

1. Warm up

Sketch the curve

$$y = 2x^3 + 9x^2 - 60x - 100.$$

To do this, you do not need to work out where the curve crosses the x -axis. Try the following steps:

- (a) Find the point where the curve crosses the y axis.
- (b) Find the two points on the curve where the gradient is zero.
- (c) Draw axes and mark in these three points (roughly, as you are only asked for a sketch).
- (d) Find the range of values of x for which the gradient of the curve is positive, and the range for which it is negative.
- (e) Consider what happens to y as $x \rightarrow +\infty$ and as $x \rightarrow -\infty$.
- (f) Complete the sketch.

2. [2010 Oxford Admission Test question 1B] (Multiple Choice)

The sum of the first $2n$ terms of

$$1, 1, 2, \frac{1}{2}, 4, \frac{1}{4}, 8, \frac{1}{8}, 16, \frac{1}{16}, \dots$$

is (a) $2^n + 1 - 2^{1-n}$ (b) $2^n + 2^{-n}$ (c) $2^{2n} - 2^{3-2n}$ (d) $\frac{2^n - 2^{-n}}{3}$

3. [2009 Oxford Admission Test question 1F] (Multiple Choice)

The equation in x

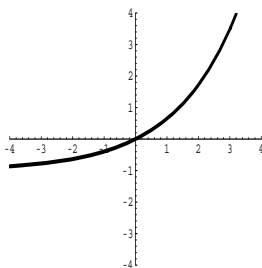
$$3x^4 - 16x^3 + 18x^2 + k = 0$$

has four real solutions

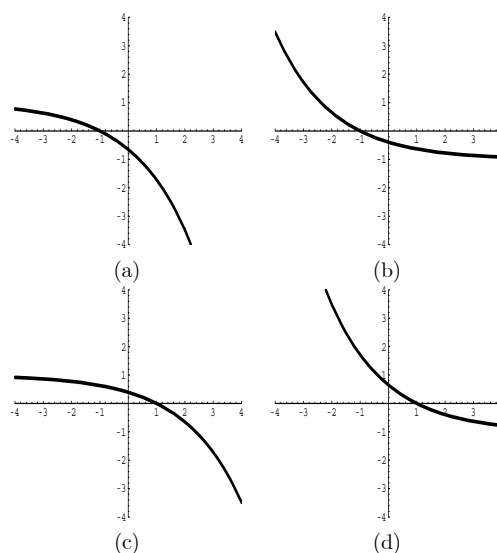
- (a) when $-27 < k < 5$
- (b) when $5 < k < 27$
- (c) when $-27 < k < -5$
- (d) when $-5 < k < 0$

4. [2009 Specimen paper 1 Oxford Admission Test question 1B] (Multiple Choice)

B. The diagram below shows the graph of the function $y = f(x)$.



The graph of the function $y = -f(x+1)$ is drawn in which of the following diagrams?



5. [2005 STEP I question 2]

The point P has coordinates $(p^2, 2p)$ and the point Q has coordinates $(q^2, 2q)$, where p and q are non zero and $p \neq q$. The curve C is given by $y^2 = 4x$. The point R is the intersection of the tangent to C at P and the tangent to C at Q . Show that R has coordinates $(pq, p+q)$.

The point S is the intersection of the normal to C at P and the normal to C at Q . If P and Q are such that $(1, 0)$ lies on the line PQ , show that S has coordinates $(p^2 + q^2 + 1, p+q)$, and that the quadrilateral $PSQR$ is a rectangle.

6. Warm up

- (a) Evaluate $1 + 4 + 7 + \cdots + 22$.
- (b) What is the remainder when $x^3 + 3x^2 - 4x + 7$ is divided by $x + 2$?

7. [2008 Oxford Admission Test question 1G] (Multiple Choice)

The function $S(n)$ is defined for positive integers n by

$$S(n) = \text{sum of the digits of } n.$$

For example, $S(723) = 7 + 2 + 3 = 12$. The sum

$$S(1) + S(2) + S(3) + \cdots + S(99)$$

equals (a) 746 (b) 862 (c) 900 (d) 924

8. [2008 Oxford Admission Test question 1D] (Multiple Choice)

When

$$1 + 3x + 5x^2 + 7x^3 + \cdots + 99x^{49}$$

is divided by $x - 1$ the remainder is

(a) 2000 (b) 2500 (c) 3000 (d) 3500

9. [2008 AEA question 2]

The points (x, y) on the curve C satisfy

$$(x + 1)(x + 2) \frac{dy}{dx} = xy.$$

The line with equation $y = 2x + 5$ is the tangent to C at a point P .

- (a) Find the coordinates of P .
- (b) Find the equation of C , giving your answer in the form $y = f(x)$.