1. [2009 STEP I question 2]

   A curve has the equation
   $$y^3 = x^3 + a^3 + b^3$$
   where a and b are positive constants.

   Show that the tangent to the curve at the point $(-a, b)$ is
   $$b^2 y - a^2 x = a^3 + b^3 .$$

   In the case $a = 1$ and $b = 2$, show that the x-coordinates of the points where the tangent meets the curve satisfy
   $$7x^3 - 3x^2 - 27x - 17 = 0 .$$

   Hence find positive integers p, q, r and s such that
   $$p^3 = q^3 + r^3 + s^3 .$$

2. [2007 STEP I question 3]

   Prove the identities $\cos^4\theta - \sin^4\theta \equiv \cos 2\theta$ and $\cos^4\theta + \sin^4\theta \equiv 1 - \frac{1}{2}\sin^2 2\theta$. Hence or otherwise evaluate
   $$\int_0^{\frac{\pi}{2}} \cos^4\theta \, \mathrm{d}\theta \quad \text{and} \quad \int_0^{\frac{\pi}{2}} \sin^4\theta \, \mathrm{d}\theta .$$

   Evaluate also
   $$\int_0^{\frac{\pi}{2}} \cos^6\theta \, \mathrm{d}\theta \quad \text{and} \quad \int_0^{\frac{\pi}{2}} \sin^6\theta \, \mathrm{d}\theta .$$

3. [1998 STEP I question 5]

   (i) In the Argand diagram, the points Q and A represent the complex numbers $4 + 6i$ and $10+2i$. If A, B, C, D, E, F are the vertices, taken in clockwise order, of a regular hexagon (regular six-sided polygon) with centre Q, find the complex number which represents B.

   (ii) Let a, b and c be real numbers. Find a condition of the form $aA + bB + cC = 0$, where A, B and C are integers, which ensures that
   $$\frac{a}{1+i} + \frac{b}{1+2i} + \frac{c}{1+3i}$$
   is real.