THE KING'S FACTOR

This file contains specific solutions to parts of some questions set in Year 12 tKF sessions, so that you can check that you are on the right track. Some questions, for instance those where the result is given and a proof is required, do not have solutions given here.

These brief solutions are NOT model answers and do not indicate how a full solution should be presented

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1. [2009 STEP I question 2]

   To show that the tangent to the curve at the point $(-a, b)$ is

   $$b^2y - a^2x = a^3 + b^3\,.$$

   first show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{x^2}{y^2}$.

   In the case $a = 1$ and $b = 2$, the equation of the curve is

   $$y^3 = x^3 + 9 \qquad (*)$$

   while the equation of the tangent to the curve at $(-1, 2)$ is

   $$4y - x = 9\,. \qquad (**)$$

   Substituting for y in terms of x (using (**)) in (*) gives after some algebra

   $$7x^3 - 3x^2 - 27x - 17 = 0\,.$$

   This has the solution $x = -1$ which we already know, which allows us to find a further (rational) solution $x = \frac{17}{7}$.

   This can be used to show that $20^3 = 17^3 + 7^3 + 14^3$.

2. [2007 STEP I question 3]

   $$\int_0^{\frac{\pi}{2}} \cos^4\theta\,\mathrm{d}\theta = \frac{3\pi}{16} \quad \text{and} \quad \int_0^{\frac{\pi}{2}} \sin^4\theta\,\mathrm{d}\theta = \frac{3\pi}{16}\,.$$

   $$\int_0^{\frac{\pi}{2}} \cos^6\theta\,\mathrm{d}\theta = \frac{5\pi}{32} \quad \text{and} \quad \int_0^{\frac{\pi}{2}} \sin^6\theta\,\mathrm{d}\theta = \frac{5\pi}{32}\,.$$

3. [1998 STEP I question 5]

   (i) B is $7 - 2\sqrt{3} + i(4 - 3\sqrt{3})$.

   (ii) $5a + 4b + 3c = 0$.