

THE KING'S FACTOR

This file contains hints for questions set in Year 12 tKF sessions, to give you a nudge if you feel really stuck

Updated February 20, 2014

1. Warm up

Hints are included with the question.

2. [2010 Oxford Admission Test question 1B] (Multiple Choice)

This sequence contains two sequences of standard form intertwined.

3. [2009 Oxford Admission Test question 1F] (Multiple Choice) This is not an equation that can easily be solved directly.

As is often the case, sketching a graph is the best approach.

Begin by finding the turning points of

$$y = 3x^4 - 16x^3 + 18x^2 + k = 0.$$

(The y coordinate of each turning point will be given in terms of k , but the x coordinates are independent of k .)

Sketch the curve for some particular value of k , say $k = 0$. Mark in the three turning points. Consider how many solutions the equation has in this case.

Now think what happens if (a) k increases (b) k decreases.

4. [2009 Specimen paper 1 Oxford Admission Test question 1B] (Multiple Choice)

Sketch the graph of the function $y = f(x + 1)$.

5. [2005 STEP I question 2]

First find an expression for $\frac{dy}{dx}$. If you have learned implicit differentiation this is straightforward.

If you have not, then once you have an expression for $\frac{dy}{dx}$ in terms of x , replace it by one in terms of y . (If you think about it, this also handles the sign ambiguities which arise from taking a square root.)

To find R , first construct the equations of the tangents to C at P and Q .

For the second part, you will need to find the condition which p and q satisfy given that the $(1, 0)$ lies on the line PQ .

6. Warm up

- (a) To evaluate $1 + 4 + 7 + \cdots + 22$ note that this is the sum of an *Arithmetic Progression*.
- (b) Either use the *Remainder Theorem* or actually carry out the long division. Better still, do both.

7. [2008 Oxford Admission Test question 1G] (Multiple Choice)

To evaluate the sum

$$S(1) + S(2) + S(3) + \cdots + S(99)$$

list the numbers from 0 to 99 systematically as

$$\begin{array}{cccccc} 00 & 01 & 02 & \cdots & 08 & 09 \\ 10 & 11 & 12 & \cdots & 18 & 19 \\ \cdots & \cdots & \cdots & & & \\ 90 & 91 & 92 & \cdots & 98 & 99 \end{array}$$

(1)

Add up all the units digits. Add up all the tens digits.

8. [2008 Oxford Admission Test question 1D] (Multiple Choice)

Use the remainder theorem to give an expression for the remainder when

$$1 + 3x + 5x^2 + 7x^3 + \cdots + 99x^{49}$$

is divided by $x - 1$.

9. [2008 AEA question 2]

- (a) Let the coordinates of P be (x, y) .

You need two simultaneous equations satisfied by x and y .

To find these equations, use the fact that you know of a line and a curve which pass through P .

- (b) [To do this part question, you will need to know how to solve differential equations. You may not yet know how to do this.]

To find the equation of C , you have to solve the differential equation, and use the coordinates of P to eliminate the arbitrary constant.