The King's Factor

This file contains hints for questions set in Year 13 tKF sessions, to give you a nudge if you feel really stuck

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1. Warm up

   (a) Note that
   $$\frac{3}{x-1} - \frac{2}{x+3} \equiv \frac{3(x+3)-2(x-1)}{(x-1)(x+3)}\,.$$

   (b) Note that
   $$\frac{A}{x+1} + \frac{B}{x+2} \equiv \frac{A(x+2)+B(x+1)}{(x+1)(x+2)}\,.$$

2. [2011 Oxford Admission Test question 1B] (Multiple Choice)

   Let the sides of the rectangle be x and y. Express A in terms of P and x.

   (Also consider the units which the various terms might be in, or (more technically) the dimensions of the terms. Only one of the relationships makes sense.)

3. [2010 AEA question 2]

   First recall the formula for the sum of the first n terms of an arithmetic progression with first term a and common difference d.

   Use this to give two equations involving p, q, a and d.

4. [2007 STEP I question 2]

   (a) You need to use the idea that these two statements are equivalent:
   $\theta = \arctan x$ and $x = \tan\theta$.
   You also need to know (don't rely on formula sheets) the expansion of $\tan(A+B)$.

   Once you have solved the part relating to $\arctan\frac{1}{2}$ and $\arctan\frac{1}{3}$, use the same idea with $\arctan\frac{1}{p} + \arctan\frac{1}{q}$, and then tidy up the algebra to show that $(p-1)(q-1) = 2$.
   To evaluate p and q, use the fact that p and q are non-zero integers. (Without this information there would be an infinite number of possible solutions.)

   (b) Using experience gained in the first part build an equation relating r, s and t.
   Using the fact that s and t are integers with no common factor other than 1, and the fact that r is an integer, you should (if you think about it) be able to see that t can only take two values, and then work out r in terms of s in each case.

5. [2004 STEP I question 7]

   (a) To simplify $f(x) - 7xf(x) + 10x^2 f(x)$:
   first write out the first few terms of the series $f(x)$ and then see how higher powers of x will cancel out in the combination $f(x) - 7xf(x) + 10x^2 f(x)$. (If you can, present a full proof of the result using Σ notation, once you see how things work.)
   To obtain $f(x) = \dfrac{1}{1-2x} + \dfrac{1}{1-5x}$, use the first part and then partial fractions.
   To show that $a_n = 2^n + 5^n$ you need to be able to recognise the sums of some infinite geometric series.
   For instance, what is $\sum_{n=0}^{\infty}(2x)^n$ if $|2x| < 1$?

   (b) This part is similar to the first part, except that you first need to evaluate p and q.

6. [2010 AEA question 5]

   (a) This part of the question instructs you to use a standard method.

   (b) It will help you if you can prove that, if $t = \tan\dfrac{\alpha}{2}$, then $\sec\alpha = \dfrac{1+t^2}{1-t^2}$.

$$I = \int \frac{1}{(x-1)\sqrt{(x^2-1)}}\,\mathrm{d}x \quad x > 1$$