m|X<h)=\frac{P\left((X>m)\cap(X<h)\right)}{P(X<h)}\,.$$