1. Calculate the square root of 24×150 (without first evaluating 24×150).

   Repeat for 147×48

   Repeat for $abc^2 \times c^4ab$

2. Prove that if n is an integer then
$$n^3 - n$$
   is divisible by 3 .

3. [2007 Oxford Admission Test question 1B] (Multiple Choice)

   The greatest value which the function
$$f(x) = \left(3\sin^2(10x+11) - 7\right)^2$$
   takes, as x varies over all real values, equals

   (a) -9 (b) 16 (c) 49 (d) 100

4. [2008 Oxford Admission Test question 1E] (Multiple Choice)

   The highest power of x in
$$f(x) = \left\{\left[\left(2x^6+7\right)^3 + \left(3x^8-12\right)^4\right]^5 + \left[\left(3x^5-12x^2\right)^5 + \left(x^7+6\right)^4\right]^6\right\}^3$$
   is

   (a) x^{424} (b) x^{450} (c) x^{500} (d) x^{504}

5. [2008 Oxford Admission Test question 2]

   (a) Find a pair of positive integers x_1 and y_1, that solve the equation
   $$(x_1)^2 - 2(y_1)^2 = 1\,.$$

   (b) Given integers a and b we define sequences $x_1, x_2, x_3, \ldots$ and $y_1, y_2, y_3, \ldots$ by setting
   $$x_{n+1} = 3x_n + 4y_n \quad \text{and} \quad y_{n+1} = ax_n + by_n \quad \text{for} \quad n \geq 1\,.$$
   Find *positive* values of a and b such that
   $$(x_n)^2 - 2(y_n)^2 = (x_{n+1})^2 - 2(y_{n+1})^2\,.$$

   (c) Find a pair of integers X, Y which satisfy $X^2 - 2Y^2 = 1$ such that $X > Y > 50$.

   (d) (Using the values of a and b found in part (b).) What is the approximate value of $\dfrac{x_n}{y_n}$ as n increases?

6. [2004 STEP I question 3]

   (a) Show that $x - 3$ is a factor of
   $$x^3 - 5x^2 + 2x^2y + xy^2 - 8xy - 3y^2 + 6x + 6y\,. \qquad (*)$$
   Express (*) in the form $(x-3)(x+ay+b)(x+cy+d)$ where a, b, c and d are integers to be determined.

   (b) Factorise $6y^3 - y^2 - 21y + 2x^2 + 12x - 4xy + x^2y - 5xy^2 + 10$ into three linear factors.

7. For each of the following statements *either* prove that it is true *or* give a counter-example to show that it is false:

   (a) The product of any two even numbers is a multiple of 4.

   (b) The product of any two even numbers is a multiple of 8.

   (c) The product of any two odd numbers is a multiple of 3.

   (d) The product of any two odd numbers is and odd number.

8. (a) Show that if

$$x^2 + y^2 - 4x + 10y + 4 = 0 \qquad (*)$$

   then the distance of the point (x, y) from the point $(2, -5)$ is equal to 5.

   (b) Sketch the set of points which satisfy $(*)$ (i.e. sketch the curve $x^2+y^2-4x+10y+4 = 0$.)

   (c) Find the equation of the tangent to this curve at the point $(5, -1)$.

9. [2009 Specimen paper Oxford Admission Test question 1A]

   The point lying between$P(2, 3)$ and $Q(8, -3)$ which divides the line PQ in the ratio $1 : 2$ has co-ordinates (a) $(4, -1)$ (b) $(6, -2)$ (c) $(\frac{14}{3}, 2)$ (d) $(4, 1)$

10. [2009 Specimen paper Oxford Admission Test question 1D]

    The numbers x and y satisfy the following inequalities

$$\begin{aligned} 2x + 3y &\le 23 \\ x + 2 &\le 3y, \\ 3y + 1 &\le 4x. \end{aligned} \qquad (1)$$

    The largest possible value of x is (a) 6 (b) 7 (c) 8 (d) 9

11. [2009 AEA question 1]

    (a) On the same diagram, sketch

    $$y = (x+1)(2-x) \quad \text{and} \quad y = x^2 - 2|x|\,.$$

    Mark clearly the coordinates of the points where these curves cross the coordinate axes.

    (b) Find the x-coordinates of the points of intersection of these two curves.

12. [2006 STEP I question 1]

    Find the integer, n, that satisfies $n^2 < 33\,127 < (n+1)^2$. Find also a small integer m such that $(n+m)^2 - 33\,127$ is a perfect square. Hence express $33\,127$ in the form pq, where p and q are integers greater than 1.

    By considering the possible factorisations of $33\,127$, show that there are exactly two values of m for which $(n+m)^2 - 33\,127$ is a perfect square, and find the other value.

13. Warm up:

    (a) Without even thinking of using a calculator evaluate $\dfrac{4^5 \times 5^{-3}}{2^8 \times 10^{-4}}$.

    (b) Express $\dfrac{(1+\sqrt{7})^2}{3+\sqrt{7}}$ in the form $a + b\sqrt{7}$.

14. [2010 Oxford Admission Test question 1A] (Multiple choice) The values of k for which the line $y = kx$ intersects the parabola $y = (x-1)^2$ are precisely

    (a) $k \leq 0$

    (b) $k \geq -4$

    (c) $k \geq 0 \quad \text{or} \quad k \leq -4$

    (d) $-4 \leq k \leq 0$

15. [2004 AEA question 3]

$$f(x) = x^3 - (k+4)x + 2k \quad \text{where k is a constant.}$$

    (a) Show that, for all values of k, the curve with equation $y = f(x)$ passes through the point $(2, 0)$.

    (b) Find the values of k for which the equation $f(x) = 0$ has exactly two distinct roots.

    (c) Given that $k > 0$, that the x-axis is a tangent to the curve with equation $y = f(x)$, and that the line $y = p$ intersects the curve in three distinct points, find the set of values that p can take.

16. [2007 Oxford Admission Test question 1A] (Multiple Choice)

    Let r and s be integers. Then
    $$\frac{6^{r+s}\times 12^{r-s}}{8^r\times 9^{r+2s}}$$
    is an integer if

    (a) $r+s\le 0$

    (b) $s\le 0$

    (c) $r\le 0$

    (d) $r\ge s$

17. [2000 STEP I question 6]

    Show that
    $$x^2-y^2+x+3y-2=(x-y+2)(x+y-1)$$
    and hence, or otherwise, indicate by means of a sketch the region of the x-y plane where
    $$x^2-y^2+x+3y>2\,.$$
    Sketch also the region of the x-y plane for which
    $$x^2-4y^2+3x-2y<-2\,.$$
    Give the coordinates of a point for which both inequalities are satisfied or explain why no such point exists.

18. [2004 STEP I question 1]

    (a) Express $(3+2\sqrt{5})^3$ in the form $(a+b\sqrt{5})$ where a and b are integers.

    (b) Find the positive integers c and d such that $\sqrt[3]{99-70\sqrt{2}}=c-d\sqrt{2}\,.$

    (c) Find the two real solutions of $x^6-198x^3+1=0\,.$

19. Warm up:

    Given that $f(x) = x^2 - 6x + 13$ show that $f(x) \geq 4$. Sketch the graph of $f(x)$.

    (If you can, do this in two ways, one using calculus and the other not using calculus.)

20. [2009 Specimen 2 Oxford Admission Test question 1F] (Multiple Choice)

    The turning point of the parabola

    $$y = x^2 - 2ax + 1$$

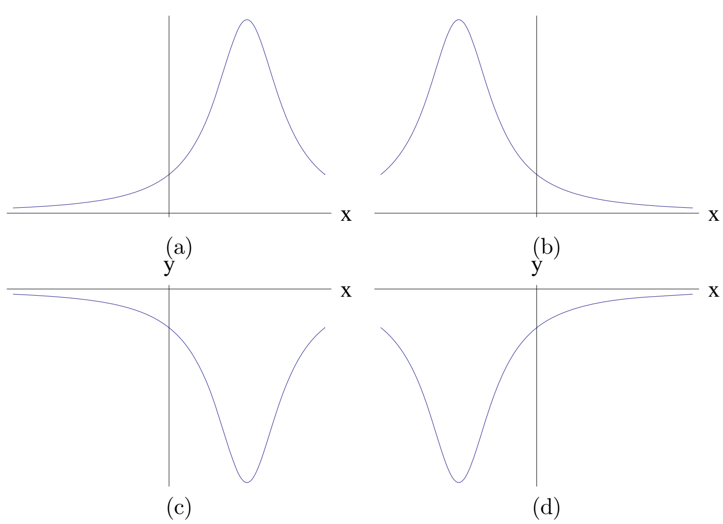
    is closest to the origin when

    (a) $a = 0$ (b) $a = \pm 1$ (c) $\pm \frac{1}{\sqrt{2}}$ or 0 (d) $\pm \frac{1}{\sqrt{2}}$

21. [2008 Oxford Admission Test question 1G] (Multiple choice)

    Which of the graphs below is a sketch of

    $$y = \frac{1}{4x - x^2 - 5} \ ?$$



22. [2007 Oxford Admission Test question 1E] (Multiple choice)

    If x and n are integers then

    $$(1-x)^n(2-x)^{2n}(3-x)^{3n}(4-x)^{4n}(5-x)^{5n}$$

    is

    (a) negative when $n > 5$ and $x < 5$
    (b) negative when n is odd and $x > 5$
    (c) negative when n is a multiple of 3 and $x > 5$
    (d) negative when n is even and $x < 5$

23. [2009 Specimen 2 Oxford Admission Test question 1G] (Multiple Choice)

    The four digit number 2652 is such that any two consecutive digits from it make a multiple of 13. Another number N has this same property, is 100 digits long, and begins in a 9. What is the last digit of N?

    (a) 2 (b) 3 (c) 6 (d) 9

24. Warm up

    Given that $g(x) = x^2$

    (a) Copy and complete this table

x	-6	-4	-2	0	2	4	6
$g(x)$							
$g(x-3)+7$							

    (b) Show that if $g(x) = x^2$ then $g(x-3)+7 = x^2 - 6x + 16$

    (c) Find the turning points of $y = g(x)$ and $y = g(x-3)+7$ and sketch (on the same axes) the graphs $y = g(x)$ and $y = g(x-3)+7$.

25. (a) If the graph of the function $y = f(x)$ has a single turning point, a minimum point at $(3, 2)$, where is the minimum point on the graphs of the following functions?

    i. $f(x-3)$

    ii. $f(x-3)+7$

    iii. $f(x+4)-8$

    (b) What can you say about the turning points of the graphs of the following functions?

    i. $-f(x)$

    ii. $\frac{1}{f(x)}$

26. [2009 Oxford Admission Test question 1C]

    Given a real constant c, the equation

$$x^4 = (x-c)^2$$

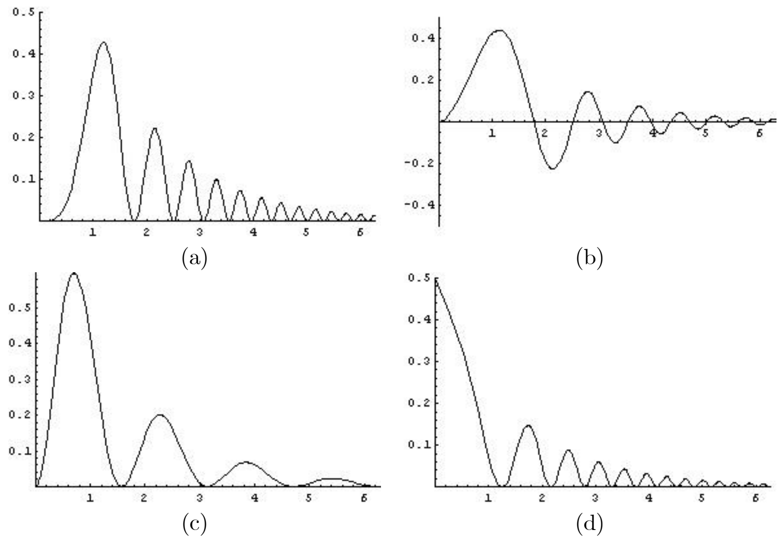
    has four real solutions (including possible repeated roots) for

    (a) $c \leq \frac{1}{4}$ (b) $-\frac{1}{4} \leq c \leq \frac{1}{4}$ (c) $c \leq -\frac{1}{4}$ (d) all values of c

27. [2007 Oxford Admission Test question 1G]

G. On which of the axes below is a sketch of the graph

$$y = 2^{-x}\sin^2\left(x^2\right)?$$



28. [2005 STEP I question 6]

(a) The point A has coordinates $(5,16)$ and the point B has coordinates $(-4,4)$. The variable point P has coordinates (x,y) and moves on a path such that $AP = 2BP$. Show that the Cartesian equation of the path of P is

$$(x+7)^2 + y^2 = 100$$

.

(b) The point C has coordinates $(a,0)$ and the point D has coordinates $(b,0)$. The variable point Q moves on a path such that

$$QC = k \times QD\,,$$

where $k > 1$. Given that the path of Q is the same as the path of P show that

$$\frac{a+7}{b+7} = \frac{a^2+51}{b^2+51}.$$

Show further that $(a+7)(b+7) = 100$ in the case $a \neq b$.